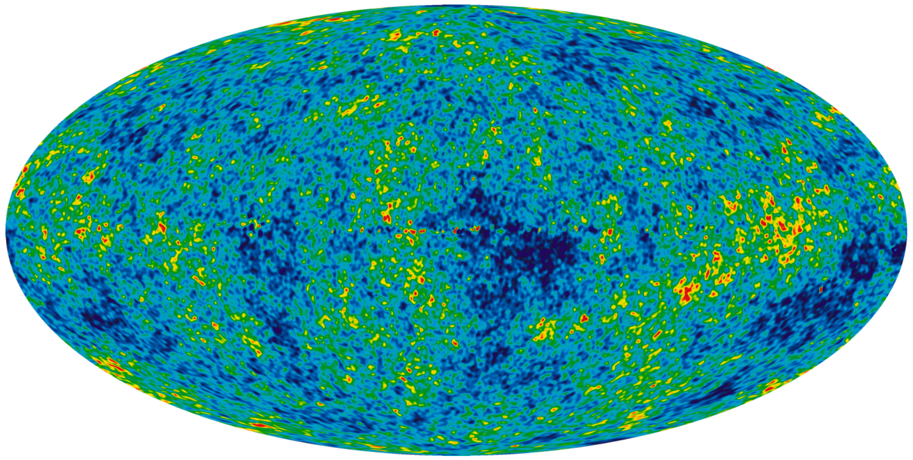
The phase diagram of quantum gravity

from diffeomorphism-invariant RG flows

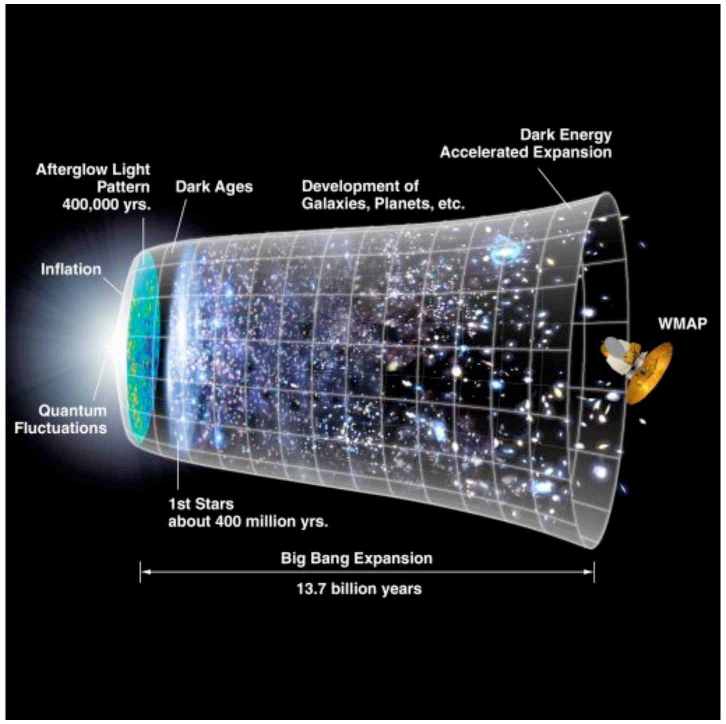
Jan M. Pawlowski
Universität Heidelberg & ExtreMe Matter Institute

Asymptotic safety seminar, May 4th 2012

Phase diagram of quantum gravity

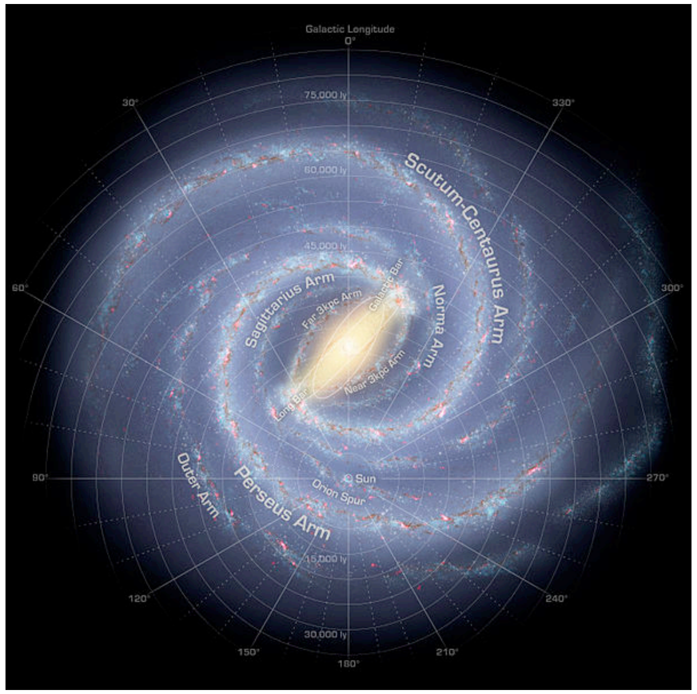


early universe

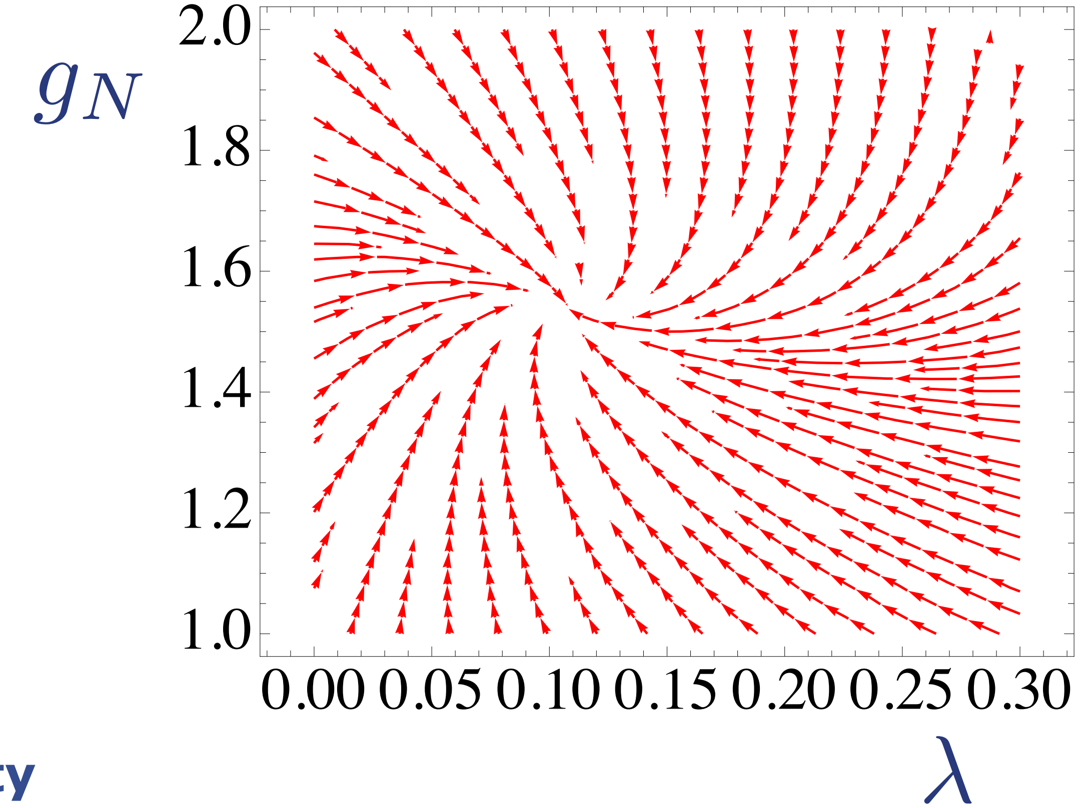


$$g_N(E) \qquad \lambda(E)$$

rotation curves



UV stability



IR stability

Functional approach to quantum gravity

Functional approach to quantum gravity

- **Einstein-Hilbert action**

Metric g **Cosmological constant** Λ

$$S[g] = \frac{1}{16\pi G_N} \int d^4x \sqrt{g} \left(-R(g) + 2\Lambda \right)$$

Newton constant G_N **Ricci scalar** $R(g)$

Functional approach to quantum gravity

- **Einstein-Hilbert action**

$$S[g] = \frac{1}{16\pi G_N} \int d^4x \sqrt{g} \Big(-R(g) + 2\Lambda \Big)$$

- **Momentum dimension of couplings**

$$\dim G_N = -2$$

perturbatively non-renormalisable

$$\dim \Lambda = 2$$

Functional approach to quantum gravity

- **correlation functions, e.g.**

$$\langle R(g(x_1)) \cdots R(g(x_n)) \rangle$$

- **fundamental correlation functions (gauge-fixed)**

$$\langle g(x_1) \cdots g(x_n) \rangle$$

not diffeomorphism-invariant

- **background formalism**

fluctuation

$$g = \bar{g} + h$$

background metric

Functional approach to quantum gravity

- **correlation functions, e.g.**

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not diffeomorphism-invariant

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fluctuation

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background metric

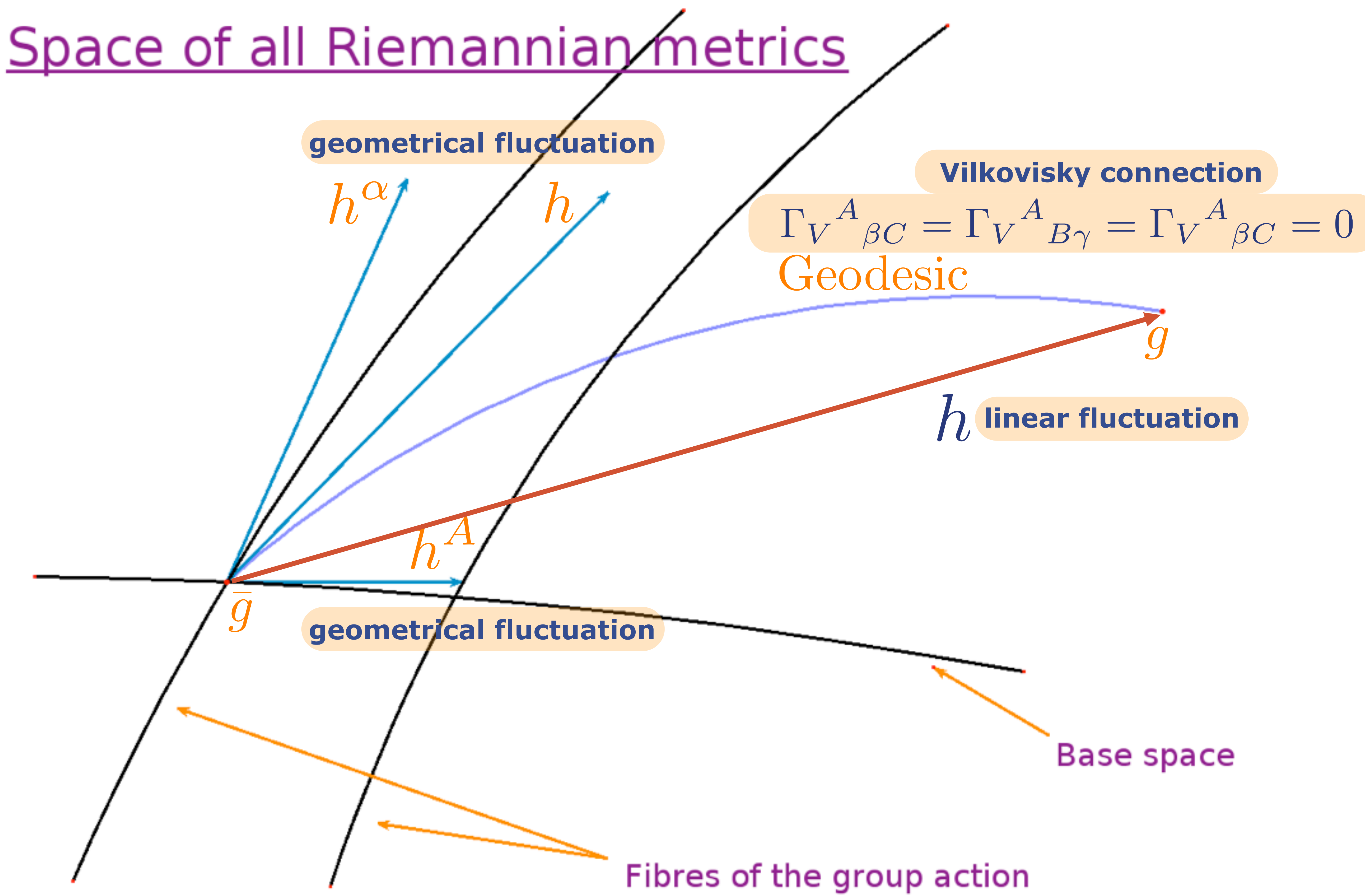
$$\langle h(x_1) \cdots h(x_n) \rangle$$

not background independent

not diffeomorphism-invariant

Functional approach to quantum gravity

geometrical approach



Functional approach to quantum gravity

geometrical approach

- **Einstein-Hilbert action**

$$S[h^A] = \frac{1}{16\pi G_N} \int d^4x \sqrt{g(h^A)} \left(-R(g(h^A)) + 2\Lambda \right)$$

no gauge fixing

effective action $\Gamma = \Gamma[\langle h^A \rangle]$

Functional approach to quantum gravity

geometrical approach

- **Einstein-Hilbert action**

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- **correlation functions at** $\langle h\rangle = 0$

fluctuation

$$g = \bar{g} + \Delta g(h)$$

background metric

Functional approach to quantum gravity

geometrical approach

- **Einstein-Hilbert action**

$$S[h^A] = \frac{1}{16\pi G_N} \int d^4x \sqrt{g(h^A)} \left(-R(g(h^A)) + 2\Lambda\right)$$

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background metric

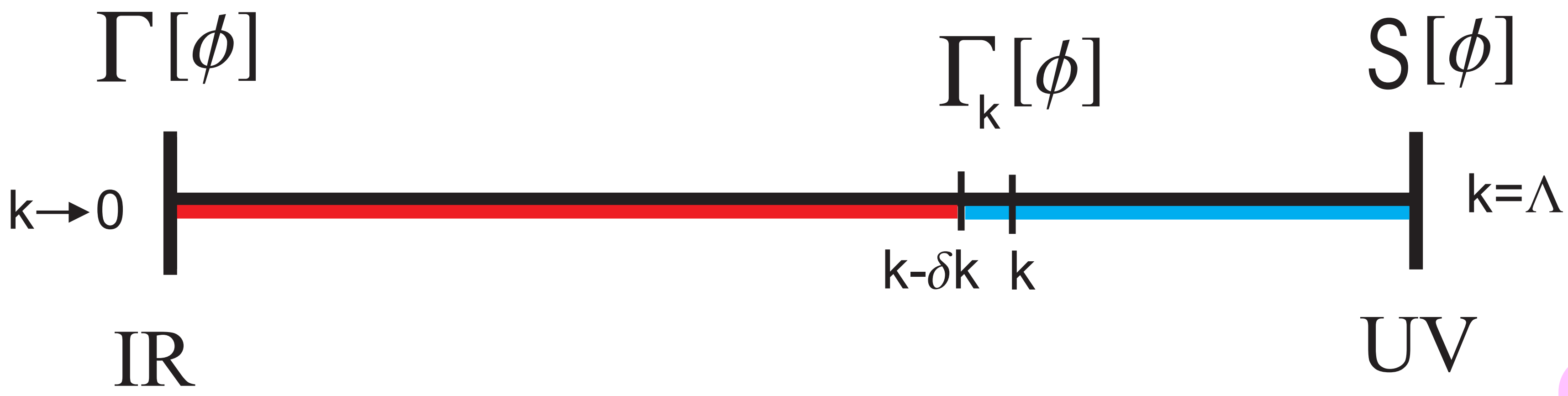
$$\langle h^{A_1} \cdots h^{A_n} \rangle$$

background independent

diffeomorphism-invariant

Functional approach to quantum gravity

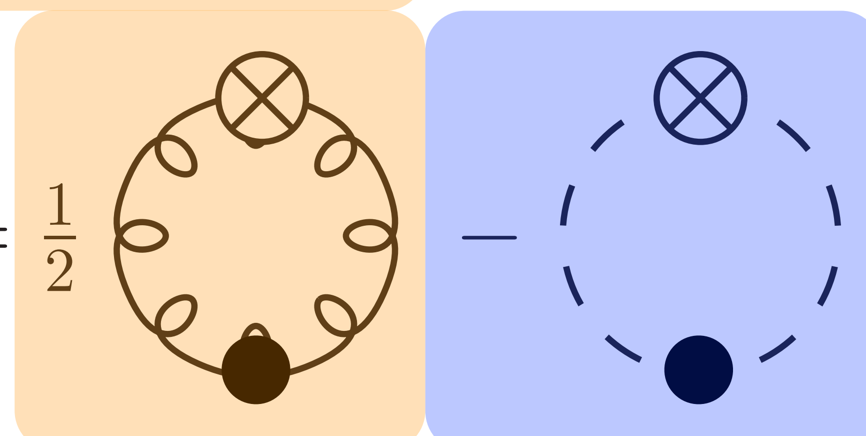
Functional RG



RG-scale k: $t = \ln k$

graviton fluctuations

$$\partial_t \Gamma_k[\phi] = \frac{1}{2}$$



free energy

ghost fluctuations

Reuter ’96

geometrical approach

Branchina, Meissner, Veneziano ’03

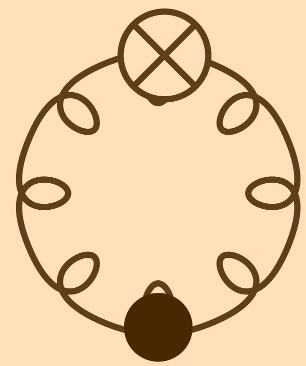
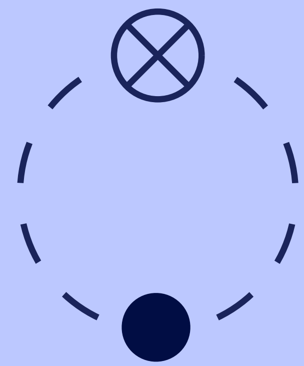
JMP ’03

$$\phi = (h^A, C, \bar{C})$$

fully diffeomorphism invariant

Functional approach to quantum gravity

Functional RG

$$\partial_t \Gamma_k[\phi] = \frac{1}{2}$$  $$-$$ 

free energy

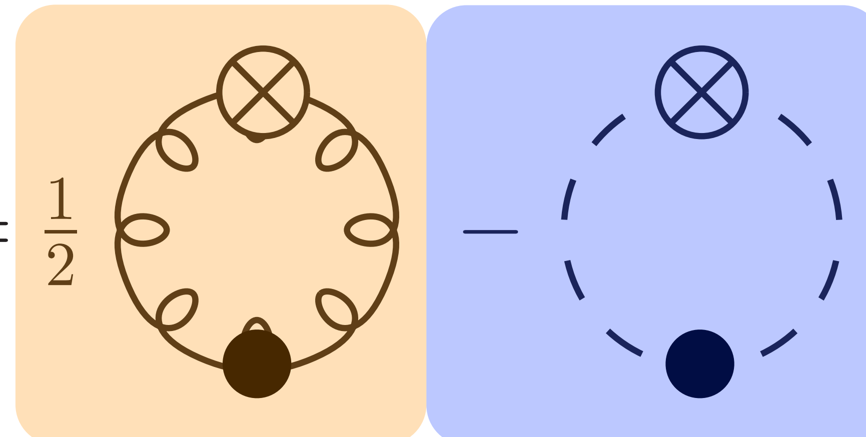
graviton fluctuations **ghost fluctuations**

$\langle hh \rangle$ $\langle C\bar{C} \rangle$

Functional approach to quantum gravity

Functional RG

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \quad (\text{graviton loop}) \quad - \quad (\text{ghost loop})$$



free energy

graviton fluctuations

ghost fluctuations

$\langle hh \rangle$

$\langle C\bar{C} \rangle$

Bi-metric approach

Manrique, Reuter, Saueressig '10

background approximation

$\langle \bar{g}\bar{g} \rangle$

- **potentially dangerous for**
  - **in Yang-Mills theory: sign of beta function & confinement**

    Litim, JMP '02 & Braun, Gies, JMP '07
  - **in Yang-Mills gravity system: stability at stake**

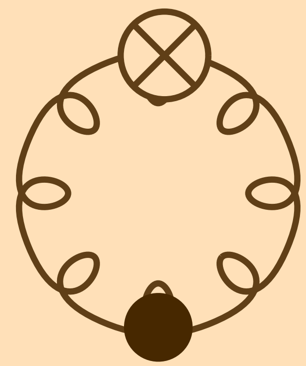
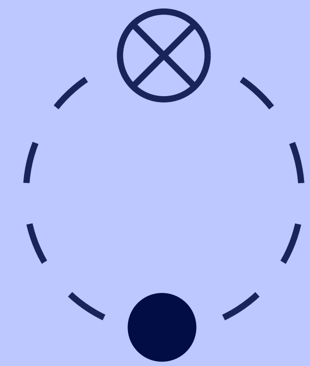
    Folkerts, Litim, JMP '11

Functional approach to quantum gravity

Geometrical RG

JMP '03

Donkin, JMP, arXiv:1203.4207

$\partial_t \Gamma_k[\phi] = \frac{1}{2}$  $-$ 

free energy

graviton fluctuations $\langle hh \rangle$

ghost fluctuations $\langle C\bar{C} \rangle$

Nielsen identities

$$\frac{\delta \Gamma}{\delta \bar{g}} = \langle Dh \rangle * \frac{\delta \Gamma}{\delta \langle h \rangle} + \mathrm{cut-off\ \ terms}$$

Dh **covariant derivative with** Γ_V

Functional approach to quantum gravity

Geometrical RG

Donkin, JMP, arXiv:1203.4207

Extended Einstein-Hilbert approximation

$$\Gamma_k[\bar{g}; h, \bar{C}, C] = \Gamma_{\rm EH}[g; \bar{C}, C; \bar{G}_N, \bar{\Lambda}] + \Delta\Gamma[\bar{g}; h]$$

$$\Delta\Gamma[\bar{g}; h] = \Delta\Gamma_1 + \Delta\Gamma_2 = \Delta\Gamma^a h_a + \frac{1}{2}\Delta\Gamma^{ab} h_a h_b$$

$$\Gamma^{ab}_{\rm EH} + \Delta\Gamma^{ab} = \Gamma^{ab}_{\rm EH}\Big|_{G_N, \Lambda}$$

Nielsen identities

$$\frac{\delta\Gamma}{\delta\bar{g}} = \langle Dh \rangle * \frac{\delta\Gamma}{\delta\langle h\rangle} + \text{cut} - \text{off terms}$$

Dh **covariant derivative with** Γ_V

Functional approach to quantum gravity

Geometrical RG

- **dimensionless couplings with** $\mathbf{E} \sim \mathbf{k}$

$$g_N = k^{d-2} G_N$$

$$\lambda = \frac{\Lambda}{k^2}$$

Functional approach to quantum gravity

Geometrical RG

- **dimensionless couplings with** $\mathbf{E} \sim \mathbf{k}$

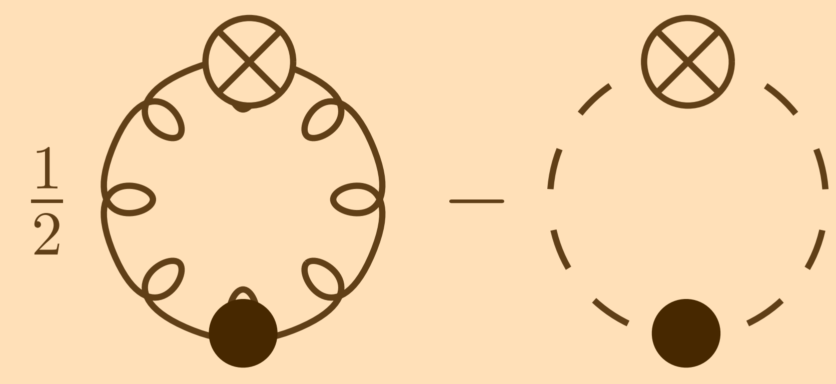
$$g_N = k^{d-2} G_N \qquad\qquad \lambda = \frac{\Lambda}{k^2}$$

- **RG-flows**

$$\partial_t g_N + (2-d) g_N = F_g(g_N, \lambda)$$

$$\partial_t \lambda + (2-\eta_N)\lambda = F_\lambda(g_N, \lambda)$$

$$\partial_t \Gamma[\bar{g}, h; \bar{g}_N, \bar{\lambda}] =$$



$$(g_N, \Lambda)$$

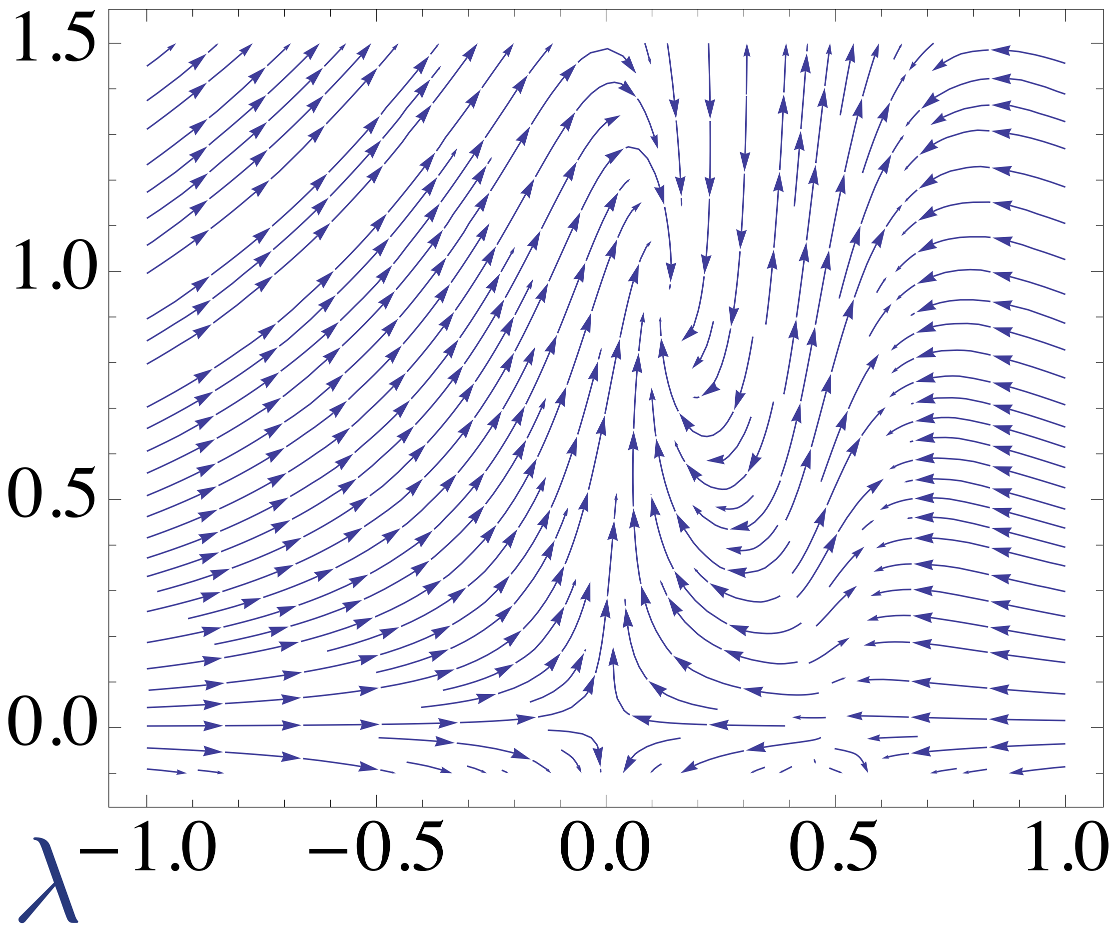
Phase diagram of quantum gravity

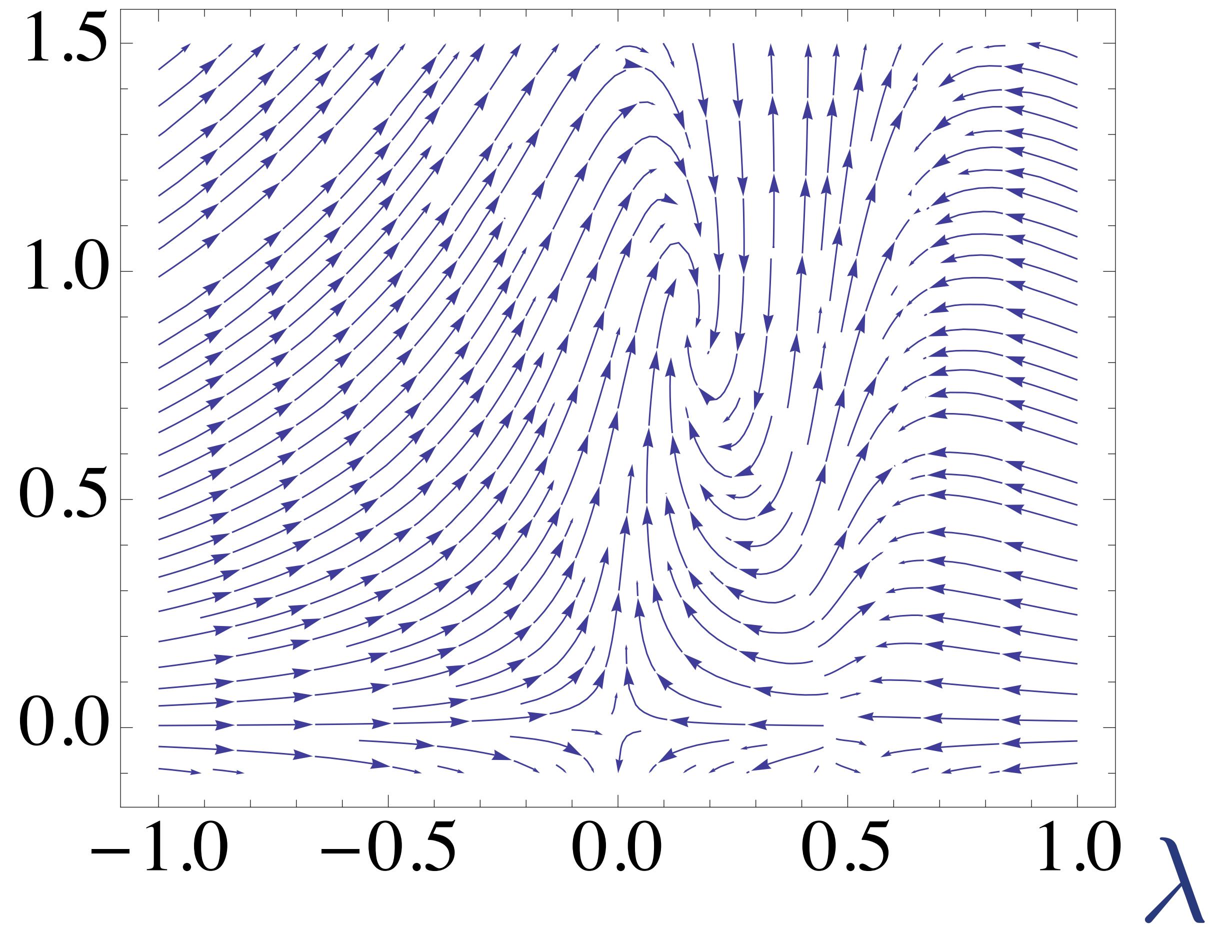
Donkin, JMP, arXiv:1203.4207

UV

Phase diagram of quantum gravity

phase portraits

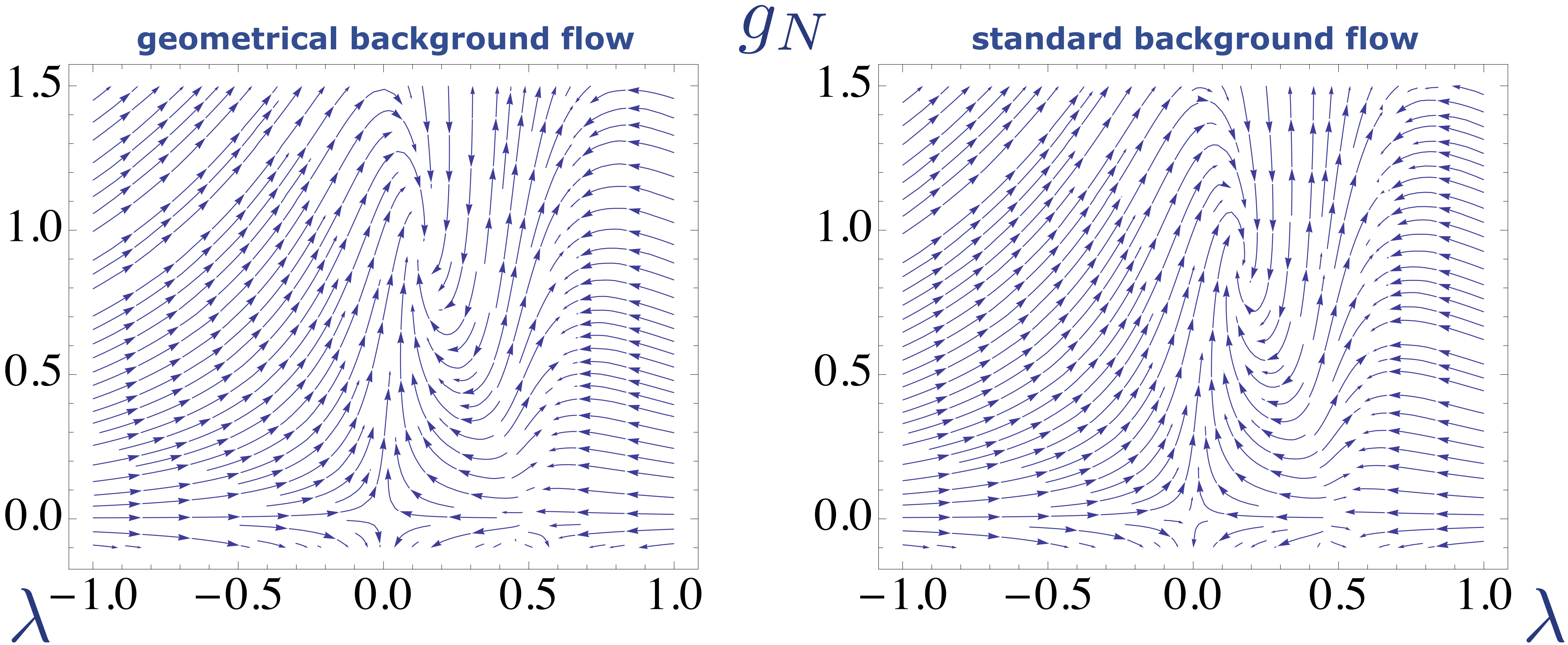


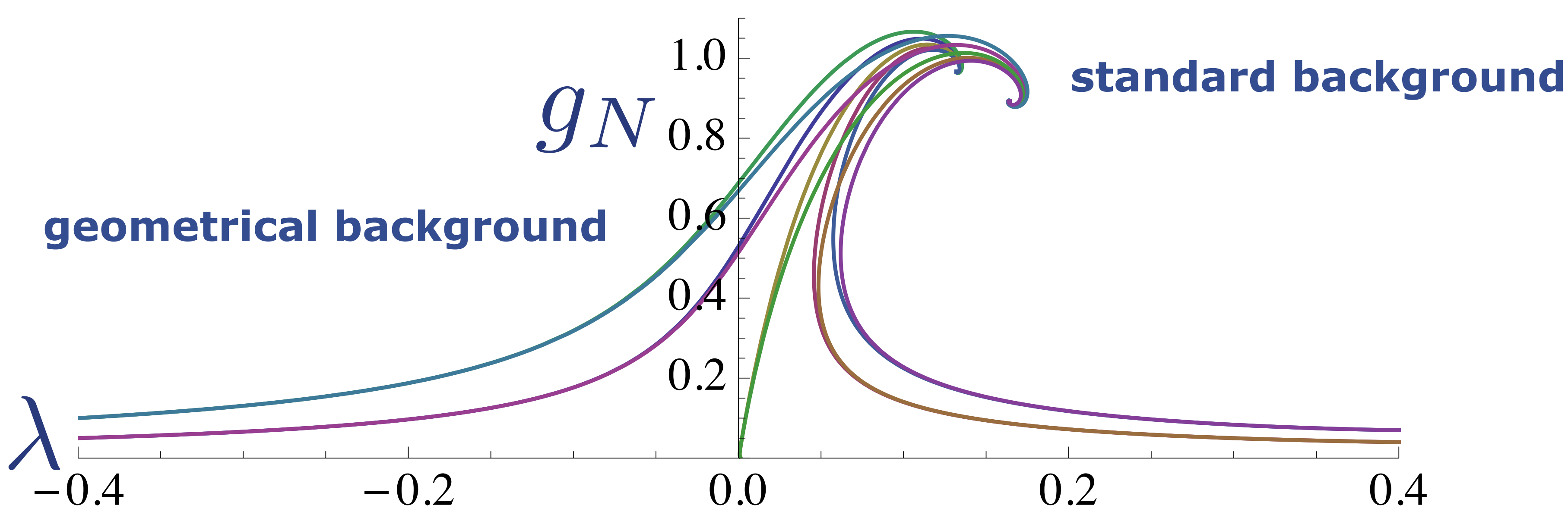


Phase diagram of quantum gravity

$\langle hh \rangle$ → $\langle \bar{g}\bar{g} \rangle$

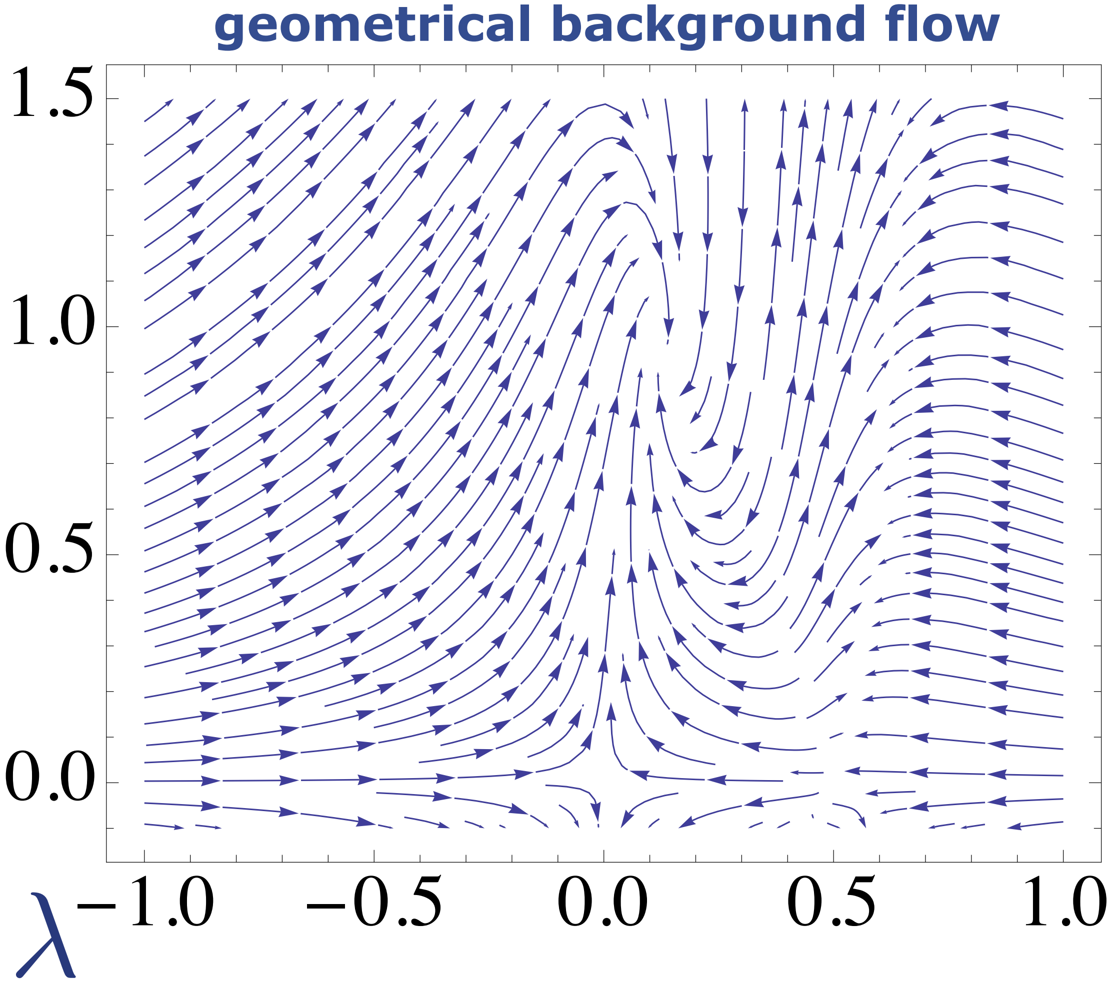
phase portraits

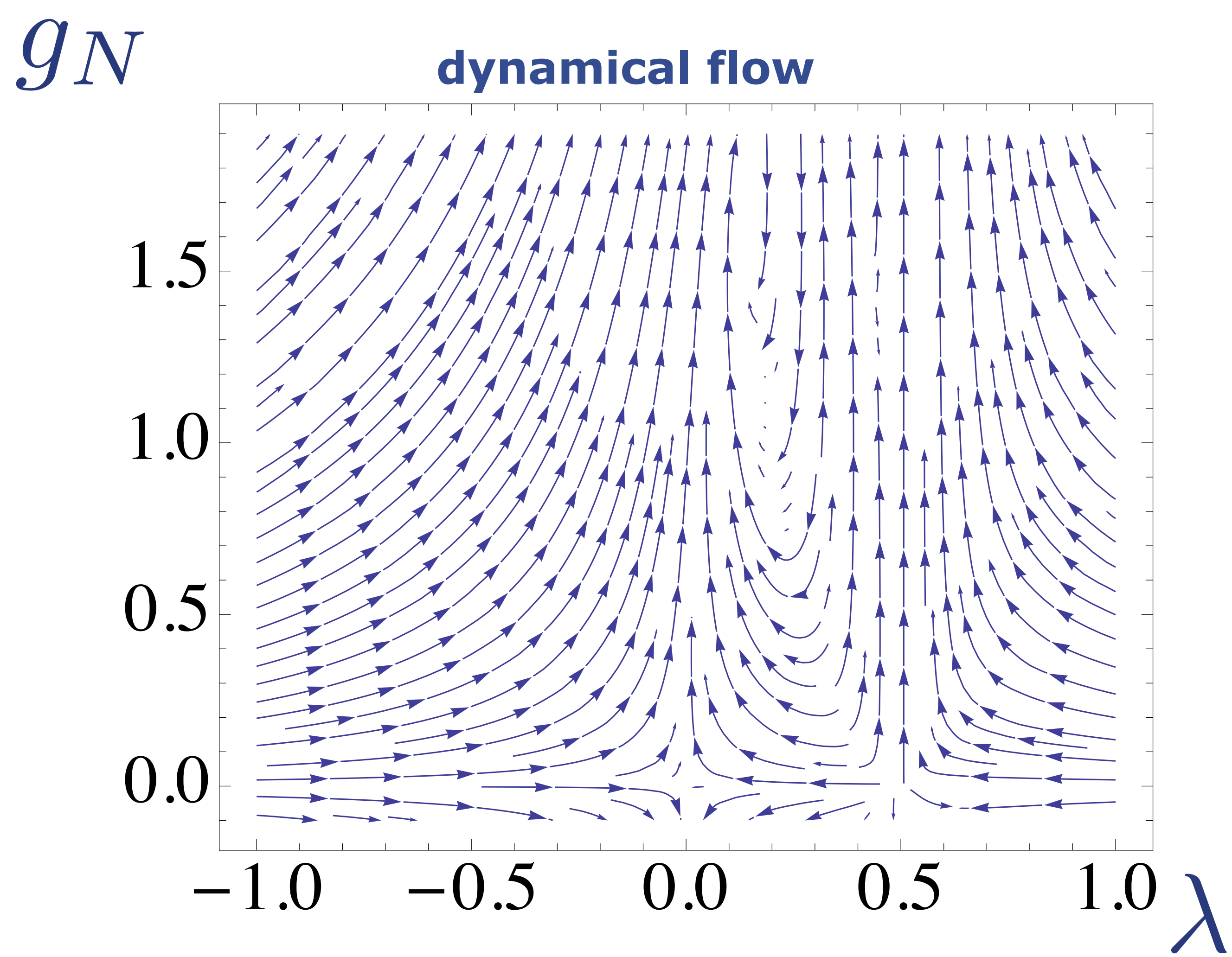


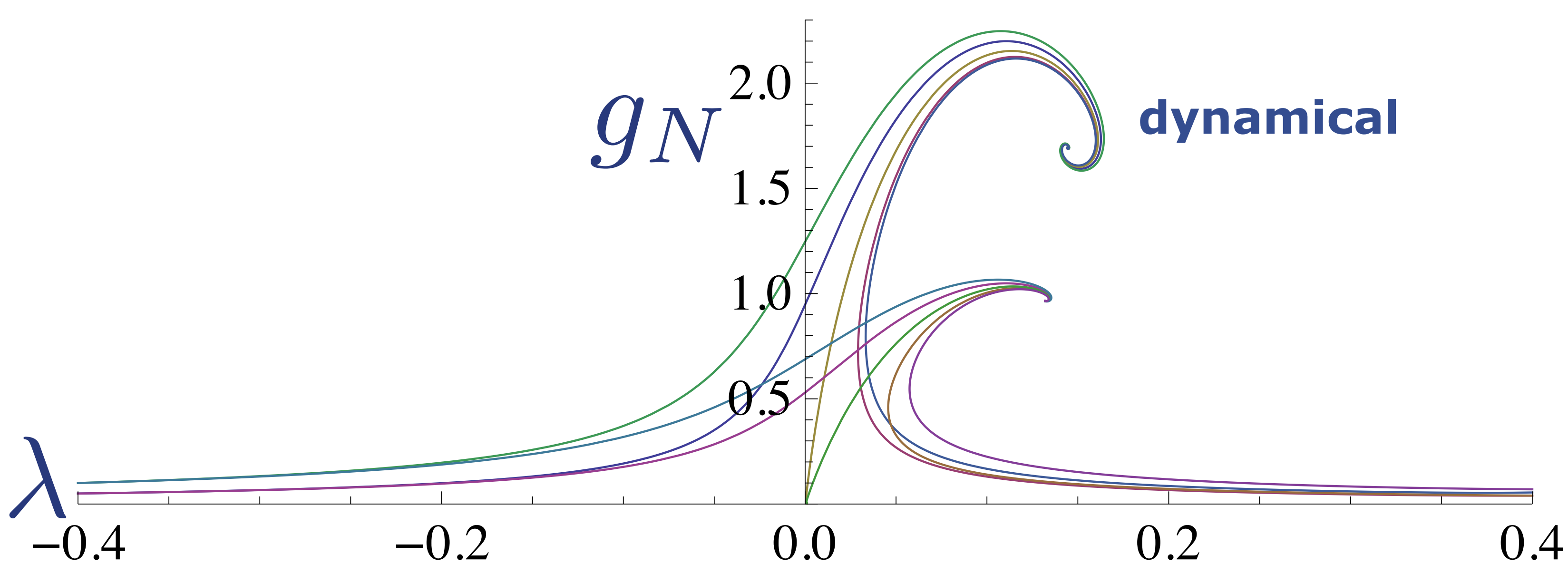


Phase diagram of quantum gravity

phase portraits







Phase diagram of quantum gravity

UV fixed point

Type of flow	g_{N*}	λ_*	$g_{N*} \times \lambda_*$
Background flow	0.893	0.164	0.146
Improved background flow	0.966	0.132	0.128
Dynamical flow	1.692	0.144	0.244
Bimetric flows	1.055	0.222	0.234

Flows	Stability matrix	Eigenvalues
Background	$\begin{pmatrix} -2.46 & -10.52 \\ 0.71 & -1.61 \end{pmatrix}$	$-2.03 + 2.69i$ $-2.03 - 2.69i$
Improved background	$\begin{pmatrix} -2.59 & -9.99 \\ 0.47 & -2.01 \end{pmatrix}$	$-2.30 - 2.16i$ $-2.30 - 2.16i$
Dynamical	$\begin{pmatrix} -1.94 & -27.9 \\ 0.26 & -0.74 \end{pmatrix}$	$-1.34 + 2.61i$ $-1.34 - 2.61i$

Phase diagram of quantum gravity

Donkin, JMP, arXiv:1203.4207

IR

Phase diagram of quantum gravity

IR stability

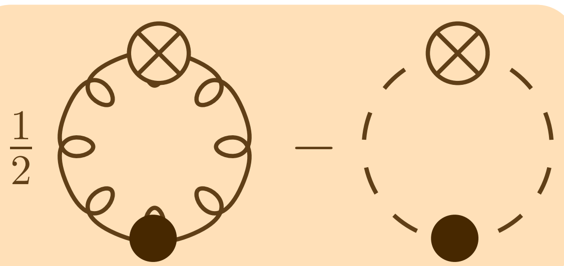
$\langle hh \rangle$

single poles in F_g & F_λ

- **propagators**

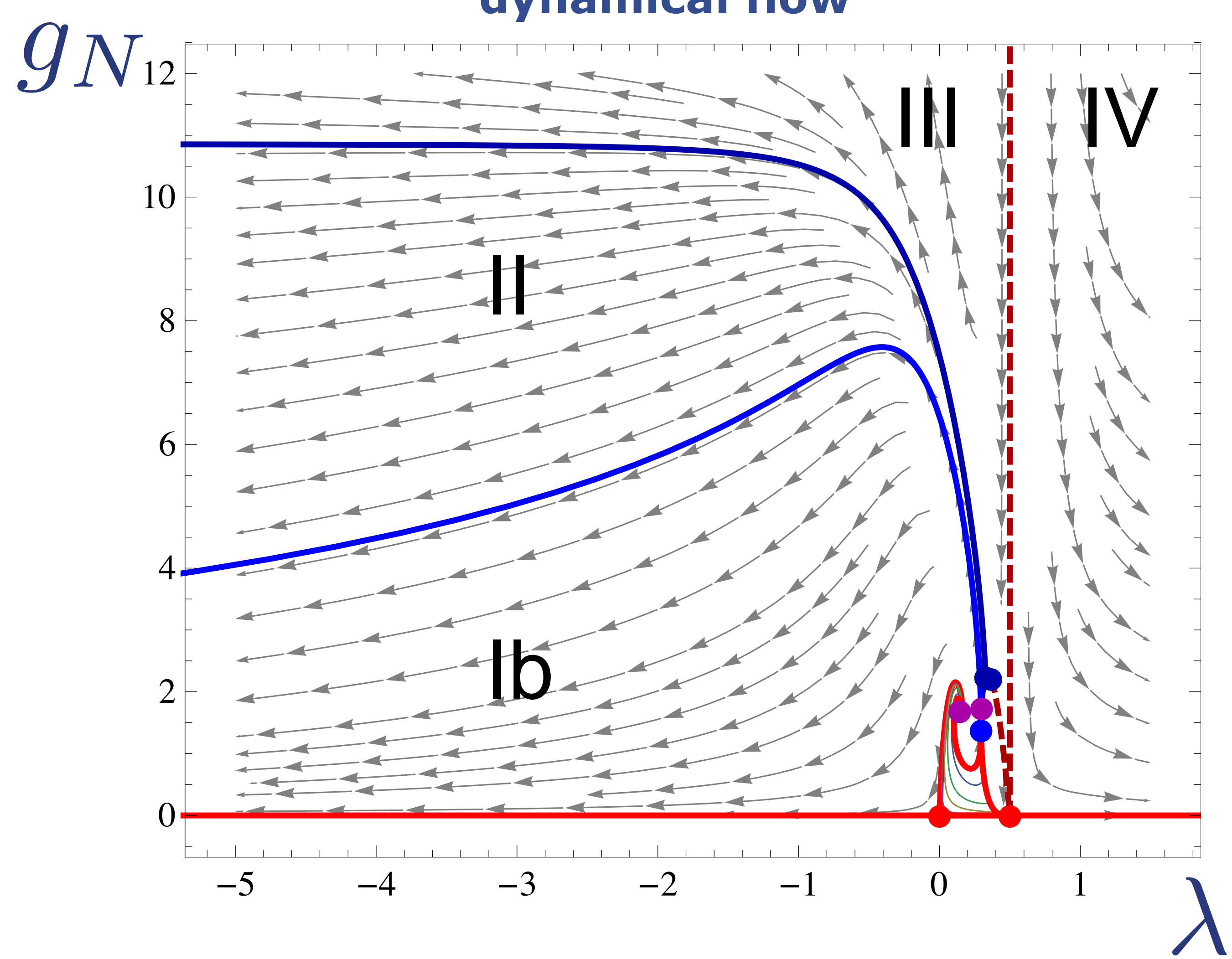
$$\frac{1}{1-2\lambda}$$

- **resummations**



$\partial_t \lambda$ **&** $\partial_t g_N$ **terms**

dynamical flow



General IR structure & background flows:
Litim, talk Bad Honnef, Feb 12
Contreras, Litim, in prep.

Phase diagram of quantum gravity

IR stability

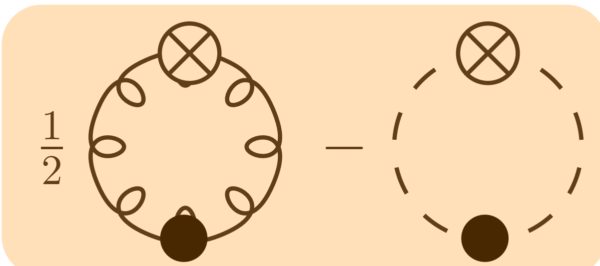
$\langle hh \rangle$

single poles in F_g & F_λ

- **propagators**

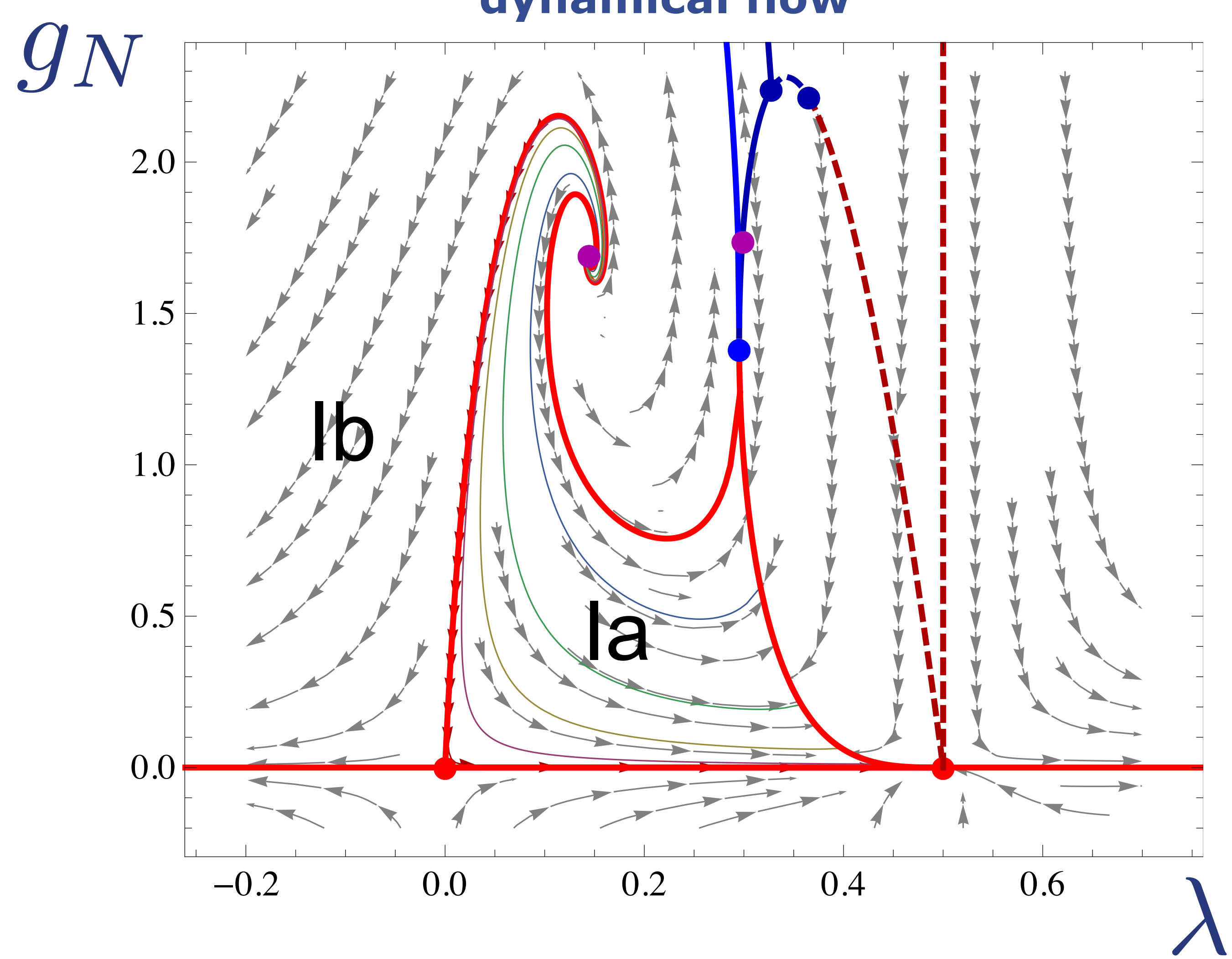
$$\frac{1}{1-2\lambda}$$

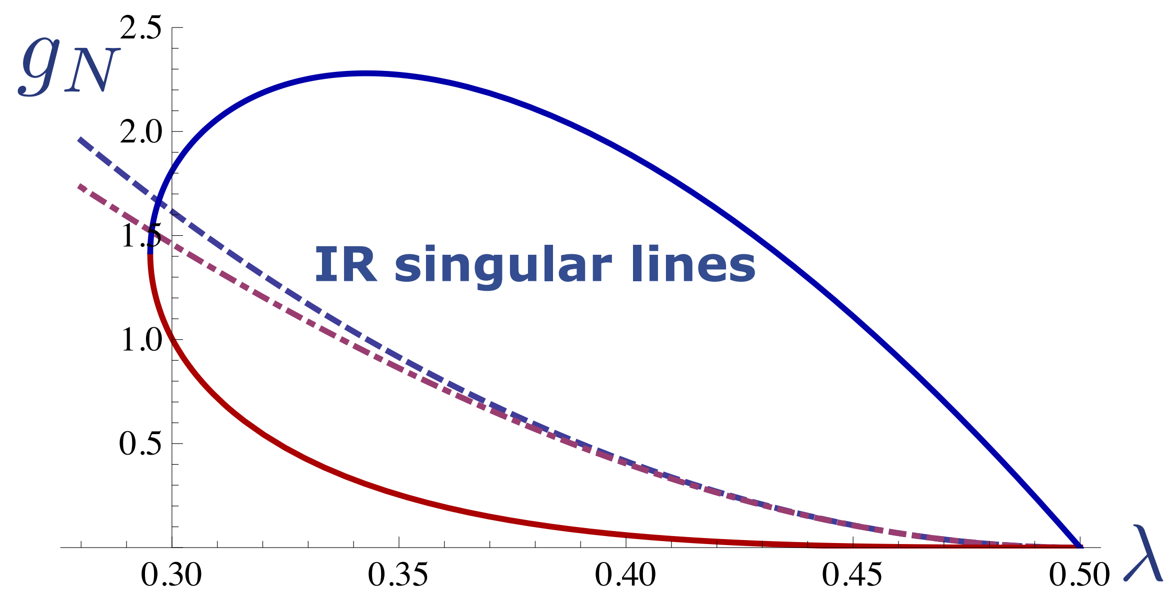
- **resummations**



$\partial_t \lambda$ & $\partial_t g_N$ terms

dynamical flow





standard background flow

geometrical background flow

Phase diagram of quantum gravity

IR stability

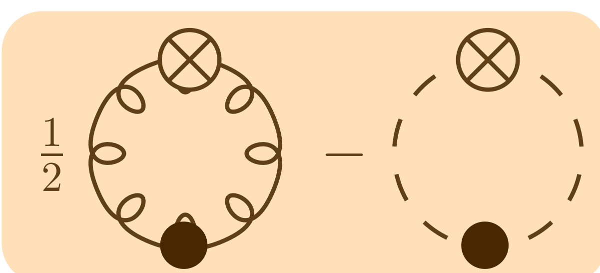
$\langle hh \rangle$

single poles in F_g & F_λ

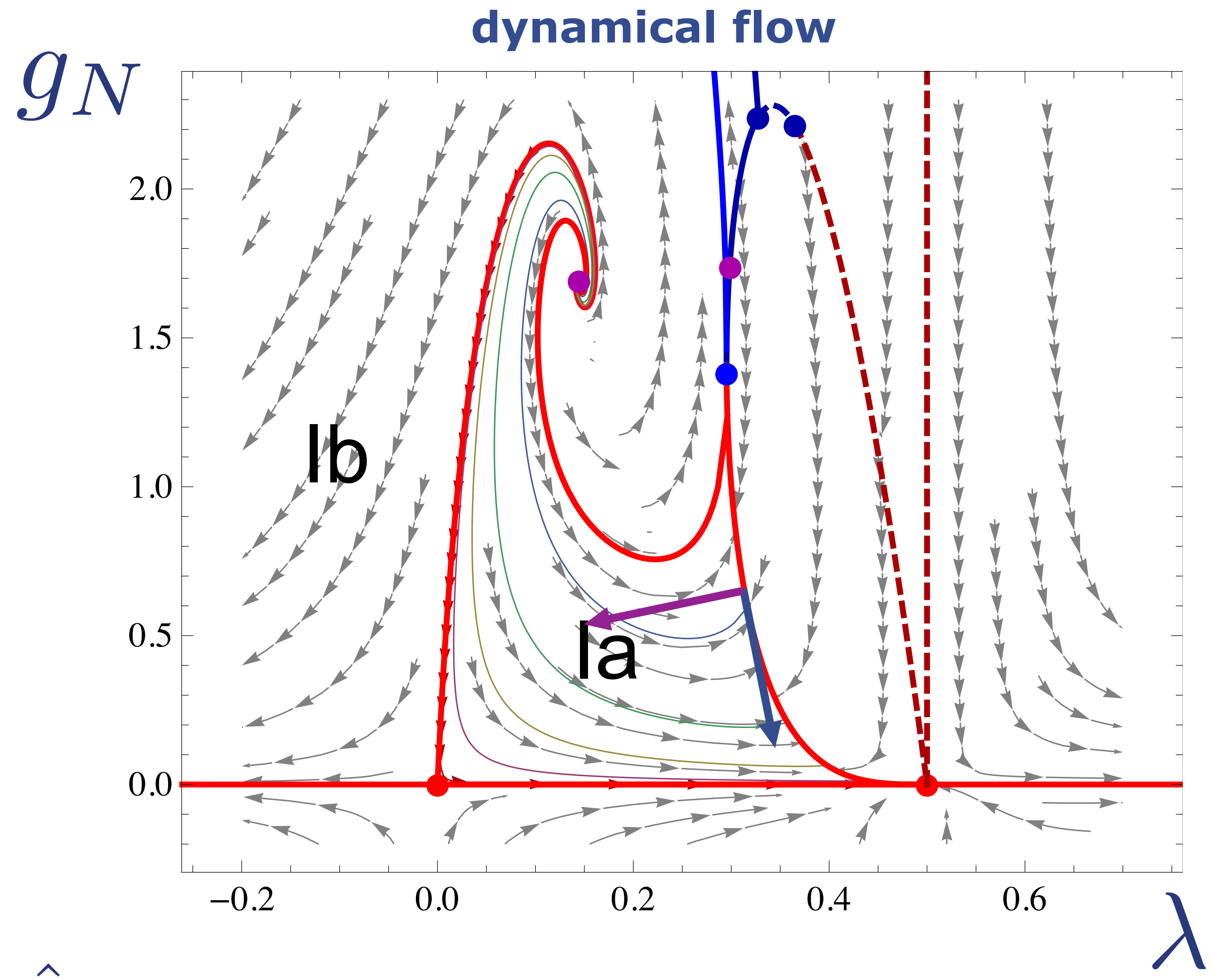
- **propagators**

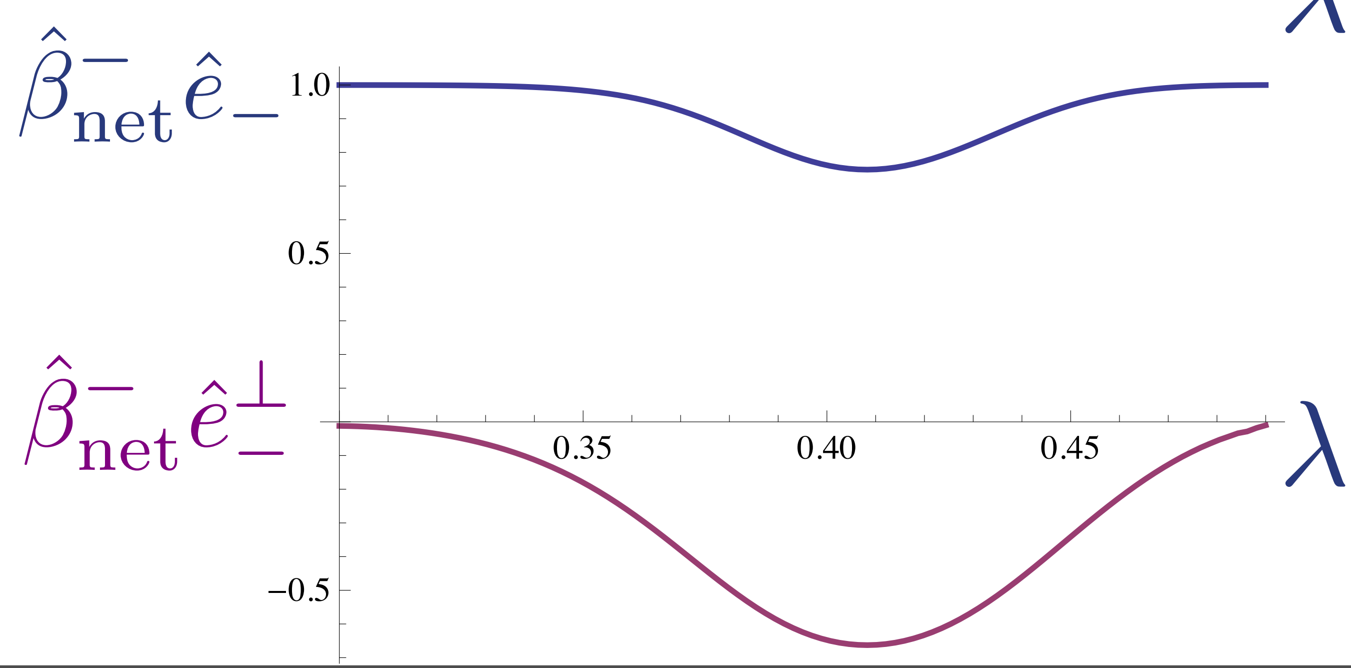
$$\frac{1}{1-2\lambda}$$

- **resummations**



$\partial_t \lambda$ & $\partial_t g_N$ terms





Summary & outlook

- **Phase diagram of quantum gravity**
  - **first results with fully diffeomorphism-invariant flows**
  - **further non-trivial confirmation of the UV fixed point scenario**
  - **IR-stability of quantum gravity**
- **Outlook**
  - **fully-coupled matter-gauge-gravity systems in the UV**
  - **long distance physics**