Running boundary actions, Asymptotic Safety, and black hole thermodynamics

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Outline

Boundary conditions in the FRG approach

- Effective average action $\Gamma_k[g, \bar{g}]$ depends on

$$g \in \{\text{metric of } \mathcal{M}\}$$
$$\bar{g} \in \{\text{metric of } \mathcal{M}\}$$

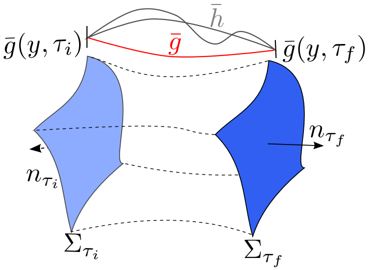
or equivalently on $\bar{g}$ and $\bar{h} = g - \bar{g}$, i.e. $\Gamma_k[\bar{h}; \bar{g}]$

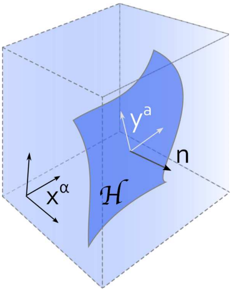
Boundary conditions in the FRG approach

- Effective average action $\Gamma_k[g, \bar{g}]$ depends on

$$g \in \{\text{metric of } \mathcal{M} \mid \quad \text{satisfies Dirichlet boundary conditions}\}$$
$$\bar{g} \in \{\text{metric of } \mathcal{M}\}$$

or equivalently on $\bar{g}$ and $\bar{h} = g - \bar{g}$, i.e. $\Gamma_k[\bar{h}; \bar{g}]$ with $\bar{h}_{\mu\nu}\big|_{\partial\mathcal{M}} = 0$.





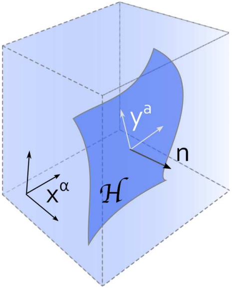
Hypersurface equation:

$$\Phi\big(x^\mu(y)\big) = 0$$

Decomposition (transverse /normal part):

- Tangential maps: $e^\alpha_a = \frac{\partial x^\alpha(y)}{\partial y^a}$
- Normal field: $n_\mu = \partial_\mu \Phi\big(x\big)$

$\Rightarrow$ Metric: $g_{\mu\nu} = n_\mu n_\nu + H_{\mu\nu}$
($H_{\mu\nu}$ transverse metric)



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Induced geometry on $\mathcal{H}$:

- Induced metric: $H_{ab} = g_{\alpha\beta}\, e^\alpha_a e^\beta_b = H_{\alpha\beta}\, e^\alpha_a e^\beta_b$
- **Extrinsic curvature**:

$$K_{ab} = \tfrac{1}{2}\left(\mathcal{L}_n g_{\alpha\beta}\right) e^\alpha_a e^\beta_b = \tfrac{1}{2}\left(D_\beta n_\alpha + D_\alpha n_\beta\right) e^\alpha_a e^\beta_b$$

- Trace extrinsic curvature: $K = H^{ab} K_{ab} = D_\alpha n^\alpha$

Ansatz for Γ_k in case of $\partial\mathcal{M} \neq 0$

- All basis invariants up to second order in ∂: [**Dirichlet** bdry. con.]

$$\Gamma_k^{\text{grav}}[g,\bar{g}] = -\frac{1}{16\pi G_k}\int_{\mathcal{M}} \mathrm{d}^d x\sqrt{g}\,(R - 2\Lambda_k) - \frac{1}{16\pi G_k^{\partial}}\int_{\partial\mathcal{M}} \mathrm{d}^{d-1} y\sqrt{H}\,\left(2K - 2\Lambda_k^{\partial}\right)$$

with $\bar{h}_{\mu\nu}\big|_{\partial\mathcal{M}} = 0 = (g_{\mu\nu} - \bar{g}_{\mu\nu})\big|_{\partial\mathcal{M}}$

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- Classical motivation: [Gibbons-Hawking]

$$S^{\text{grav}}[g] = -\frac{1}{16\pi G}\int_{\mathcal{M}} \mathrm{d}^d x\sqrt{g}\,R - \frac{1}{16\pi G^{\partial}}\int_{\partial\mathcal{M}} \mathrm{d}^{d-1} y\sqrt{H}\,2K$$

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$$\delta_g S^{\text{grav}}[g] = \frac{1}{16\pi G}\int_{\mathcal{M}} \mathrm{d}^d x\sqrt{g}\left[G^{\mu\nu} + D_\sigma\left(g^{\mu\nu}D^\sigma - g^{\nu\sigma}D^\mu\right)\right]\delta g_{\mu\nu} - \frac{1}{16\pi G^{\partial}}\int_{\partial\mathcal{M}} \mathrm{d}^{d-1}y\sqrt{H}\; H^{\alpha\beta}n^\lambda\partial_\lambda\,\delta g_{\alpha\beta}$$

Ansatz for Γ_k in case of $\partial\mathcal{M} \neq 0$

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$$\Gamma_k^{\text{grav}}[g,\bar{g}] = -\frac{1}{16\pi G_k}\int_{\mathcal{M}} \mathrm{d}^d x\sqrt{g}\,(R - 2\Lambda_k) - \frac{1}{16\pi G_k^\partial}\int_{\partial\mathcal{M}} \mathrm{d}^{d-1}y\sqrt{H}\,(2K - 2\Lambda_k^\partial)$$

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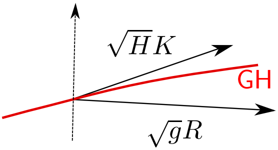


$$\delta_g S^{\text{grav}}[g] = \frac{1}{16\pi G}\int_{\mathcal{M}} \mathrm{d}^d x\sqrt{g}\,G^{\mu\nu}\,\delta g_{\mu\nu} + \frac{1}{16\pi}\left(\frac{1}{G} - \frac{1}{G^\partial}\right)\int_{\partial\mathcal{M}} \mathrm{d}^{d-1}y\sqrt{H}\,H^{\alpha\beta} n^\lambda \partial_\lambda\,\delta g_{\alpha\beta}$$

Our truncations

1. **Single-metric** truncation for full fledged QEG
   Investigation of
   - potential problems and conceptual issues related to the new setting
   - existence of a Non-Gaussian fixed point
   - presence of a '**Gibbons-Hawking** trajectory'

2. **Bi-metric** truncation – induced gravity approximation
   - scalar fields A coupled to gravity induce $\Gamma_k[A, g, \bar{g}]$
   - quantum fluctuations of gravity itself neglected

Single-Metric Truncation

The single-metric effective average action ansatz

- Purely **gravitational** part: $\delta g_{\mu\nu}|_{\partial\mathcal{M}} = 0$

$$\Gamma_k^{\text{grav}}[g] = -\frac{1}{16\pi G_k}\int_{\mathcal{M}} \mathrm{d}^d x\sqrt{g}\,(R - 2\Lambda_k) - \frac{1}{16\pi G_k^{\partial}}\int_{\partial\mathcal{M}} \mathrm{d}^{d-1} y\sqrt{H}\,(2K - 2\Lambda_k^{\partial})$$

- **Ghost** and **gauge fixing** part: $\Gamma_k^{\text{gf}}[\bar{g}, g]$, $\Gamma_k^{\text{gh}}[\xi, \bar{\xi}, \bar{g}, g]$

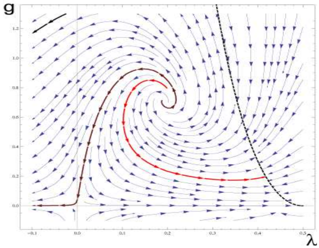
Functional renormalization group equation

$$k\partial_k\, \Gamma_k\big|_{g=\bar{g}} = +\tfrac{1}{2}\mathrm{STr}\left[\left(\Gamma_k^{(2)} + R_k\right)^{-1}\Big|_{g=\bar{g}} k\partial_k R_k\right]$$

Results: single-metric truncation

- Existence of a Non-Gaussian fixed point:

Hierarchy	NG-FP
g_k	0.707
λ_k	0.193
$\Downarrow$	
g_k^∂	-2.29
$\Downarrow$	
λ_k^∂	1.201



- Newton type couplings g_k, g_k^∂ $(d = 4$, with $g \lesssim 3$, $\lambda < 1/2)$

$$\eta_{\mathrm{N}} = -\,\alpha_0\, g_k\,, \qquad \alpha_0(g_k, \lambda_k) > 0$$

$$\eta_{\mathrm{N}}^\partial = +\left(\alpha_1^\partial - \eta_{\mathrm{N}}\,\alpha_2^\partial\right) g_k^\partial\,, \qquad \alpha_{1,2}^\partial(g_k, \lambda_k) > 0$$

Bi-metric matter induced truncation

The bi-metric $(\bar{h}, \bar{g})$ matter induced ansatz

$$\Gamma_k[\bar{h}, A; \bar{g}] = \Gamma_k^{\mathsf{B}}[A; \bar{g}] + \Gamma_k^{\mathsf{lin}}[\bar{h}, A; \bar{g}]\,, \quad \textbf{with} \quad \bar{h}_{\mu\nu}\big|_{\partial\mathcal{M}} = 0 = \delta A|_{\partial\mathcal{M}}$$

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The level-(0) part (Background part)

$$\begin{aligned}
\Gamma_k^{\mathsf{B}}[A; \bar{g}] = &-\frac{1}{16\pi G_k^{(0)}} \int_{\mathcal{M}} \mathsf{d}^d x \sqrt{\bar{g}}\, \left(\bar{R} - 2\Lambda_k^{(0)}\right) \\
&- \frac{1}{16\pi G_k^{(0,\partial)}} \int_{\partial\mathcal{M}} \mathsf{d}^{d-1} x \sqrt{\bar{H}}\, \left(2\bar{K} - 2\Lambda_k^{(0,\partial)}\right) \\
&+ \int_{\mathcal{M}} \mathsf{d}^d x \sqrt{\bar{g}} \left\{ \frac{1}{2}\, \bar{g}^{\mu\nu} \partial_\mu A \partial_\nu A + \frac{1}{2} \xi_k^{(0)}\, \bar{R} A^2 + V_k^{(0)}(A) \right\} \\
&+ \int_{\partial\mathcal{M}} \mathsf{d}^{d-1} x \sqrt{\bar{H}}\, \xi_k^{(0,\partial)}\, \bar{K} A^2
\end{aligned}$$

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The level-(1) part (Linear part)

$$\Gamma_k^{\mathsf{lin}}[\bar{h}, A; \bar{g}] = \frac{1}{16\pi G_k^{(1)}} \int_{\mathcal{M}} \sqrt{\bar{g}}\; \mathcal{E}_k^{\mu\nu}[\bar{g}, A]\; \bar{h}_{\mu\nu} + \int_{\partial\mathcal{M}} \sqrt{\bar{H}}\; c_k^{\partial\mathcal{M}} n^\lambda \partial_\lambda \bar{h}^\mu{}_\mu$$

with

$$\mathcal{E}_k^{\mu\nu}[\bar{g}, A] \equiv \bar{G}^{\mu\nu} - \tfrac{1}{2}\, E_k\, \bar{g}^{\mu\nu} \bar{R} + \Lambda_k^{(1)}\, \bar{g}^{\mu\nu} - 8\pi G_k^{(1)} \mathcal{T}_k^{\mu\nu}[A; \bar{g}]$$

and an **energy-momentum tensor**

$$\mathcal{T}_k^{\mu\nu}[A; \bar{g}] \equiv \left(\bar{g}^{\mu\rho}\bar{g}^{\nu\rho} - \tfrac{1}{2}\, \bar{g}^{\mu\nu}\bar{g}^{\rho\sigma}\right) \partial_\rho A \partial_\sigma A - \bar{g}^{\mu\nu} V_k^{(1)}(A) - \tfrac{1}{2}\, \bar{g}^{\mu\nu} \xi_k^{(1,\mathsf{I})}\, \bar{R} A^2 + \xi_k^{(1,\mathsf{II})} \left\{ \bar{g}^{\mu\nu} \bar{D}^2(A^2) - \bar{D}^\mu \bar{D}^\nu (A^2) + \bar{R}^{\mu\nu} A^2 \right\}$$

The bi-metric $(\bar{h}, \bar{g})$ matter induced ansatz

$$\Gamma_k[\bar{h}, A; \bar{g}] = \Gamma_k^{\mathsf{B}}[A; \bar{g}] + \Gamma_k^{\mathsf{lin}}[\bar{h}, A; \bar{g}] \,, \quad \textbf{with} \;\; \bar{h}_{\mu\nu}\big|_{\partial\mathcal{M}} = 0 = \delta A|_{\partial\mathcal{M}}$$

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$$\Gamma_k^{\mathsf{lin}}[\bar{h}, A; \bar{g}] = \frac{1}{16\pi G_k^{(1)}} \int_{\mathcal{M}} \sqrt{\bar{g}}\; \mathcal{E}_k^{\mu\nu}[\bar{g}, A]\; \bar{h}_{\mu\nu} + \int_{\partial\mathcal{M}} \sqrt{\bar{H}}\; c_k^{\partial\mathcal{M}} n^\lambda \partial_\lambda \bar{h}^\mu{}_\mu$$

and the **boundary coefficient**:

$$c_k^{\partial\mathcal{M}} \equiv \frac{1}{16\pi} \left(\frac{1}{G_k^{(1)}} - \frac{1}{G_k^{(1,\partial)}} \right) - \frac{1}{2} \left(\xi_k^{(1,\mathsf{II})} - \xi_k^{(1,\partial)} \right) A^2$$

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Inspiration for this structure: split symmetry

$$\xi_k^{(0)} = \xi_k^{(1,\mathsf{I})} = \xi_k^{(1,\mathsf{II})}\,, \qquad \xi_k^{(1,\partial)} = \xi_k^{(0,\partial)}\,,$$

$$g_k^{(0)} = g_k^{(1)}\,, \qquad g_k^{(0,\partial)} = g_k^{(1,\partial)}\,,$$

$$\Lambda_k^{(0)} = \Lambda_k^{(1)}\,, \qquad V_k^{(0)} = V_k^{(1)}\,, \qquad E_k = 0$$

Split symmetry **intact** $\Rightarrow$

$$\Gamma_k^{\mathsf{B}}[A; \bar{g}] + \Gamma_k^{\mathsf{lin}}[\bar{h}, A; \bar{g}] \equiv \Gamma_k^{\mathsf{B}}[A; \bar{g} + \bar{h}]$$

Results

Bi-metric truncation

Split symmetry

- **Split symmetry** stable under RG flow if

$$\partial_t \xi_k^{(0)} = \partial_t \xi_k^{(1,\mathrm{I})} = \partial_t \xi_k^{(1,\mathrm{II})}\,, \qquad \partial_t \xi_k^{(1,\partial)} = \partial_t \xi_k^{(0,\partial)}\,,$$

$$\partial_t g_k^{(0)} = \partial_t g_k^{(1)}\,, \qquad \partial_t g_k^{(0,\partial)} = \partial_t g_k^{(1,\partial)}$$

Split symmetry

- **Split symmetry** explicitly **violated** by RG flow

$$\partial_t \xi_k^{(0)} \neq \partial_t \xi_k^{(1,\mathrm{I})} \neq \partial_t \xi_k^{(1,\mathrm{II})}\,, \qquad \partial_t \xi_k^{(1,\partial)} \neq \partial_t \xi_k^{(0,\partial)}\,,$$

$$\partial_t g_k^{(0)} \neq \partial_t g_k^{(1)}\,, \qquad \partial_t g_k^{(0,\partial)} \neq \partial_t g_k^{(1,\partial)} \quad \text{(in general)}$$

- $\partial_t g_k^{(0)} = \partial_t g_k^{(1)}$ **requires** $\xi_k^{(0)} > \xi_k^{(1,\mathrm{II})}$

$\Rightarrow$ bi-metric structure of Γ_k must be retained in the ansatz

Existence of a **Non-Gaussian** fixed point

- Matter couplings (trivial values)

$$m_*^{(0)} = 0 = m_*^{(1)} \qquad u_*^{(0)} = 0 = u_*^{(1)}$$

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$$\xi_*^{(0,\partial)} = \text{arbitrary}\,, \qquad \xi_*^{(1,\partial)} = \text{arbitrary}\,,$$

$$\xi_*^{(0)} = \text{arbitrary} \neq \frac{1}{6}\,\frac{\Phi_1^1(0)}{\Phi_2^2(0)}\,, \quad \xi_*^{(1,\mathrm{II})} = \text{arbitrary} \neq \frac{1}{6}\,, \quad \xi_*^{(1,\mathrm{I})} = \text{arbitrary}\,,$$

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- Newton type couplings

$$g_*^{(0,\partial)} = -\frac{12\pi}{n_\mathsf{s}\Phi_1^1(0)}\,, \qquad g_*^{(0)} = -\frac{12\pi}{n_\mathsf{s}\left(\Phi_1^1(0) - 6\Phi_2^2(0)\,\xi_*^{(0)}\right)}\,,$$

$$g_*^{(1,\partial)} = +\frac{24\pi}{n_\mathsf{s}\Phi_2^2(0)}\,, \qquad g_*^{(1)} = +\frac{24\pi}{n_\mathsf{s}\left(2 - 12\,\xi_*^{(1,\mathrm{II})}\right)\Phi_2^2(0)}\,,$$

Level-(0) Newton couplings

Bi-metric matter induced | **Single-metric** $(g \lesssim 3, \lambda < 1/2)$

$$\eta^{(0)} = +2\,\gamma_4\,\Phi_1^1 \left(1 - 6\,\xi^{(0)}\,\frac{\Phi_2^2}{\Phi_1^1}\right) g_k^{(0)} \quad \| \quad \eta_{\mathrm{N}} = -\alpha_0\, g_k$$

$$\eta^{(0,\partial)} = +2\,\gamma_4\,\Phi_1^1\, g_k^{(0,\partial)} \quad \| \quad \eta_{\mathrm{N}}^{\partial} = +\left(\alpha_1^{\partial} + \alpha_3^{\partial}\, g_k\right) g_k^{\partial}$$

Comparison

- **For** $g,\ g^{(0)} > 0,\ \xi^{(0)} \gtrsim 1/3$: $\eta^{(0)},\ \eta_N < 0$
- **For** $g > 0,\ g^{\partial} > 0\ (< 0)$: $\eta^{(0,\partial)},\ \eta_N^{\partial} > 0\ (< 0)$

Bulk-Boundary Matching

$$\cdots + \int_{\partial\mathcal{M}} \sqrt{\bar{H}}\, c_k^{\partial\mathcal{M}}\, n^\lambda \partial_\lambda \delta \bar{h}^\mu{}_\mu \overset{!}{=} 0$$

❶ Require $c_k^{\partial\mathcal{M}} \equiv \frac{1}{16\pi}\left(\frac{1}{G_k^{(1)}} - \frac{1}{G_k^{(1,\partial)}}\right) - \frac{1}{2}\left(\xi_k^{(1,\mathrm{II})} - \xi_k^{(1,\partial)}\right) A^2 \overset{!}{=} 0$:

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   - Newton type couplings

$$\eta^{(1)} = +\gamma\left(2 - 12\,\xi^{(1,\mathrm{II})}\right)\Phi_2^2 \cdot g^{(1)} \qquad , \gamma > 0$$

$$\eta^{(1,\partial)} = -\gamma\left(\Phi_2^2 + 3m^{(1)\,2}\,\Phi_1^2\right) \cdot g^{(1,\partial)}$$

   - Non-minimal parameters

$$\xi_k^{(1,\mathrm{II})} = \left(\xi_{k_0}^{(1,\mathrm{II})} - \frac{1}{6}\right)\left(\frac{k}{k_0}\right)^{\alpha u^{(0)}} + \frac{1}{6} \qquad , \alpha > 0$$

$$\xi_k^{(1,\partial)} = \mathrm{const} \cdot u^{(0/1)} \ln\left(\frac{k}{k_0}\right) + \xi_{k_0}^{(1,\partial)}$$

Bulk-Boundary Matching

$$\cdots + \int_{\partial\mathcal{M}} \sqrt{\bar{H}}\, c_k^{\partial\mathcal{M}}\, n^\lambda \partial_\lambda \delta\bar{h}^\mu{}_\mu \stackrel{!}{=} 0$$

1. Require $c_k^{\partial\mathcal{M}} = 0$: satisfied at certain **points** in theory space
   - At only one scale $k = k_0$ (physical scale $k_0 = 0$)
   - At the **fixed point**: $\xi_*^{(0)} = 0 = \xi_*^{(0,\partial)}$ and $\xi_*^{(1,\mathrm{II})} = 1/12 = \xi_*^{(1,\partial)}$

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2. Require $n^\lambda \partial_\lambda \delta\bar{h}^\mu{}_\mu \overset{!}{=} 0$
   - $\bar{h}_{\mu\nu} \in \mathcal{F} \equiv \{f_{\mu\nu} \text{ tensor on } \mathcal{M} \,|\, f_{\mu\nu} = 0 \text{ on } \partial\mathcal{M}\}$
   - $\delta\bar{h}_{\mu\nu} \in \mathcal{F} \equiv \{f_{\mu\nu} \text{ tensor on } \mathcal{M} \,|\, f_{\mu\nu} = 0 \text{ on } \partial\mathcal{M}\}$

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   - $\delta\bar{h}_{\mu\nu} \in \mathcal{F}' \equiv \left\{f_{\mu\nu} \text{ tensor on } \mathcal{M} \,|\, f_{\mu\nu} = 0 \text{ and } \bar{D}_n f_{\mu\nu} = 0 \text{ on } \partial\mathcal{M}\right\}$

Motivation

- Self-consistent backgrounds; $\bar{h}_{\mu\nu} = 0$ on *entire* $\mathcal{M}$, not 'lost' when $\delta\bar{h} \in \mathcal{F}'$

$$\delta_{\bar{h}}\Gamma_k|_{\bar{h}=0}\,[\bar{g}_k^{\text{self-con}}] = 0 = \delta_{\bar{h}}\Gamma_k^{\text{lin}}[\bar{g}_k^{\text{self-con}}]$$

- **Here**: $\Gamma_k = B^0 + B^1\bar{h} \Rightarrow \bar{h}_{\mu\nu}$ auxilliary field enforcing $B^1[\bar{g}] = 0$

Summary of the results

- **Split symmetry** broken: Γ_k functional of $\bar{g}$ and $\bar{h}$ separately
  (Bi-metric truncation is compulsory)
- NG fixed point exists in both truncation
- $\eta^{(0)},\ \eta_N\ < 0$ and $\eta^{(0,\partial)},\ \eta_N^{\partial} > 0$
  For $\xi^{(0)} \gtrsim 1/3,\ g \in (0,3),\ \lambda < 1/2\,,\ g^{\partial} \in (0,\infty)$
- Well posed **variational principle**: $\int_{\partial\mathcal{M}} \sqrt{\bar{H}}\, c_k^{\partial\mathcal{M}}\, n^{\lambda}\partial_{\lambda}\bar{h}^{\mu}{}_{\mu} \stackrel{!}{=} 0$
  - We found: $c_k^{\partial\mathcal{M}} \neq 0$
  - $\bar{h}_{\mu\nu}\big|_{\partial\mathcal{M}} = 0$ and $n^{\lambda}\partial_{\lambda}\delta\bar{h}^{\mu}{}_{\mu} = 0$ required

Running on-shell actions, black hole thermodynamics

Running on-shell actions

Exact functional integro-differential equation: | $\widetilde{S} = S + S_{\mathsf{gf}} + S_{\mathsf{gh}}$

$$e^{-\Gamma_k[\Phi;\bar{g}]} = \int \mathcal{D}\hat{\Phi} \, \exp\left[-\widetilde{S}[\hat{\Phi}] + \int \mathsf{d}^d x \, \left(\hat{\Phi}_a - \Phi_a\right) \frac{\delta}{\delta \Phi_a} \Gamma_k[\Phi]\right] e^{-\Delta_k S[\hat{\Phi}-\Phi]}$$

quantum fields $\hat{\Phi} = \{\hat{h}, \hat{A}, \hat{\xi}^\mu, \hat{\bar{\xi}}_\mu\}$, expectation values $\Phi = \{\bar{h}, A, \xi^\mu, \bar{\xi}_\mu\}$

Running on-shell actions

Exact functional integro-differential equation: | $\widetilde{S} = S + S_{\text{gf}} + S_{\text{gh}}$

$$e^{-\Gamma_k[\Phi;\bar{g}]} = \int \mathcal{D}\hat{\Phi}\, \exp\left[-\widetilde{S}[\hat{\Phi}] + \int \mathrm{d}^d x\, \left(\hat{\Phi}_a - \Phi_a\right) \frac{\delta}{\delta \Phi_a} \Gamma_k[\Phi]\right] e^{-\Delta_k S[\hat{\Phi}-\Phi]}$$

quantum fields $\hat{\Phi} = \{\hat{h}, \hat{A}, \hat{\xi}^\mu, \hat{\bar{\xi}}_\mu\}$, expectation values $\Phi = \{\bar{h}, A, \xi^\mu, \bar{\xi}_\mu\}$

On-shell: $\Phi(x) \equiv \Phi_k^{\text{SP}}[\bar{g}](x)$ | running stationary point

$$e^{-\Gamma_k[\Phi_k^{\text{SP}};\bar{g}]} = \int \mathcal{D}\hat{\Phi}\, \exp\left[-\widetilde{S}[\hat{\Phi}] - \Delta_k S[\hat{\Phi} - \Phi_k^{\text{SP}}]\right]$$

Special case: $\bar{g}_k^{\text{selfcon}} \in \mathcal{F}_{\text{selfcon}} \equiv \{\bar{g}_{\mu\nu} \text{ metric } \mid \bar{h}[\bar{g}] = 0\}$

$\mathbb{Z}_k \equiv e^{-\Gamma_k[0;\bar{g}_k^{\text{selfcon}}]}$ | only level-(0) contributions

Quantum statistical interpretation

If $\bar{g}_{\mu\nu}$ self-consistent and β-periodic in (eucl. !) time:

$$\mathbb{Z}_k[\bar{g}] = \int \mathcal{D}\hat{\Phi}\, \exp\left[-S[\hat{\Phi};\bar{g}] - \Delta_k S[\hat{\Phi};\bar{g}]\right] \hat{=} \mathrm{Tr}\left[e^{-\beta H}\right] \equiv Z^{\mathrm{Stat.}}(\beta)$$

- **Thermodynamical quantities** | **Temperature** $T = 1/\beta$

Free energy

$$F_k = -\beta^{-1} \ln \mathbb{Z}_k$$

Entropy

$$S_k = \beta^2 \partial_\beta F_k$$

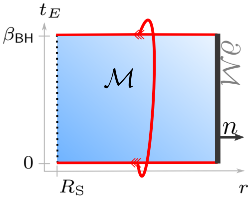
Internal energy

$$U_k = \partial_\beta (\beta F_k)$$

Specific heat capacity

$$C_k = -\beta^2 \partial_\beta U_k$$

The Euclidean Schwarzschild black hole



Solution of field equation $\Lambda^{(0,1)} = 0 = \Lambda^{(0,1,\partial)}$:

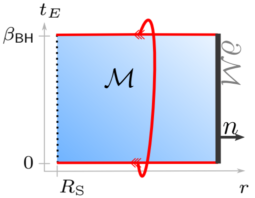
$$G^{\mu\nu}(\bar{g}_{\text{S}}) = 0 \quad \Leftrightarrow \quad R(\bar{g}_{\text{S}}) = 0$$

Euclidean Schwarzschild geometry $(d = 4)$

$$\mathrm{d}s^2 = + \left(1 - \frac{R_{\text{S}}}{r}\right) \mathrm{d}t_E^2 + \left(1 - \frac{R_{\text{S}}}{r}\right)^{-1} \mathrm{d}r^2 + r^2 \mathrm{d}\Omega^2$$

with $t_E \in [0, \beta_{\text{BH}}]$, and $r \in (R_{\text{S}}, \infty)$

The Euclidean Schwarzschild black hole



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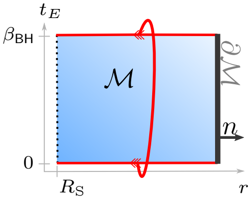
with $t_E \in [0, 4\pi R_{\text{S}}]$, and $r \in (R_{\text{S}}, \infty)$

- $\bar{g}_{\text{S}}$ has period $\beta_{\text{BH}} = 4\pi R_{\text{S}}$ | Kruskal coordinates

$$v = \sqrt{(r/R_{\text{S}} - 1)}\, e^{r/2R_{\text{S}}} \sin\left(\frac{2\pi t_E}{4\pi R_{\text{S}}}\right)$$

$$u = \sqrt{(r/R_{\text{S}} - 1)}\, e^{r/2R_{\text{S}}} \cos\left(\frac{2\pi t_E}{4\pi R_{\text{S}}}\right)$$

The Euclidean Schwarzschild black hole



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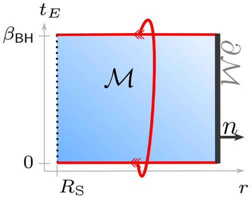
with $t_E \in [0, 4\pi R_S]$, and $r \in (R_S, \infty)$

- **Thus**, Γ_k evaluated at $\bar{g}_S$

$$-\ln \mathbb{Z}_k[\bar{g}_S] = \Gamma_k[0; \bar{g}_S] = -\frac{1}{8\pi G_k^{(0,\partial)}} \int_{\partial\mathcal{M}} \mathrm{d}^3x \sqrt{\bar{H}_S}\, \bar{K}(\bar{g}_S)$$

with $\partial\mathcal{M} = [0, \beta_{\text{BH}}] \times S^2_\infty$

The Euclidean Schwarzschild black hole



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with $t_E \in [0, 4\pi R_{\rm S}]$, and $r \in (R_{\rm S}, \infty)$

- **Thus**, Γ_k evaluated at $\bar{g}_{\rm S}$

$$-\ln \mathbb{Z}_k[\bar{g}_{\rm S}] = \Gamma_k[0; \bar{g}_{\rm S}] = \frac{\beta_{\rm BH} R_{\rm S}}{4G_k^{(0,\partial)}} = \frac{\mathcal{A}}{4G_k^{(0,\partial)}}$$

with $\mathcal{A} \equiv 4\pi R_{\rm S}^2$

The Euclidean Schwarzschild black hole $\beta_{\text{BH}} = 4\pi R_{\text{S}}$

$$-\ln \mathbb{Z}_k[\bar{g}_{\text{S}}] = \frac{\beta_{\text{BH}}^2}{16\pi G_k^{(0,\partial)}} = \frac{\mathcal{A}}{4G_k^{(0,\partial)}} \quad \Big| \quad \frac{1}{G_k^{(0,\partial)}} = \frac{1}{G_0^{(0,\partial)}} - |\omega_4^{(0,\partial)}| k^2$$

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The Euclidean Schwarzschild black hole $\beta_{\mathrm{BH}} = 4\pi R_{\mathrm{S}}$

$$-\ln \mathbb{Z}_k[\bar{g}_{\mathrm{S}}] = \frac{\beta_{\mathrm{BH}}^2}{16\pi G_k^{(0,\partial)}} = \frac{\mathcal{A}}{4G_k^{(0,\partial)}} \quad \Big| \quad \frac{1}{G_k^{(0,\partial)}} = \frac{1}{G_0^{(0,\partial)}} - |\omega_4^{(0,\partial)}| k^2$$

with $\mathcal{A} \equiv 4\pi R_{\mathrm{S}}^2$

Thermodynamical quantities | Temperature $T = 1/4\pi R_{\mathrm{S}}$

Free energy

$$F_k = \frac{1}{16\pi G_k^{(0,\partial)}\, T}$$

Entropy

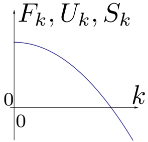
$$S_k = \frac{1}{16\pi G_k^{(0,\partial)}\, T^2}$$

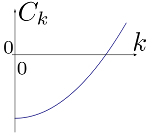
Internal energy

$$U_k = 2F_k$$

Specific heat capacity

$$C_k = -\frac{1}{8\pi G_k^{(0,\partial)}\, T^2}$$





The Euclidean Schwarzschild black hole $\quad \beta_{\text{BH}} = 4\pi R_{\text{S}}$

$$-\ln \mathbb{Z}_k[\bar{g}_{\text{S}}] = \frac{\beta_{\text{BH}}^2}{16\pi G_k^{(0,\partial)}} = \frac{\mathcal{A}}{4G_k^{(0,\partial)}} \quad \Bigg| \quad \frac{1}{G_k^{(0,\partial)}} = \frac{1}{G_0^{(0,\partial)}} - |\omega_4^{(0,\partial)}| k^2$$

with $\mathcal{A} \equiv 4\pi R_{\text{S}}^2$

Thermodynamical quantities | Temperature $\quad T = 1/4\pi R_{\text{S}}$

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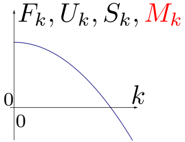
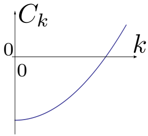




Running ADM mass M_k

$$M_k \equiv -\int_{S^2_\infty} \frac{(K - K_0)}{8\pi G_k^{(0,\partial)}} = \frac{R_{\text{S}}}{2G_k^{(0,\partial)}}$$

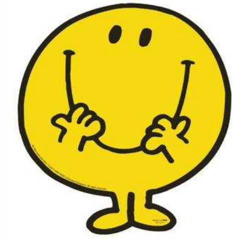
Conclusion and Outlook

1. Truncations
   - Bi-metric truncation compulsory
   - Existence of a Non-Gaussian fixed point
   - **No** 'Gibbons-Hawking' trajectory
   - Opposite running of $G_k^{(0)}$ and $G_k^{(0,\partial)}$
2. Application: black hole thermodynamics
   - Running on-shell action: $\mathbb{Z}_k \equiv e^{-\Gamma_k[0;\bar{g}_k^{\text{selfcon}}]}$
   - Running ADM mass for BH: $M_k \equiv \frac{R_S}{2G_k^{(0,\partial)}}$

Outlook

- 'Improve' the RG-improved Black Holes
  A. Bonanno, M. Reuter (1999,2000,2006)
  M. Reuter, E. Tuiran (2006,2011)
- Employ surface terms in 'multi-scale Riemannian structure' approach
  M. Reuter, J. Schwindt (2006,2007)

The End



‘Have a very happy day’

Mr Happy | Roger Hargreaves