Light fermions in quantum gravity

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Research Training Group
Quantum and Gravitational Fields

Phenomenology of quantum gravity

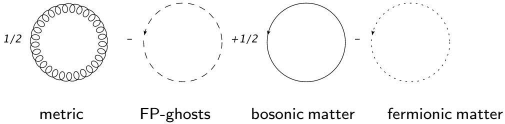
- new (to be observed) effects from QG
  - problem: (expected) typical scale of QG effects $M_{\mathrm{Planck}} \sim 10^{19}\mathrm{GeV}$
  - cosmology/ astrophysics?
  - extra dimensions?

- compatibility of QG with physics that *has* already been observed
  - existence of semiclassical regime, where GR description holds
  - compatibility with existence of matter, in particular preserve observed low-energy properties of matter

$\rightarrow$ *mandatory* requirements! $\rightarrow$ can be used to rule out QG proposals

Matter and asymptotically safe QG

coupling to matter: $\partial_t \Gamma_k =$



- What happens to the NGFP under the inclusion of matter?

  → minimally coupled matter compatible with NGFP at $G_* > 0$ for many matter models (e.g. Standard Model) [Percacci,Perini (2002,2003)]

- What happens to matter coupled to asymptotically safe quantum gravity?

  → YM theory probably remains asymptotically free

  [Daum, Harst, Reuter (2009); Folkerts, Litim, Pawlowski (2011)]

  QED possibly asymptotically free/safe

  [Harst, Reuter, 2011]

  Higgs sector possibly asymptotically safe

  [Zanusso, Zambelli, Vacca, Percacci (2009); Vacca, Zanusso (2010)]

Fermion masses

Standard Model:

- no explicit mass terms in microscopic action (quark sector: chiral symmetry $SU(\mathrm{N}_f)_L \times SU(\mathrm{N}_f)_R$)
- strong fermionic correlations (e.g. in QCD by strong gluon coupling) generate condensate $\langle\bar{\psi}\psi\rangle$ and break chiral symmetry
- $\rightarrow$ fermion masses $<< M_{\mathrm{Planck}}$

similarity between Yang-Mills theory/ QCD and gravity:

CAN METRIC FLUCTUATIONS INDUCE STRONG FERMIONIC CORRELATIONS AND BREAK CHIRAL SYMMETRY?

$\rightarrow$ fermion masses $\simeq M_{\mathrm{Planck}}$

not compatible with observation of light fermions in our universe!

$\rightarrow$ existence of chiral symmetry breaking mechanism in quantum gravity might rule out quantum gravity models!

Framework

QG parametrised by metric fluctuations

- *effective* description below $k_0 \lesssim M_{\mathrm{Planck}}$

  metric: *effective* degree of freedom
  fundamental degrees of freedom can be very different (LQG, strings, causal sets ...)

  microscopic UV completion for gravity determines values of couplings on this initial scale k_0
  below k_0 our description applies $\rightarrow$ can study compatibility of initial conditions with existence of light fermions at low energies
- *fundamental* description (valid far beyond Planck scale) within asymptotic-safety scenario

How to detect χSB from the RG flow?

$\mathrm{SU}(\mathrm{N}_f)_L \times \mathrm{SU}(\mathrm{N}_f)_R$ chirally symmetric 4-fermion interaction:

$$\lambda_\pm \left[\left(\bar\psi^i \gamma_\mu \psi^i\right)^2 \pm \left(\bar\psi^i \gamma_\mu \gamma_5 \psi^i\right)^2 \right] \qquad \text{(Fierz complete) } i = 1, \ldots, \mathrm{N}_f$$

Criterion for χSB

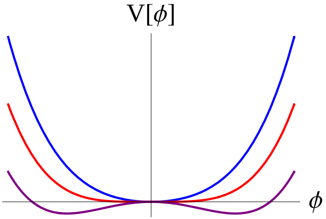
χSB: $\beta_{\lambda_\pm}$ have no real fixed points!

$$\lambda_+ \left[(\bar\psi^i \gamma_\mu \psi^i)^2 - (\bar\psi^i \gamma_\mu \gamma_5 \psi^i)^2 \right] = \lambda_\sigma \left[(\bar\psi^i \psi^j)^2 - (\bar\psi^i \gamma_5 \psi^j)^2 \right], \; \lambda_\sigma = -\tfrac{1}{2}\lambda_+$$

composite boson field $\phi \sim \bar\psi\psi$: $\lambda(\bar\psi\psi)^2 \rightarrow -\frac{2}{\lambda}\phi^2 - h\phi\bar\psi\psi$

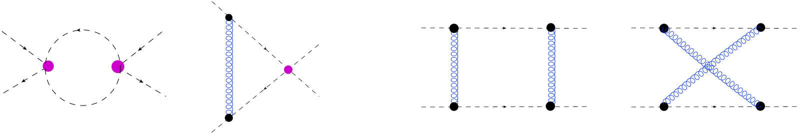
χSB:$\langle\bar\psi\psi\rangle \neq 0 \Leftrightarrow \langle\phi\rangle \neq 0$

onset of χSB: $m = 0 \Leftrightarrow \lambda \rightarrow \infty$

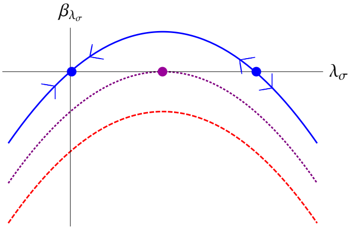


χSB in QCD

fermions coupled to gauge field: $\bar{\psi}^i \not{D}[A]\psi^i$



$$\beta_{\lambda_i} = (2+\eta_\psi)\lambda_i + \lambda_m A_{mn}\lambda_n + b_n\lambda_n g^2 + c g^4 \;\; (i,m,n = \pm,\sigma)$$



gauge coupling $g = 0$: χSB depends on initial conditions

gauge coupling $g = g_{\rm crit}$

gauge coupling $g > g_{\rm crit}$: no initial conditions exist in UV such that χ symmetry remains unbroken

Gies, Jaeckel, Wetterich (2004), Gies, Jaeckel (2006), Braun, Gies (2010)

induced masses: $m_{\rm proton} \sim \Lambda_{\rm QCD}$

Coupling fermions to gravity

vierbein formalism:
$e^a_\mu(x)$ transition from local orthonormal frame to coordinate frame
$g_{\mu\nu}(x) = e^a_\mu(x) e^b_\nu(x) \eta_{ab}$

$$\nabla_\mu \psi = \partial_\mu \psi + \frac{1}{8}[\gamma^a, \gamma^b] \omega_{\mu\, ab} \psi, \qquad \gamma^\mu(x) = e^\mu_a(x) \gamma^a$$

spin connection from: $\nabla_\mu e^a_\nu = 0 \to \omega^{ab}_\mu = -e^{\nu\, b} \left(\partial_\mu e^a_\nu - \Gamma^\lambda_{\mu\nu} e^a_\lambda \right)$

Truncation

$$\Gamma_k = \int d^4x \sqrt{g} \left[\frac{Z_N(k)}{16\pi G_N} (-R + 2\bar{\lambda}(k)) \right] + \Gamma_{gf} + \Gamma_{gh}$$
$$+ \int d^4x \sqrt{g} \left[i\, Z_\psi(k) \bar{\psi}^i \gamma^\mu \nabla_\mu \psi^i + \lambda_\pm(k) \left(\left(\bar{\psi}^i \gamma_\mu \psi^i \right)^2 \pm \left(\bar{\psi}^i \gamma_\mu \gamma_5 \psi^i \right)^2 \right) \right]$$

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- G_* and λ_* as in [Percacci, Perini (2002)]
- $\beta_{\lambda_\pm}$ and η_ψ: flat background sufficient

background field gauge: $g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu} \longrightarrow \delta_{\mu\nu} + h_{\mu\nu}$

$$F_\mu = \frac{\sqrt{2}}{32\pi G_N}\left(\bar{D}^\nu h_{\mu\nu} - \frac{1+\rho}{d}\bar{D}_\mu h^\nu{}_\nu\right)$$

local O(4): $G_{ab} = e_{\mu a}\bar{g}^{\mu\nu}\bar{e}_{\nu b} - e_{\mu b}\bar{g}^{\mu\nu}\bar{e}_{\nu a}$ van Nieuwenhuizen (1981), Woodard (1984)

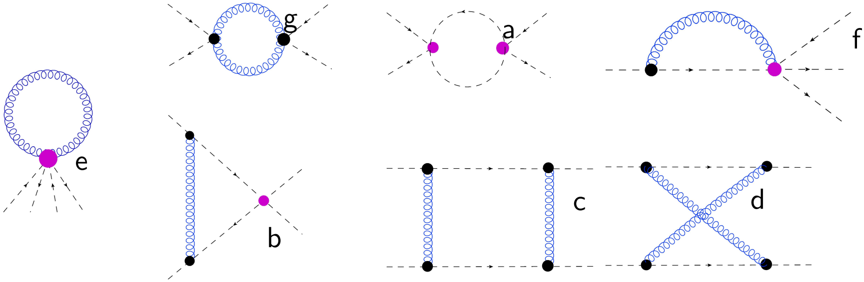
$\rightarrow \delta e^a_\mu = \frac{1}{2}h_{\mu\nu}e_{\kappa a}\delta^{\nu\kappa} + \ldots \rightarrow$ metric fluctuation propagators

χSB in gravity: Similarities to QCD?

$$\Gamma_k = \int d^4x\sqrt{g}\left[\frac{Z_N(k)}{16\pi G_N}(-R+2\bar{\lambda}(k))\right] + \Gamma_{gf} + \Gamma_{gh}$$

$$+\int d^4x\sqrt{g}\left[i\,Z_\psi\bar{\psi}^i\gamma^\mu\nabla_\mu\psi^i + \lambda_\pm(k)\left(\left(\bar{\psi}^i\gamma_\mu\psi^i\right)^2 \pm \left(\bar{\psi}^i\gamma_\mu\gamma_5\psi^i\right)^2\right)\right]$$

$\partial_t\lambda_\pm$ from $\partial_t\Gamma_k|_{4-\mathrm{fermion}}$ $\rightarrow$ diagrammatically:

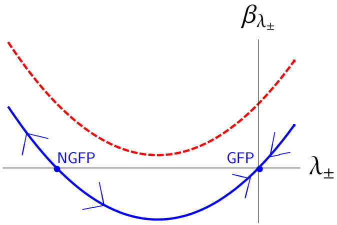


$$\beta_{\lambda_\pm} = (2+\eta_\psi)\lambda_\pm + \lambda_m A_{mn}\lambda_n + b_{\mathrm{grav}}\, G\, \lambda_\pm f_1(\lambda) + c_{\mathrm{grav}}\, G^2 f_2(\lambda)$$

cf. QCD: $\beta_{\lambda_\pm} = 2\lambda_\pm + \lambda_m A_{mn}\lambda_n + b_n\lambda_n g^2 + cg^4$

χSB in gravity: Similarities to QCD?

$$\beta_{\lambda_\pm} = (2+\eta_\psi)\lambda_\pm + \lambda_m A_{mn}\lambda_n + b_{\mathrm{grav}}\, G\, \lambda_\pm f_1(\lambda) + c_{\mathrm{grav}}\, G^2 f_2(\lambda)$$

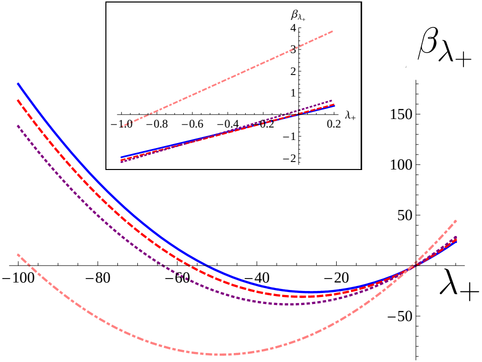


expect: if metric fluctuations break chiral symmetry: masses $\sim M_{\mathrm{Planck}}$

severe conflict with observations!

Questions?

Results *: Existence of fixed points



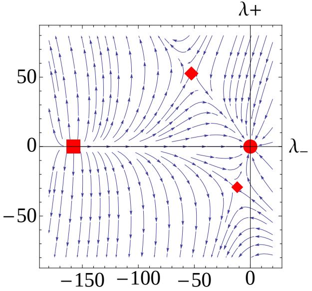
G=0, G=0.2, G=0.5, G=2

- FPs exist even for large $G \longrightarrow \chi$SB can remain unbroken
- Gaußian fixed point is shifted and becomes interacting

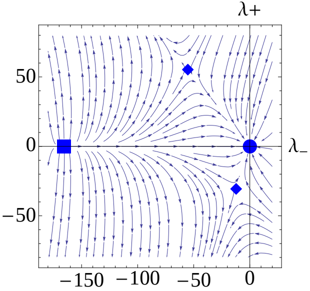
$*$ Landau deWitt gauge $\alpha = 0 = \rho$, regulator $R_k(p^2) = \Gamma_k^{(2)}(p^2) r_k(p^2)$, where $r_{k\,\mathrm{grav}}(p^2) = \left(\frac{\Gamma_k^{(2)}(k^2)}{\Gamma_k^{(2)}(p^2)} - 1\right)\theta(k^2 - p^2)$ and $r_{k\,\mathrm{ferm}}(p^2) = \left(\sqrt{\frac{k^2}{p^2}} - 1\right)\theta(k^2 - p^2)$

Results: Asymptotic safety

RG flow without gravity



RG flow with gravity ($N_f = 2$)



G_*, λ_* from [Percacci, Perini, 2002]

asymptotically safe quantum gravity supports universes with light fermions

Decoupling mechanism:

metric propagator $\frac{1}{p^2 - 2\lambda}$

fermions shift the fixed point $\lambda \ll 0$ [Percacci, Perini, 2002]

$\Rightarrow$ metric fluctuations are strongly suppressed

Universality classes

critical exponents $\{\theta\} = \mathrm{spec}(-B_{ij}) = \mathrm{spec}\left(\frac{-\partial\beta_{g_i}}{\partial g_j}\right)\Big|_{g=g_*}$

$\theta_i > 0$: relevant direction: UV attractive (IR repulsive)

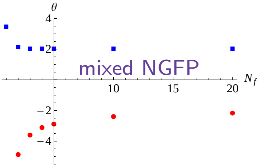
$\rightarrow$ IR value has to be fixed by experiment $\rightarrow$ free parameter

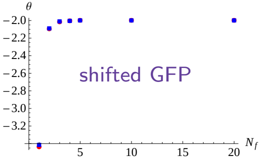
$\theta_i < 0$: irrelevant direction: UV repulsive (IR attractive)

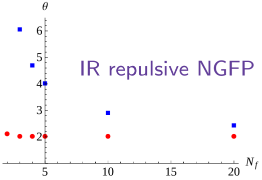
$\rightarrow$ no free parameter, since complete RG trajectory in critical hypersurface of UV fixed point

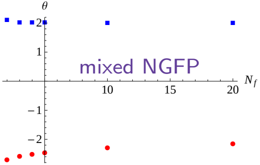
Here: $\frac{-\partial\beta_G}{\partial\lambda_\pm} = 0 = \frac{-\partial\beta_\lambda}{\partial\lambda_\pm}$

critical exponents in fermionic subsector:







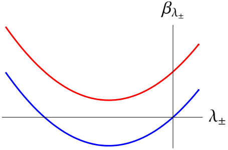


Difference between gluons and gravitons

fixed-point structure: $\lambda_\pm^2$ vs. $\lambda_\pm$

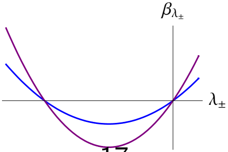
QCD: $\beta_{\lambda_\pm} = 2\lambda_\pm + \lambda_i A_{ij} \lambda_j + b_{\mathrm{glue}\,j} \lambda_j g^2 + c_{\mathrm{glue}} g^4$

gluonic fluctuation mainly strengthen fermionic fluctuations (terms $\sim g^4$ dominate over terms $\sim g^2 \lambda_\pm$)

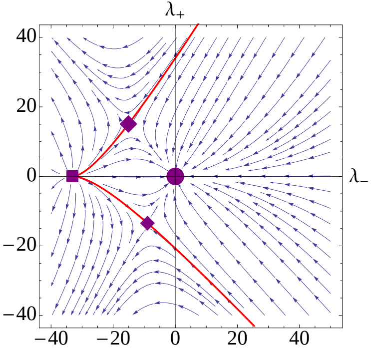


gravity: $\beta_{\lambda_\pm} = (2 + \eta_\psi)\lambda_\pm + \lambda_i A_{ij} \lambda_j + b_{\mathrm{grav}}\, G\, \lambda_\pm f_1(\lambda) + c_{\mathrm{grav}}\, G^2 f_2(\lambda)$

metric fluctuations mainly enhance anomalous scaling (terms $\sim G\lambda_\pm$ dominate over terms $\sim G^2$)



Results: Effective theory



$G = 0.1, \eta_N = 0, \lambda = 0.1, N_f = 6$

UV completion for gravity determines initial conditions for the RG flow at scales $k_0 \leq M_{\text{Planck}}$

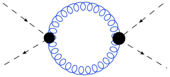
to avoid χSB: initial conditions within basin of attraction of Gaußian fixed point

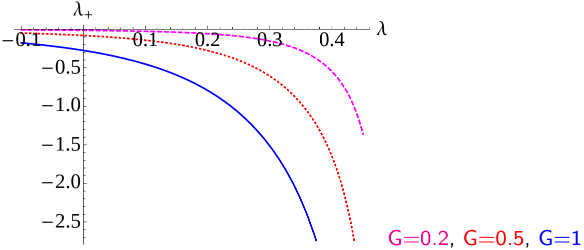
$\rightarrow$ in principle restriction on generic UV completion for gravity

Results: Shifted GFP

$$\beta_{\lambda_\pm} = 2\lambda_\pm + \lambda_i A_{ij} \lambda_j + b\, G\, \lambda_\pm f_1(\lambda) + c\, G^2 f_2(\lambda) \overset{\lambda_\pm \to 0}{\Longrightarrow} c\, G^2 f_2(\lambda) \neq 0$$

diagrammatically:





→ possibly strong interactions even at shifted GFP

general remark:

similarly NGFP for G ⇒ shift of GFP for other operators

→ non-minimal couplings will be generated (truncation not closed)

Summary

- quantum gravity must be consistent with observed low-energy properties of matter
- quantum gravity should not break chiral symmetry and induce fermion masses $\sim M_{\text{Planck}}$
- in fRG: χSB signalled by divergent four-fermion couplings

- If quantum gravity fluctuations are metric fluctuations, light fermions can exist! *

- asymptotically safe quantum gravity supports universes with light fermions
- decoupling mechanism in asymptotic safety: metric fluctuations do not alter properties of fermionic sector strongly
- any UV completion for gravity has to provide initial conditions for the RG flow that avoid χ SB

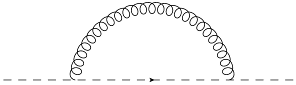
*this claim is within a *truncation* of the full fRG equations...

(similar truncation in QCD is sufficient to detect χSB; all channels for χSB included in pointlike and minimally coupled limit)

Outlook

terms beyond the truncation:
non-minimal couplings

e.g. $\xi \int d^4x \sqrt{g} R \bar{\psi}^i i \not{\nabla} \psi^i$

∂_R 

expect $\xi_* \neq 0$ at $G_* \neq 0$

$\Rightarrow$ non-trivial backreaction on the whole system

possible influence on fixed point structure in fermionic as well as metric subsystem

$\beta_{\lambda_\pm} \sim \xi^2 \rightarrow$ may destroy dominance of $\sim \lambda_\pm G$ terms

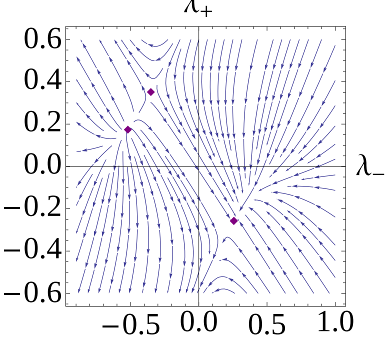
Outlook

terms beyond the truncation:

anomalous dimension η_ψ

here: η_ψ parameter

$\eta_{\psi\,crit}$: fixed points merge



Thank you for your attention!