QEG Corrections in Yang-Mills Theories and QED

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Outline

- Introduction
- Einstein-Yang-Mills system
  - Construction of a background gauge invariant ghost action
  - Discussion of the gravitational correction at 1-loop level
- QED coupled to QEG
  - Discussion of the fixed point structure
  - Predictability of the fine structure constant
    $\leftrightarrow$ Comparison to SM values
- Summary

Quantum Einstein Gravity (QEG)

Quantum Einstein Gravity = Asymptotically Safe theory of quantum gravity with the metric as fundamental degree of freedom.

Asymptotic Safety = UV completion of a QFT at a non-Gaussian fixed point of the RG flow with a finite dimensional UV critical hypersurface $\mathscr{S}_{\mathrm{UV}}$

- NGFP = FP of the flow with at least one non-zero coupling
- $\mathscr{S}_{\mathrm{UV}}$ = set of all points mapped onto the FP by the inverse RG flow
- $\dim \mathscr{S}_{\mathrm{UV}}$ = number of critical exponents with positive real part $\leftrightarrow$ predictivity of the theory

Quantum Einstein Gravity

How to find out whether such a theory exists?

Tool to analyze the RG flow: ERGE for Gravity

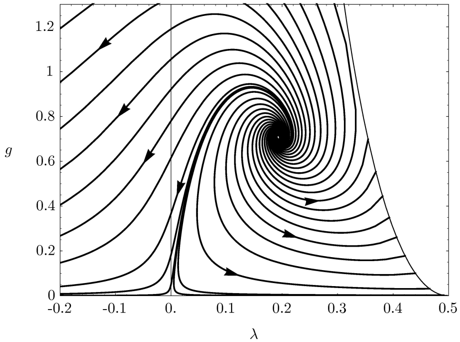
$$\partial_t \Gamma_k = \frac{1}{2} \mathrm{STr}\left[\left(\Gamma_k^{(2)} + \mathcal{R}_k(\Delta)\right)^{-1} \partial_t \mathcal{R}_k(\Delta)\right]$$

- exact equation suitable for non-perturbative calculations
- usually it can be only solved for truncations of $\Gamma_k \Rightarrow$ can only collect evidence for a NGFP in different truncations

Example for gravity: Einstein-Hilbert truncation

$$\Gamma_k^{\mathrm{EH}}[g] + \Gamma_k^{\mathrm{gf}} + S_{\mathrm{gh}} = \frac{Z_N(k)}{16\pi \hat{G}} \int \mathrm{d}^4x \sqrt{g} \left(-R(g) + 2\bar{\lambda}(k)\right) + \Gamma_k^{\mathrm{gf}} + S_{\mathrm{gh}}$$

RG Flow of the Einstein-Hilbert Truncation



- we find a NGFP in the EH truncation
- its UV critical hypersurface is two dimensional

The Einstein-Yang-Mills System

$$\Gamma_k = \Gamma_k^{\mathrm{EH}} + \Gamma_k^{\mathrm{YM}} + \Gamma_k^{\mathrm{gf}} + S_{\mathrm{gh}} = \breve{\Gamma}_k + S_{\mathrm{gh}}$$

with

$$\Gamma_k^{\mathrm{YM}}[g, A] = \frac{Z_F(k)}{4\,\hat{g}_{\mathrm{YM}}^2} \int \mathrm{d}^4x\, \sqrt{g}\, g^{\mu\rho} g^{\nu\sigma} F^a_{\mu\nu} F^a_{\rho\sigma}$$

$$\Gamma_k^{\mathrm{gf}}[\bar{h}, \bar{a}; \bar{g}, \bar{A}] = \int \mathrm{d}^4x\, \sqrt{\bar{g}}\, \left(\frac{Z_N(k)}{2\alpha_{\mathrm{D}}} \bar{g}^{\mu\nu} \mathrm{F}_\mu \mathrm{F}_\nu + \frac{Z_F(k)}{2\alpha_{\mathrm{YM}}} \mathrm{G}^a \mathrm{G}^a \right)$$

Motivation:

- Γ_k^{EH} contains the all essential features of gravity close to the non-Gaussian fixed point (NGFP)
- in pure YM theory Γ_k^{YM} approximates the perturbative 2-loop result within a few percent

Gauge Fixing

Gauge conditions:

$$\mathrm{F}_\mu(\bar{h};\bar{g}) = \frac{1}{\sqrt{16\pi\hat{G}}}\left(\delta^\beta_\mu \bar{g}^{\alpha\gamma}\bar{D}_\gamma - \frac{1}{2}\bar{g}^{\alpha\beta}\bar{D}_\mu\right)\bar{h}_{\alpha\beta}$$

$$\mathrm{G}^a(\bar{a};\bar{g},\bar{A}) = \hat{g}_{\mathrm{YM}}^{-1}\,\bar{g}^{\mu\nu}\bar{\mathcal{D}}_\mu \bar{a}^a_\nu$$

Gauge parameters chosen to $\alpha_{\mathrm{D}} = 1 = \alpha_{\mathrm{YM}}$

Notational remark: Different covariant derivatives

- $\nabla = \partial + A$
- $D = \partial + \Gamma$
- $\mathcal{D} = \partial + \Gamma + A$

Problems with Ghost Action

Standard ghost action is of the form ($\mathcal{A} = \bar{A} + a,\ \gamma = \bar{g} + h$):

$$S_{\text{gh}}[h, a, \mathcal{C}, \bar{\mathcal{C}}, \Sigma, \bar{\Sigma}; \bar{g}, \bar{A}] = -\int \mathrm{d}^4x\, \sqrt{\bar{g}} \left(\bar{\mathcal{C}}^\nu \frac{\partial \mathrm{F}_\nu}{\partial h_{\rho\sigma}} \delta_{\mathrm{D}}(\mathcal{C}) \gamma_{\rho\sigma} + \bar{\mathcal{C}}^\nu \frac{\partial \mathrm{F}_\nu}{\partial h_{\rho\sigma}} \delta_{\mathrm{YM}}(\Sigma) \gamma_{\rho\sigma} + \bar{\Sigma}^a \frac{\partial \mathrm{G}^a}{\partial a_\mu^b} \delta_{\mathrm{D}}(\mathcal{C}) \mathcal{A}_\mu^b + \bar{\Sigma}^a \frac{\partial \mathrm{G}^a}{\partial a_\mu^b} \delta_{\mathrm{YM}}(\Sigma) \mathcal{A}_\mu^b \right)$$

Problem:

$$\delta_{\mathrm{D}}(\mathcal{C}) \mathcal{A}_\mu^b = \mathcal{L}_{\mathcal{C}} \mathcal{A}_\mu^b = \mathcal{C}^\rho \partial_\rho \mathcal{A}_\mu^a + (\partial_\mu \mathcal{C}^\rho) \mathcal{A}_\rho^a$$

is not an SU(N)-vector $\Rightarrow$ background gauge invariance is broken!

Ward Operators

Ward operators that generate gauge transformations of the fields $\Phi = (\mathcal{A}, \gamma, \mathcal{C}, \bar{\mathcal{C}}, \Sigma, \bar{\Sigma})$ are defined as

$$\mathcal{W}_{\mathrm{D}}(v) = -\int \mathrm{d}^4x \left(\delta_{\mathrm{D}}(v)\Phi^i(x) \frac{\delta}{\delta \Phi^i(x)} \right)$$
$$\mathcal{W}_{\mathrm{YM}}(\lambda) = -\int \mathrm{d}^4x \left(\delta_{\mathrm{YM}}(\lambda)\Phi^i(x) \frac{\delta}{\delta \Phi^i(x)} \right)$$

They satisfy the algebra

$$\begin{aligned}
[\mathcal{W}_{\mathrm{D}}(v_1), \mathcal{W}_{\mathrm{D}}(v_2)] &= \mathcal{W}_{\mathrm{D}}([v_1, v_2]) \\
[\mathcal{W}_{\mathrm{YM}}(\lambda_1), \mathcal{W}_{\mathrm{YM}}(\lambda_2)] &= \mathcal{W}_{\mathrm{YM}}(f\lambda_1\lambda_2) \\
[\mathcal{W}_{\mathrm{D}}(v), \mathcal{W}_{\mathrm{YM}}(\lambda)] &= \mathcal{W}_{\mathrm{YM}}(\mathcal{L}_v\lambda)
\end{aligned}$$

Structure of full gauge group: $\mathbf{G} = \mathsf{Diff} \ltimes \mathsf{SU(N)}_{\mathrm{loc}}$.

Back to the Ghost Action

YM gauge transformation of the problematic term:

$$\begin{aligned} \delta_{\mathrm{YM}}(\lambda)\delta_{\mathrm{D}}(\mathcal{C})\mathcal{A}_\mu^b &= \mathcal{W}_{\mathrm{YM}}(\lambda)\mathcal{W}_{\mathrm{D}}(\mathcal{C})\mathcal{A}_\mu^a \\ &= \mathcal{C}^\rho\partial_\rho(f^{abc}\lambda^b\mathcal{A}_\mu^c) + (\partial_\mu\mathcal{C}^\rho)f^{abc}\lambda^b\mathcal{A}_\rho^c \\ &\neq f^{abc}\lambda^b\left(\mathcal{C}^\rho\partial_\rho\mathcal{A}_\mu^c + (\partial_\mu\mathcal{C}^\rho)\mathcal{A}_\rho^c\right) \\ &= \mathcal{W}_{\mathrm{D}}(\mathcal{C})\mathcal{W}_{\mathrm{YM}}(\lambda)\mathcal{A}_\mu^a \end{aligned}$$

broken gauge invariance of ghost action

$\leftrightarrow$ non-vanishing commutator $[\mathcal{W}_{\mathrm{D}}(v), \mathcal{W}_{\mathrm{YM}}(\lambda)]$

Idea: reparametrize the gauge group such that the mixed commutator vanishes.

Modified Diffeomorphisms

$$\widetilde{\mathcal{W}_{\mathrm{D}}}(v) \equiv \mathcal{W}_{\mathrm{D}}(v) + \mathcal{W}_{\mathrm{YM}}(\mathcal{A} \cdot v)$$

Definition of invariant functional F is unchanged

$$\begin{aligned} &\widetilde{\mathcal{W}_{\mathrm{D}}}(v)F = 0 \ \wedge \ \mathcal{W}_{\mathrm{YM}}(\lambda)F = 0 \\ \Leftrightarrow\ &\mathcal{W}_{\mathrm{D}}(v)F = 0 \ \wedge \ \mathcal{W}_{\mathrm{YM}}(\lambda)F = 0 \end{aligned}$$

Modified algebra:

$$\begin{aligned} [\widetilde{\mathcal{W}_{\mathrm{D}}}(v_1), \widetilde{\mathcal{W}_{\mathrm{D}}}(v_2)] &= \widetilde{\mathcal{W}_{\mathrm{D}}}([v_1, v_2]) - \mathcal{W}_{\mathrm{YM}}(v_1 v_2 \cdot F) \\ [\mathcal{W}_{\mathrm{YM}}(\lambda_1), \mathcal{W}_{\mathrm{YM}}(\lambda_2)] &= \mathcal{W}_{\mathrm{YM}}(f \lambda_1 \lambda_2) \\ [\widetilde{\mathcal{W}_{\mathrm{D}}}(v), \mathcal{W}_{\mathrm{YM}}(\lambda)] &= 0 \end{aligned}$$

With these modified generators we can construct a background gauge invariant ghost action the standard way!

Background Gauge Invariant Ghost Action

$$S_{\mathrm{gh}}[h, a, \mathcal{C}, \bar{\mathcal{C}}, \Sigma, \bar{\Sigma}; \bar{g}, \bar{A}] = -\int \mathrm{d}^4x\, \sqrt{\bar{g}} \Big(\bar{\mathcal{C}}^{\nu} \frac{\partial \mathrm{F}_{\nu}}{\partial h_{\rho\sigma}} \widetilde{\delta_{\mathrm{D}}}(\mathcal{C}) \gamma_{\rho\sigma} + \bar{\mathcal{C}}^{\nu} \frac{\partial \mathrm{F}_{\nu}}{\partial h_{\rho\sigma}} \delta_{\mathrm{YM}}(\Sigma) \gamma_{\rho\sigma} + \bar{\Sigma}^{a} \frac{\partial \mathrm{G}^{a}}{\partial a^{b}_{\mu}} \widetilde{\delta_{\mathrm{D}}}(\mathcal{C}) \mathcal{A}^{b}_{\mu} + \bar{\Sigma}^{a} \frac{\partial \mathrm{G}^{a}}{\partial a^{b}_{\mu}} \delta_{\mathrm{YM}}(\Sigma) \mathcal{A}^{b}_{\mu} \Big)$$

Now we have

$$\begin{aligned} \widetilde{\delta_{\mathrm{D}}}(\mathcal{C}) \mathcal{A}^{b}_{\mu} &= \mathcal{L}_{\mathcal{C}} \mathcal{A}^{b}_{\mu} - \nabla_{\mu} (\mathcal{A}^{b}_{\nu} \mathcal{C}^{\nu}) \\ &= \mathcal{C}^{\rho} \partial_{\rho} \mathcal{A}^{b}_{\mu} + (\partial_{\mu} \mathcal{C}^{\rho}) \mathcal{A}^{a}_{\rho} - \partial_{\mu} (\mathcal{A}^{b}_{\rho} \mathcal{C}^{\rho}) - f^{bcd} \mathcal{A}^{c}_{\mu} \mathcal{A}^{d}_{\rho} \mathcal{C}^{\rho} \\ &= \mathcal{F}^{b}_{\rho\mu} \mathcal{C}^{\rho} \end{aligned}$$

which manifestly transforms tensorial under SU(N) transformations and diffeomorphisms!

Some Computational Remarks

- Classical ghost sector (no evolution)

$$\partial_t \Gamma_k = \frac{1}{2}\,\mathrm{Tr}\left[\frac{\partial_t \mathcal{R}_k\big(\breve{\Delta}\big)}{\breve{\Gamma}_k^{(2)} + \mathcal{R}_k\big(\breve{\Delta}\big)}\right] - \mathrm{Tr}\left[\frac{\partial_t \mathcal{R}_k^{\mathrm{gh}}\big(\Delta_{\mathrm{gh}}\big)}{S_{\mathrm{gh}}^{(2)} + \mathcal{R}_k^{\mathrm{gh}}\big(\Delta_{\mathrm{gh}}\big)}\right]$$

- spectrally adjusted cutoff operator

$$\breve{\Delta} = \mathcal{Z}_k^{-1}\breve{\Gamma}_k^{(2)} \qquad \Delta_{\mathrm{gh}} = \mathcal{Z}_{\mathrm{gh}}^{-1} S_{\mathrm{gh}}^{(2)}$$

$\longrightarrow$ admits a simple spectral representation of the RHS

Different degrees of RG improvement:

- ∂_t acts only on explicit k-dependence $\leftrightarrow$ 1-loop calculation
- ∂_t acts in addition on $\mathcal{Z}_k$-factors
- ∂_t acts also on $\breve{\Gamma}_k^{(2)}$ in $\breve{\Delta}$

1-Loop Result

Switching to dimensionless couplings

$$g_{\mathrm{YM}}^2(k) \equiv \frac{\hat{g}_{\mathrm{YM}}^2}{Z_F(k)}, \quad g(k) \equiv k^2 \frac{\hat{G}}{Z_N(k)}, \quad \lambda(k) \equiv k^{-2}\bar{\lambda}(k)$$

we obtain the 1-loop result

$$\boxed{\partial_t g_{\mathrm{YM}}^2 = -\frac{6\,\Phi_1^1(0)}{\pi}\; g\, g_{\mathrm{YM}}^2 - \frac{11\,N}{24\,\pi^2}\; g_{\mathrm{YM}}^4}$$

where

$$\Phi_1^1(w) = \int_0^\infty \mathrm{d}z\, \frac{R^{(0)}(z) - zR^{(0)\prime}(z)}{z + R^{(0)}(z) + w}$$

Classical Regime

Newton's constant

$$G(k) \approx G_0 = \text{const}, \quad g(k) = G_0 k^2$$

For an Abelian field ($N = 0$) we obtain

$$\partial_t g_{\mathrm{YM}}^2 = -\frac{6\,\Phi_1^1(0)}{\pi}\, G_0\, k^2\, g_{\mathrm{YM}}^2$$

with solution

$$\begin{aligned} g_{\mathrm{YM}}^2(k) &= g_{\mathrm{YM}}^2(0) \cdot \exp\left(-\omega_{\mathrm{YM}}(k/m_{\mathrm{Pl}})^2\right) \\ &= g_{\mathrm{YM}}^2(0) \cdot \left[1 - \omega_{\mathrm{YM}}(k/m_{\mathrm{Pl}})^2 + \mathcal{O}(k^4/m_{\mathrm{Pl}}^4)\right] \end{aligned}$$

where $m_{\mathrm{Pl}} = G_0^{-1/2}$ and $\omega_{\mathrm{YM}} = 3\Phi_1^1(0)/\pi$.

Asymptotic Safety

Free Maxwell field does not destroy NGFP of the EH truncation. Therefore in the UV

$$g(k) \to g^* \quad \Longrightarrow \quad G(k) = g^*/k^2 \to 0 \quad \text{as} \quad k \to \infty$$

which implies

$$\partial_t g_{\rm YM}^2 = -\frac{6\,\Phi_1^1(0)}{\pi}\, g^*\, g_{\rm YM}^2$$

with solution

$$g_{\rm YM}^2(k) \propto k^{-\Theta_{\rm YM}}, \quad \Theta_{\rm YM} = \frac{6\,\Phi_1^1(0)}{\pi}\, g^*$$

- total system has a NGFP with $(g_{\rm YM}^* = 0, g^* > 0, \lambda^* > 0)$
- gravity speeds up approach of asymptotic freedom in the YM sector (power law instead of logarithmic)

Gravity Correction to the β-function of QED

1-loop QED β-function with gravity correction:

$$\partial_t \alpha = \beta_\alpha = \left(A h(\alpha) - \frac{6}{\pi} \Phi_1^1(0) g \right) \alpha \quad \text{with} \quad A = \frac{2}{3\pi} n_F$$

- fermionic and gravitational contributions compete in effect
- in the perturbative approach: $g = G_0 k^2$. Gravitational contributions will always dominate for large k, so that $\alpha \to \alpha^* = 0$ for $k \to \infty$
- in the Asymptotic Safety scenario we find a second fixed point: As $g \to g^*$, α can approach $\alpha^* \neq 0$ with $Ah(\alpha^*) = \frac{6g^*}{\pi} \Phi_1^1(0)$

Fermionic Contributions

Perturbative calculations in pure QED show

$$\beta_\alpha(\alpha)\big|_{g=0} \equiv A\, h(\alpha)\, \alpha = \alpha \left[\frac{2}{3} \left(\frac{\alpha}{\pi} \right) + \frac{1}{2} \left(\frac{\alpha}{\pi} \right)^2 + \mathcal{O}(\alpha^3) \right]$$

Properties of $h(\alpha)$:

- $h'(\alpha) > 0 \Rightarrow$ FP condition satisfied for some α^*
- $h(\alpha) \approx \alpha$ for $\alpha \lesssim 1$

$\Rightarrow$ using the one-loop approximation $h(\alpha) = \alpha$ will give a qualitatively reliable picture of the full beta-function.

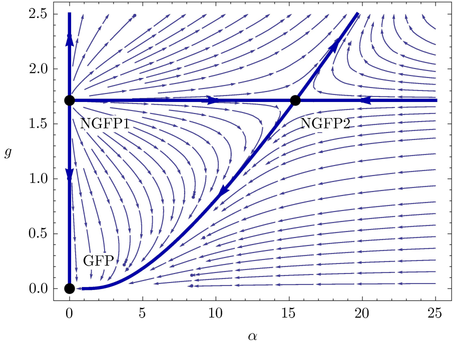
A Simple 2D Truncation

$$\partial_t g = [2 + B_1(0) g] g$$

$$\partial_t \alpha = \left(A\alpha - \frac{6}{\pi} \Phi_1^1(0) g \right) \alpha$$

- backreaction to the gravitational β-function is neglected
- flow can be solved analytically
- results are in agreement with full EH-truncation in the gravity sector

Flow of the 2D Truncation



- Three kinds of possible UV behavior
- Enhanced predictivity of AS scenario w.r.t. $\mathbf{NGFP}_2$

Asymptotically Safe Trajectory

β_g can be solved in isolation:

$$g(k) = \frac{G_0 k^2}{1 + \frac{G_0 k^2}{g^*}}$$

G_0 is the IR value of the running Newton constant.

Unique trajectory which is asymptotically safe w.r.t. $\mathbf{NGFP}_2$:

$$\frac{1}{\alpha(k)} = \frac{1}{\alpha^*}\left(1 + \frac{g^*}{G_0 k^2}\right) {}_2F_1\left(1, 1, 1 + \frac{3}{\pi}\Phi_1^1 g^*; -\frac{g^*}{G_0 k^2}\right)$$

$\alpha(k)$ is a prediction, when G_0 was determined by experiment!

Asymptotically Safe Trajectory for $k \approx m_e$

For $k = m_e$ we have $\frac{g^*}{G_0 m_e^2} = g^* \left(\frac{m_{\rm Pl}}{m_e}\right)^2 \approx 10^{44} \Rightarrow$ expand ${}_2F_1$ for large arguments:

$$\frac{1}{\alpha(k)} = \frac{A}{2}\left[\ln\left(\frac{g^*}{G_0 k^2}\right) - \gamma - \psi\left(\frac{3}{\pi}\Phi_1^1 g^*\right)\right]$$

- in the IR the result is of the same form as in pure QED $\alpha(k)^{-1} = -A\ln(k) + \text{const}$
- putting in numbers we obtain $\alpha_{\rm IR}^{-1} \approx 10.91\, n_F$ (cp. to $\alpha_{\rm IR}^{-1} \approx 137$ in real Nature).
  $\Rightarrow$ for $n_F \approx 13$ prediction consistent with real value

Including Other Charged Particles

Within the SM or MSSM the same argumentation holds true for the weak hypercharge α_1 with 1-loop β-function $\partial_t \alpha_1 = A\alpha_1^2$:

	A	$\alpha_1(M_Z)^{-1}$
SM	$\frac{41}{20\pi}$	25.7
MSSM	$\frac{33}{10\pi}$	41.3

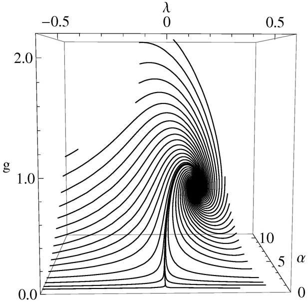
$$\alpha_1^{\exp}(M_Z)^{-1} \approx 59.5$$

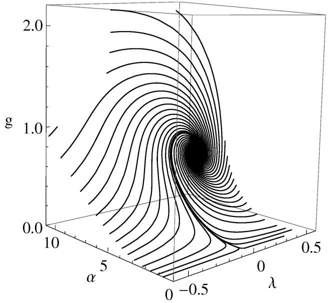
These estimates for α_1 turn out too large compared to the experimental value. Possible conclusions:

- Asymptotic Safety w.r.t. $\mathbf{NGFP}_2$ is possible if there exist (not too many) more $U(1)$ charged particles.
- the known particle content of the SM suggests Asymptotic Safety w.r.t. $\mathbf{NGFP}_1$ when coupled to QEG.

Einstein-Hilbert + QED: Numerical Results

$\mathscr{S}_{\mathrm{UV}}$ of $\mathbf{NGFP_2}$ in g-λ-α–coupling space:





- long classical regime needed to drive α to small values
- numerical result for "realistic trajectory": $\alpha_{\mathrm{IR}}^{-1} \approx 10.93$
  $\Rightarrow$ running of λ negligible

Summary

Yang-Mills Theories

- we find a non-vanishing gravity contribution to the YM-beta function setting in already at one-loop order
- the correction is consistent with the Asymptotic Safety scenario of QEG and supports asymptotic freedom of YM

Quantum Electrodynamics

- Fermionic and gravitational contributions to the running of α compete in effect
- for SM values the gravitational ones are dominant, resulting in Asymptotic Safety w.r.t. $\mathbf{NGFP}_1 \Rightarrow \alpha$ is a free parameter
- an exact balancing of both contributions is possible $\leftrightarrow$ Asymptotic Safety w.r.t. $\mathbf{NGFP}_2 \Rightarrow \alpha$ can be predicted
- coupling a theory to Asymptotically Safe gravity may provide a mechanism able to reduce the number of free parameters.