First steps toward an asymptotically safe model of electroweak interactions

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Motivations

The minimal realization of the SM would include only fundamental particles already discovered (no Higgs boson).

EW symmetry implemented by coupling fermions and gauge bosons to the corresponding Nambu-Goldstone bosons described by a Nonlinear σ Model.

Two strong arguments against this picture:

- violation of unitarity;
- EW precision measurements;

Both these arguments may be avoided if this EW symmetry breaking sector is Asymptotically Safe.

The Nonlinear σ Model (NLσM)

The NLσM action describes the physics of the Goldstone bosons (Gb) of a spontaneously broken symmetry $(G \to H)$:

$$\mathcal{S} = \frac{1}{2f^2} \int d^d x \, h_{\alpha\beta} \partial_\mu \varphi^\alpha \partial^\mu \varphi^\beta$$

- f is the Gb coupling with mass dimension $(2-d)/2$.
- the flow of $\frac{1}{f^2} h_{\alpha\beta}$ is governed by the Ricci tensor:

$$\frac{d}{dt} \left(\frac{1}{f^2} h_{\alpha\beta} \right) = 2 c_d k^{d-2} R_{\alpha\beta} \, .$$

- one-loop UV non-trivial fixed point $(d > 2)$:

$$\tilde{f}^2{}_* = \frac{d-2}{2} \frac{D}{c_d \, R} . \qquad (\tilde{f}^2 = k^{d-2} f^2)$$

(R Ricci scalar) [A. Codello and R. Percacci, PLB **672** (2009) 280]

Gauging the NLσM

Gauged $SU(N)$ chiral NLσM Euclidean action (the left part of the isometry group $SU(N)_L \times SU(N)_R$ is gauged):

$$\mathcal{S} = \frac{1}{2f^2} \int d^d x \, h_{\alpha\beta} D_\mu \varphi^\alpha D^\mu \varphi^\beta + \frac{1}{4g^2} \int d^d x \, F^i_{\mu\nu} F^{\mu\nu}_i$$

- g is the gauge coupling.
- $D_\mu \varphi^\alpha = \partial_\mu \varphi^\alpha + A^i_\mu R^\alpha_i(\varphi)$ is the gauge covariant derivative, R^α_i-right invariant Killing vectors $(\alpha\, , i = 1, .., N^2 - 1)$.
- $F^i_{\mu\nu} = \partial_\mu A^i_\nu - \partial_\nu A^i_\mu + f^i{}_{jl} A^j_\mu A^l_\nu$ is the gauge field strength.
- the action is invariant under local $SU(N)_L$ infinitesimal transformations

$$\delta_\epsilon \varphi^\alpha = -\epsilon^i_L R^\alpha_i(\varphi) \qquad \delta_\epsilon A^i_\mu = \partial_\mu \epsilon^i_L + f_{jl}{}^i A^j_\mu \epsilon^l_L \, .$$

Background field expansion and gauge fixing

Expand the fieds around nonconstant backgrounds $\bar{\varphi}$ and $\bar{A}$:

$$\varphi^\alpha = \bar{\varphi}^\alpha + \xi^\alpha - \frac{1}{2}\Gamma_\beta{}^\alpha{}_\gamma \xi^\beta \xi^\gamma + \ldots \qquad A^i_\mu = \bar{A}^i_\mu + a^i_\mu \,.$$

ξ^α normal coordinates (preserve background invariance).
Functional Taylor series expansion of the action:

$$\mathcal{S}(\varphi, A) = \mathcal{S}(\bar{\varphi}, \bar{A}) + \mathcal{S}^{(1)}(\bar{\varphi}, \bar{A}; \xi, a) + \mathcal{S}^{(2)}(\bar{\varphi}, \bar{A}; \xi, a) + \ldots$$

The second order piece is:

$$\begin{aligned}
\mathcal{S}^{(2)} = & \frac{1}{2f^2}\int d^d x\, \xi^\alpha \left(-\bar{D}^2 h_{\alpha\beta} - D_\mu \bar{\varphi}^\epsilon D^\mu \bar{\varphi}^\eta R_{\epsilon\alpha\eta\beta}\right) \xi^\beta \\
& + \frac{1}{2g^2}\int d^d x\, a^i_\mu \left(-\bar{D}^2 \delta_{ij}\delta^{\mu\nu} + \bar{D}^\nu \bar{D}^\mu \delta_{ij} + \bar{F}^{\ell\mu\nu} f_{\ell ij} + \frac{g^2}{f^2}\delta_{ij}\delta^{\mu\nu}\right) a^j_\nu \\
& + \frac{1}{f^2}\int d^d x\, a^i_\mu \left(h_{\alpha\gamma} D^\mu \bar{\varphi}^\alpha \nabla_\beta R^\gamma_i + h_{\alpha\beta} R^\alpha_i \bar{D}^\mu\right) \xi^\beta \,,
\end{aligned}$$

where $\bar{D}_\mu \xi^\alpha = \nabla_\mu \xi^\alpha + \bar{A}^i_\mu \nabla_\beta R^\alpha_i \xi^\beta$.

Background gauge fixing term:

$$\mathcal{S}_{gf} = \frac{1}{2\alpha g^2} \int d^d x \, \delta_{ij} \chi^i \chi^j \qquad \text{with} \qquad \chi^i = \bar{D}^\mu a^i_\mu + \beta \frac{g^2}{f^2} R^i_\alpha \xi^\alpha$$

The second order piece becomes:

$$\begin{aligned}
\mathcal{S}^{(2)} &= \frac{1}{2f^2} \int d^d x \, \xi^\alpha \left(-D^2 h_{\alpha\beta} - D_\mu \varphi^\epsilon D^\mu \varphi^\eta R_{\epsilon\alpha\eta\beta} + \frac{\beta^2}{\alpha} \frac{g^2}{f^2} h_{\alpha\beta} \right) \xi^\beta \\
&+ \frac{1}{2g^2} \int d^d x \, a^i_\mu \left(-D^2 \delta^{\mu\nu}_{ij} + \left(1 - \frac{1}{\alpha}\right) \delta_{ij} D^\mu D^\nu - 2 F^{\ell\mu\nu} f_{i\ell j} + \frac{g^2}{f^2} \delta^{\mu\nu}_{ij} \right) a^j_\nu \\
&+ 2 \frac{1}{f^2} \int d^d x \, a^{\mu i} D_\mu \varphi^\alpha h_{\alpha\gamma} \nabla_\beta R^\gamma_i \xi^\beta + \frac{1}{f^2} \left(\frac{\beta}{\alpha} - 1 \right) \int d^d x \, D^\mu a^i_\mu \delta_{ij} R^j_\beta \xi^\beta \\
&+ \mathcal{S}_{gh} \, .
\end{aligned}$$

where $\mathcal{S}_{gh}$ is the ghost action.

The case $\alpha = \beta = 1$ is the generalization of the 't Hooft-Feynman gauge fixing to the background field method.

Functional RG study

RG study is done by using ERGE and heat-kernel techniques:

$$\dot{\Gamma}_k = \frac{d\Gamma_k}{dt} = \frac{1}{2}\mathrm{Tr}\left(\frac{\delta^2\Gamma_k}{\delta\theta\delta\theta} + \mathcal{R}_k^\theta\right)^{-1}\dot{\mathcal{R}}_k^\theta - \mathrm{Tr}\left(\frac{\delta^2\Gamma_k}{\delta\bar{c}\delta c} + \mathcal{R}_k^c\right)^{-1}\dot{\mathcal{R}}_k^c$$

where $\theta^T = (\xi^i, a_\mu^i)$ and $t = \log(k/k_0)$.
Optimized cutoff kernels $\mathcal{R}_k^\theta$ and $\mathcal{R}_k^c$:

$$\mathcal{R}_k^\theta = \begin{pmatrix} \frac{1}{f^2}R_k(-D_\xi^2) & 0 \\ 0 & \frac{1}{g^2}R_k(-D_a^2) \end{pmatrix} \qquad \mathcal{R}_k^c = R_k(-D_c^2)$$

The t-derivative of the cutoff is

$$\frac{d\mathcal{R}_k^\theta}{dt} = \begin{pmatrix} \frac{1}{f^2}\left[\partial_t R_k(-D_\xi^2) + \eta_\xi R_k(-D_\xi^2)\right] & 0 \\ 0 & \frac{1}{g^2}\left[\partial_t R_k(-D_a^2) + \eta_a R_k(-D_a^2)\right] \end{pmatrix}$$

where $\eta_\xi = -2\partial_t \log f$ and $\eta_a = -2\partial_t \log g$.

β-functions in $d = 4$

System of linear equations for the β functions:

$$\frac{d}{dt}\frac{1}{g^2} = \frac{N}{(4\pi)^2}\left[\frac{8}{(1+\frac{g^2}{\tilde{f}^2})^3}\left(1+\frac{\eta_a}{6}\right) - \frac{1}{3}\left(\frac{9}{4}+2\eta_a+\frac{1}{8}\eta_\xi\right)\frac{1}{1+\frac{g^2}{\tilde{f}^2}}\right]$$

$$\frac{d}{dt}\frac{1}{f^2} = \frac{N}{(4\pi)^2}\frac{k^2}{4}\frac{1}{(1+\frac{g^2}{\tilde{f}^2})^2}\left[1+\frac{\eta_\xi}{6}+\frac{4\frac{g^2}{\tilde{f}^2}}{1+\frac{g^2}{\tilde{f}^2}}\left(2+\frac{\eta_\xi+\eta_a}{6}\right)\right].$$

One loop flow equations for $g^2 \ll \tilde{f}^2$:

$$\frac{d}{dt}g^2 = -\frac{N}{(4\pi)^2}\frac{29}{4}g^4$$

$$\frac{d}{dt}\tilde{f}^2 = 2\left(1-\frac{3N}{4(4\pi)^2}g^2\right)\tilde{f}^2 - \frac{1}{(4\pi)^2}\frac{N}{4}\tilde{f}^4$$

Fixed point - numerical result

Table:

cutoff and gauge	$\tilde{f}_*$	g_*
type I, $\alpha = 1$	$4\pi\sqrt{6/N}$	0
type II, $\alpha = 1$	$8\pi\sqrt{2/3N}$	0
type II, $\alpha = 0$	$8\pi\sqrt{2/3N}$	0

Figure:

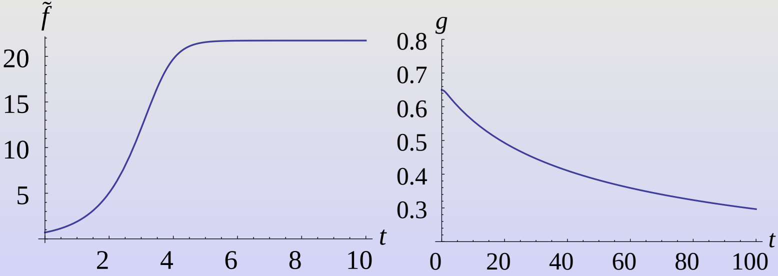


Figure: Running of $\tilde{f}$ and g for $N = 2$ in $d = 4$, $\tilde{f}_* \simeq 21.7$.

$SU(2) \times U(1)$ gauged NLσM

If no Higgs, the spontaneous breaking of $SU(2) \times U(1) \to U(1)$ would be implemented by coupling the gauge bosons to the NLσM Nambu-Goldstone bosons:

$$\mathcal{S} = \frac{1}{2f^2}\int d^4x\, h_{\alpha\beta} D_\mu \phi^\alpha D^\mu \phi^\beta + \frac{1}{4g^2}\int d^4x\, W^i_{\mu\nu} W_i^{\mu\nu} + \frac{1}{4g'^2}\int d^4x\, B_{\mu\nu} B^{\mu\nu} \qquad \text{(Euclidean action)}$$

- $1/f^2 = v^2/4$ (v is the EW VEV), g and g' are the gauge couplings.
- the gauge covariant derivative is

$$D_\mu \phi^\alpha = \partial_\mu \phi^\alpha + W^i_\mu R^\alpha_i - B_\mu L^\alpha_3 \qquad \alpha\,, i = 1, 2, 3\,.$$

- R/L are right/left invariant Killing vectors.

β-functions

The system of RGEs for $\tilde{f}^2$, g^2 and g'^2 is (at one-loop):

$$\frac{d\tilde{f}^2}{dt} = 2\tilde{f}^2 - \frac{1}{(4\pi)^2}\left\{\frac{1}{4}\frac{\tilde{f}^4}{(1+\tilde{m}_W^2)^2} + \frac{1}{4}\frac{\tilde{f}^4}{(1+\tilde{m}_Z^2)^2} + \frac{2g^2\tilde{f}^2}{(1+\tilde{m}_W^2)^3}\right.$$
$$+ \frac{g'^2\tilde{f}^2}{(1+\tilde{m}_W^2)(1+\tilde{m}_B^2)}\left[\frac{1}{(1+\tilde{m}_W^2)} + \frac{1}{(1+\tilde{m}_B^2)}\right]$$
$$\left. + \frac{g^2\tilde{f}^2}{(1+\tilde{m}_W^2)(1+\tilde{m}_Z^2)}\left[\frac{1}{(1+\tilde{m}_W^2)} + \frac{1}{(1+\tilde{m}_Z^2)}\right]\right\}$$
$$\frac{dg^2}{dt} = \frac{g^4}{(4\pi)^2}\frac{1}{1+\tilde{m}_W^2}\left[-\frac{16}{(1+\tilde{m}_W^2)^2} + \frac{3}{2}\right]$$
$$\frac{dg'^2}{dt} = \frac{1}{6}\frac{g'^4}{(4\pi)^2}\frac{1}{1+\tilde{m}_W^2}$$

where $\tilde{m}_W^2 = m_W^2/k^2 = g^2/\tilde{f}^2$, $\tilde{m}_B^2 = g'^2/\tilde{f}^2$ and $\tilde{m}_Z^2 = (g^2+g'^2)/\tilde{f}^2$.

U.V. limit ($g^2, g'^2 \ll \tilde{f}^2$) of the RGEs:

$$\frac{d\tilde{f}^2}{dt} = 2\tilde{f}^2 - \frac{1}{2}\frac{\tilde{f}^2}{(4\pi)^2}\left(\tilde{f}^2 + 6g^2 + 3g'^2\right)$$
$$\frac{dg^2}{dt} = -\frac{g^4}{(4\pi)^2}\frac{29}{2}$$
$$\frac{dg'^2}{dt} = \frac{1}{6}\frac{g'^4}{(4\pi)^2}$$

Approximation of constant gauge couplings ($g = g_* = 0.65$, $g' = g'_* = 0.35$):

$$\tilde{f}_* = \sqrt{64\pi^2 - 6g_*^2 - 3g_*'^2} \simeq 25.06$$

Approximate solution for $f^2(k)$:

$$f^2(k) = \frac{\tilde{f}_*^2 f_0^2}{\tilde{f}_*^2 + (k^2 - k_0^2) f_0^2} \, .$$

Scattering amplitude and perturbative unitarity

Elastic pion-pion scattering amplitude $\pi^i\pi^j \to \pi^k\pi^l$:

$A(\pi^i\pi^j \to \pi^k\pi^l) = A(s,t,u)\delta^{ij}\delta^{kl} + A(t,s,u)\delta^{ik}\delta^{jl} + A(u,s,t)\delta^{il}\delta^{jk}$

where $A(s,t,u) = sf^2/4$.
Isospin amplitudes:

$$
\begin{aligned}
A_0 &= 3A(s,t,u) + A(t,s,u) + A(u,s,t) \\
A_1 &= A(t,s,u) - A(u,s,t) \\
A_2 &= A(t,s,u) + A(u,s,t)
\end{aligned}
$$

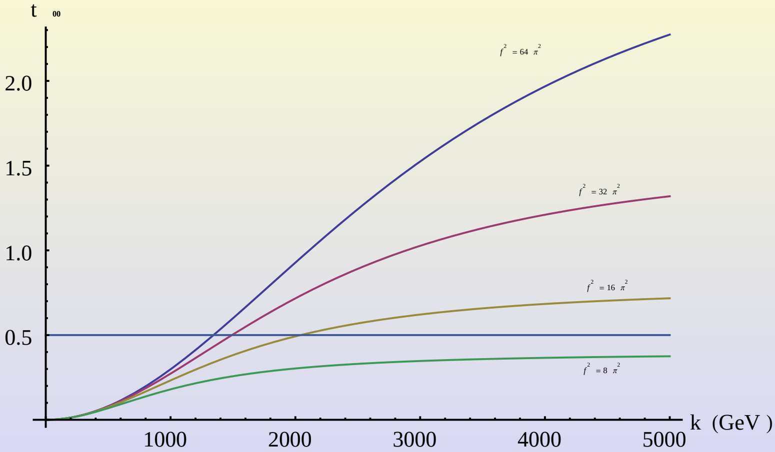
Partial wave decomposition of A_I $(I = 0,1,2)$:

$$t_{IJ} = \frac{1}{64\pi}\int_{-1}^{1} \mathrm{d}\cos\theta\; P_J(\cos\theta)\, A_I$$

The unitarity bound on the first partial wave:

$$t_{00} = \frac{sf^2}{64\pi} < \frac{1}{2}\,.$$

RG improved unitarity bound



The position of the fixed point is regularization scheme dependent, inclusion of higher derivative operators may also move the fixed point [R. Percacci and O. Zanusso, Phys. Rev. D **81** (2010) 065012].

S and T parameters

$\mathcal{L}^{(0)}$ and $\mathcal{L}^{(1)}$ operators that give contribution to electroweak oblique parameters:

$$\mathcal{L} = \frac{1}{2f^2} h_{\alpha\beta} D_\mu \varphi^\alpha D^\mu \varphi^\beta + \frac{1}{4g^2} W^i_{\mu\nu} W_i^{\mu\nu} + \frac{1}{4g'^2} B_{\mu\nu} B^{\mu\nu}$$
$$- \frac{a_0}{f^2} D_\mu \varphi^\alpha D^\mu \varphi^\beta L^3_\alpha L^3_\beta - \frac{a_1}{2} B^{\mu\nu} W^i_{\mu\nu} R_{i\alpha} L^\alpha_3$$

S and T parameters computed in our model:

$$S = -16\pi a_1(m_Z) + \frac{1}{6\pi}\left[\frac{5}{12} - \log\left(\frac{m_H}{m_Z}\right)\right],$$
$$T = \frac{2}{\alpha_{em}} a_0(m_Z) - \frac{3}{8\pi \cos^2\theta_W}\left[\frac{5}{12} - \log\left(\frac{m_H}{m_Z}\right)\right].$$

β functions I (ungauged case)

Lagrangian in absence of gauge interactions:

$$\mathcal{L} = \frac{1}{2f^2}\hat{h}_{\alpha\beta}\partial_\mu\varphi^\alpha\partial^\mu\varphi^\beta \,.$$

New metric $\hat{h}_{\alpha\beta} = L^1_\alpha L^1_\beta + L^2_\alpha L^2_\beta + (1-2a_0)L^3_\alpha L^3_\beta$.
RG flow of the NLσM:

$$\frac{d}{dt}\left(\frac{1}{f^2}\hat{h}_{\alpha\beta}\right) = \frac{1}{(4\pi)^2}k^2\hat{R}_{\alpha\beta}$$

In the basis of the right-invariant vectorfields:

$$\hat{R}_{11} = \hat{R}_{22} = \frac{1}{2} + a_0, \qquad \hat{R}_{33} = \frac{1}{2} - a_0 \,.$$

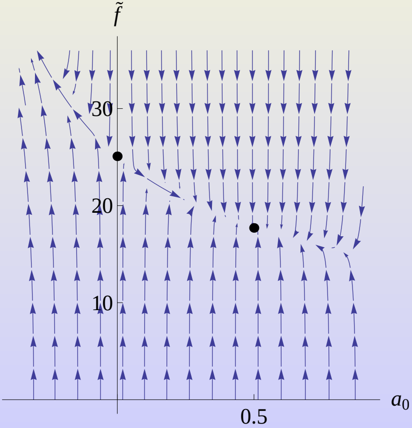
One-loop beta functions of $\tilde{f}^2$ and a_0:

$$\begin{aligned}
\frac{d\tilde{f}^2}{dt} &= 2\tilde{f}^2 - \frac{1}{(4\pi)^2}\tilde{f}^4\left(\frac{1}{2} + a_0\right) \\
\frac{da_0}{dt} &= \frac{1}{2}\frac{1}{(4\pi)^2}\tilde{f}^2 a_0(1-2a_0) \,.
\end{aligned}$$

Two nontrivial Fixed Points:

FPI: $\tilde{f}_* = 8\pi \qquad a_{0*} = 0 \qquad SU(2)_R$ symmetric

FPII: $\tilde{f}_* = 4\sqrt{2}\pi \qquad a_{0*} = 1/2 \qquad SU(2)_R$ broken



β functions II (gauged case)

One-loop beta functions ($g^2, g'^2 \ll \tilde{f}^2$):

$$\frac{d\tilde{f}^2}{dt} = 2\tilde{f}^2 - \frac{1}{2}\frac{\tilde{f}^2}{(4\pi)^2}\left(\tilde{f}^2(1+2a_0) + 6g^2 + 3g'^2\right) ,$$
$$\frac{da_0}{dt} = \frac{1}{2}\frac{1}{(4\pi)^2}\left(\tilde{f}^2 a_0(1-2a_0) + \frac{3}{2}g'^2\right) ,$$
$$\frac{da_1}{dt} = \frac{1}{(4\pi)^2}\left(\tilde{f}^2 a_1 + \frac{1}{6}\right) .$$

The two nontrivial Fixed Points of the ungauged case are slightly shifted:

FPI: $\tilde{f}_* = 25.1 \qquad a_{0*} = -0.000292 \qquad a_{1*} = -0.000265$
(1 relevant and 2 irrelevant directions)

FPII: $\tilde{f}_* = 17.7 \qquad a_{0*} = 0.501 \qquad a_{1*} = -0.000530$
(2 relevant and 1 irrelevant directions)

Comparison with experimental bounds

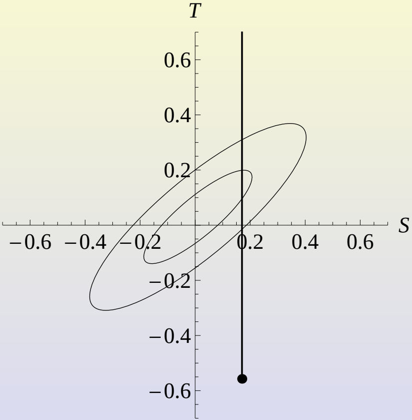


Figure: The half-line (FPII endpoints) and the dot (FPI endpoint) show the values permitted by asymptotic safety. The ellipses show the 1 and 2 σ experimental bounds with m_H=117GeV [PDG, J. Phys. G, 37, 075021 (2010)].

Fermions and Goldstone bosons

$SU(N)_L \times SU(N)_R$ invariant nonlinear sigma model lagrangian coupled to fermions:

$$\mathcal{L} = -\frac{1}{f^2}\mathrm{Tr}\left(U^\dagger \partial_\mu U U^\dagger \partial^\mu U\right) + \bar{\psi}_L i\gamma^\mu \partial_\mu \psi_L + \bar{\psi}_R i\gamma^\mu \partial_\mu \psi_R$$
$$- \frac{2h}{f}\left(\bar{\psi}_L^{ia} U^{ij} \psi_R^{ja} + \mathrm{h.c.}\right) . \qquad (1/f = \upsilon/2)$$

$U = e^{if\pi^a T_a}$ is $SU(N)$ valued scalar field, π^a Goldstone bosons.
$\psi^{ia}_{L/R}$ in the fundamental of $SU(N)_{L/R}$ and $SU(N_c)$

Degenerate fermion multiplet of mass

$$m = 2\frac{h}{f} = h\upsilon \,,$$

h is the Yukawa coupling.

Beta functions

One-loop RG equations for $\tilde{f}$ and h using sharp cutoff regularization:

$$
\begin{aligned}
\frac{d\tilde{f}}{dt} &= \tilde{f} - \frac{N}{64\pi^2}\tilde{f}^3 + \frac{N_c}{4\pi^2}h^2\tilde{f} \ , \\
\frac{dh}{dt} &= \frac{1}{16\pi^2}\left(4N_c - 2\frac{N^2-1}{N}\right)h^3 + \frac{1}{64\pi^2}\frac{N^2-2}{N}h\tilde{f}^2 \, .
\end{aligned}
$$

Fixed Points:

FPI $(h_* = 0, \tilde{f}_* = 0) \Rightarrow$ trivial

FPII $(h_* = 0, \tilde{f}_* = 8\pi/\sqrt{N}) \Rightarrow h = 0$ at all scales

FPIII $(h_* \neq 0, \tilde{f}_* \neq 0) \Rightarrow N > 2N_c$ (not true for the most phenomenologically important case $N = 2$, $N_c = 3$)

Four-fermion interactions

Fix $N = 2$, we add to the lagrangian a complete set of $SU(2)_L \times SU(2)_R$ four fermion operators:

$$
\begin{aligned}
\mathcal{L}_{\psi^4} &= \lambda_1 \left(\bar{\psi}_L^{ia}\psi_R^{ja}\bar{\psi}_R^{jb}\psi_L^{ib}\right) + \lambda_2 \left(\bar{\psi}_L^{ia}\psi_R^{jb}\bar{\psi}_R^{jb}\psi_L^{ia}\right) \\
&+ \lambda_3 \left(\bar{\psi}_L^{ia}\gamma_\mu\psi_L^{ia}\bar{\psi}_L^{jb}\gamma^\mu\psi_L^{jb} + \bar{\psi}_R^{ia}\gamma_\mu\psi_R^{ia}\bar{\psi}_R^{jb}\gamma^\mu\psi_R^{jb}\right) \\
&+ \lambda_4 \left(\bar{\psi}_L^{ia}\gamma_\mu\psi_L^{ib}\bar{\psi}_L^{jb}\gamma^\mu\psi_L^{ja} + \bar{\psi}_R^{ia}\gamma_\mu\psi_R^{ib}\bar{\psi}_R^{jb}\gamma^\mu\psi_R^{ja}\right) .
\end{aligned}
$$

- These four-fermion interactions have been studied also by Gies, Jaeckel and Wetterich [PRD 69 105008 (2004)];
- We do not seek to model chiral symmetry breaking;
- In our RG analysis we consider only the third family of quarks, $\psi^t = (t\ b)$;
- In the case of $SU(2) \times U(1)$ there would be 10 operator.

Beta functions

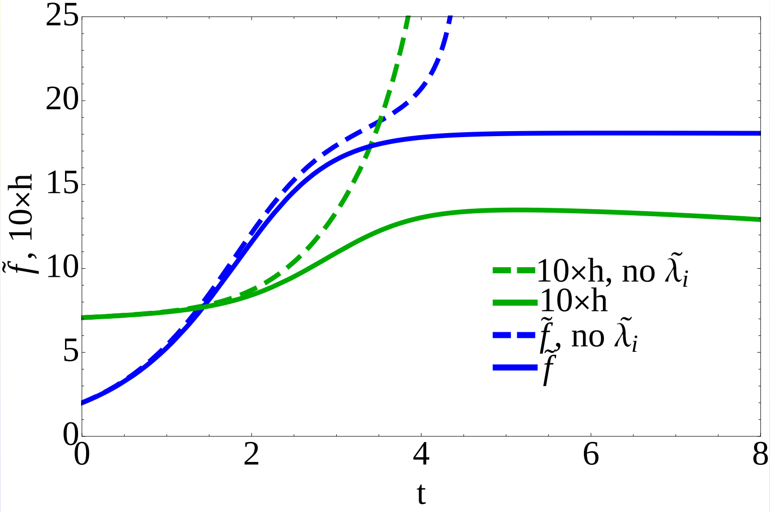
One-loop RG equations for $\tilde{f}, h$ and $\tilde{\lambda}_i = k^2 \lambda_i$ (sharp cutoff):

$$
\begin{aligned}
\frac{d\tilde{f}}{dt} &= \tilde{f} - \frac{1}{32\pi^2}\tilde{f}^3 + \frac{N_c}{4\pi^2}h^2\tilde{f} \\
\frac{dh}{dt} &= \frac{1}{16\pi^2}\left[4N_c - 3 + \frac{16}{\tilde{f}^2}(N_c\tilde{\lambda}_1 + \tilde{\lambda}_2)\right]h^3 \\
&+ \frac{1}{64\pi^2}\left[\tilde{f}^2 - 16(N_c\tilde{\lambda}_1 + \tilde{\lambda}_2)\right]h \\
\frac{d\tilde{\lambda}_1}{dt} &= 2\tilde{\lambda}_1 - \frac{1}{4\pi^2}\left[N_c\tilde{\lambda}_1^2 + \frac{3}{2}\tilde{\lambda}_1\tilde{\lambda}_2 - 2\tilde{\lambda}_1\tilde{\lambda}_3 - 4\tilde{\lambda}_1\tilde{\lambda}_4\right] \\
\frac{d\tilde{\lambda}_2}{dt} &= 2\tilde{\lambda}_2 + \frac{1}{4\pi^2}\left[\frac{1}{4}\tilde{\lambda}_1^2 + 4\tilde{\lambda}_1\tilde{\lambda}_3 + 2\tilde{\lambda}_1\tilde{\lambda}_4 - \frac{3}{4}\tilde{\lambda}_2^2 + 2(2N_c - 1)\tilde{\lambda}_2\tilde{\lambda}_3\right] \\
\frac{d\tilde{\lambda}_3}{dt} &= 2\tilde{\lambda}_3 + \frac{1}{4\pi^2}\left[\frac{1}{4}\tilde{\lambda}_1\tilde{\lambda}_2 + \frac{N_c}{8}\tilde{\lambda}_2^2 + (2N_c - 1)\tilde{\lambda}_3^2 + 2(N_c + 2)\tilde{\lambda}_3\tilde{\lambda}_4\right] \\
\frac{d\tilde{\lambda}_4}{dt} &= 2\tilde{\lambda}_4 + \frac{1}{4\pi^2}\left[\frac{1}{8}\tilde{\lambda}_1^2 - 4\tilde{\lambda}_3\tilde{\lambda}_4 + (N_c + 2)\tilde{\lambda}_4^2\right].
\end{aligned}
$$

Fixed points table ($\tilde{f}_* = 17.78$, $h_* = 0$)

	$\tilde{\lambda}_1$	$\tilde{\lambda}_2$	$\tilde{\lambda}_3$	$\tilde{\lambda}_4$	ϵ_h
fp0	0	0	0	0	0.5
fp1a	0	-28.71	-7.18	0	1.22
fp1b	0	0	7.85	-9.51	0.5
fp1c	0	25.61	-4.27	0	-0.15
fp1d	25.80	-1.77	0.19	-1.15	-1.42
fp2a	13.41	20.10	-3.80	-0.24	-1.03
fp2b	20.86	-3.56	7.04	-8.94	-1.00
fp2c	0	-36.55	2.34	-13.92	1.43
fp2d	0	0	-15.79	0	0.5
fp2e	37.17	-37.36	-8.43	-1.65	-1.38
fp2f	-2.92	32.59	4.67	-12.04	-0.10
fp3a	$0.$	31.67	4.67	-12.06	-0.30
fp3b	19.95	-8.59	-15.27	-0.36	-0.80
fp3c	31.22	-44.52	0.73	-13.38	-0.74
fp3d	-4.87	1.54	-5.42	-20.10	0.83
fp4	0	0	-5.42	-20.13	0.5

Numerical solution, initial conditions $h_0 = m_t/v$ and $\tilde{f}_0 = 2$:



Running of $\tilde{f}$ and h for $N = 2$ and $N_c = 3$. Without four-fermion interactions, the AS behavior of $\tilde{f}$ is destabilized around $t = 3.5$ ($\sim 8,3$ TeV).

Experimental constraints

Current bounds on contact interactions have been published for the case in which only one operator is considered [E. Eichten, K. D. Lane, M. E. Peskin, PRL **50** (1983) 811], here $\psi^t = (u\ d)$:

$$\mathcal{L}_{qqqq} = \frac{4\pi A}{2\Lambda^2}\,\bar{\psi}_L^{ia}\gamma_\mu\psi_L^{ia}\bar{\psi}_L^{jb}\gamma^\mu\psi_L^{jb} \qquad (A = \pm 1)\,.$$

The experimental bound is a lower bound of the so-called contact interaction scale Λ:

$$\lambda(k) = \frac{2\pi}{\Lambda^2}\,.$$

Current published bound [ATLAS Collaboration, arXiv:1103.3864 hep-ex]:

$$\Lambda > 9.5\text{TeV} \qquad \text{with } 36\,\text{pb}^{-1}.$$

Future expected bound [ATLAS and CMS, arXiv:0709.2518 hep-ph]:

$$\Lambda > 30\text{TeV} \qquad \text{with } 100\,\text{fb}^{-1}.$$

We enforce the same bound on all the coefficients (conservative but unrealistic), $\tilde{\lambda}_i(k) < 2\pi k^2/\Lambda_{bound}$:

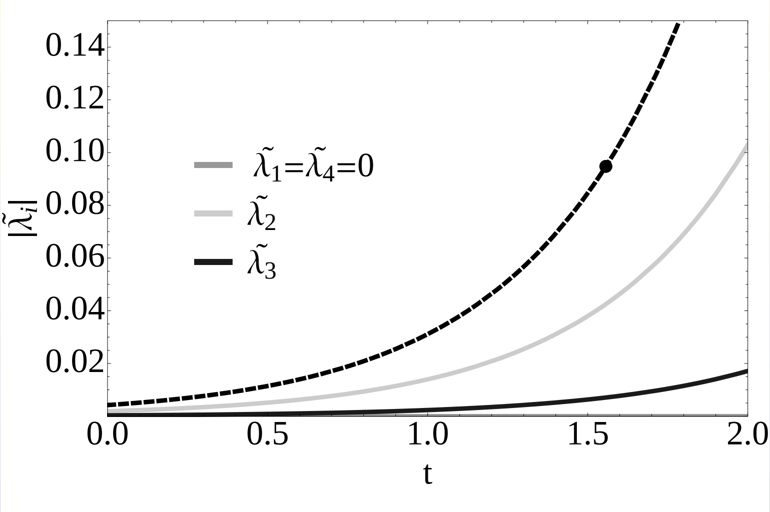


Figure: RG evolution of $\tilde{\lambda}_i$ towards the IR for the point fp1c.

Summary and Conclusions

- There seem to exist fixed point for the gauged NLσM in which only the leading two-derivative operator is considered.
- The position of the fixed point depends on the scheme of regularization and inclusion of higher derivative operators may also move the fixed point.
- In the case of the electroweak chiral lagrangian we were able to study some phenomenology emerging from the AS picture.
- AS seems to be compatible with electroweak precision measurement. It is possible to obtain estimations of S and T parameters in agreement with experimental data.
- Coupling the NLσM to fermion we have that, in the case of $SU(2) \times U(1)$, the model is no more AS.
- AS can be restored introducing effective four-fermion interactions that satisfy current LHC experimental bounds on contact interactions.