Functional Renormalization with fermions and tetrads

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Functional Renormalization with fermions and tetrads,
P.D., R. Percacci - arXiv:1209.3649v2

Overview of the talk

1. Scheme dependence and fermions
   - A puzzling ambiguity and its solution
   - General discussion
2. Constraints on matter content
   - One loop results
3. FRGE and tetrad gravity
   - Type Ia cutoff
   - Type Ib cutoff
   - Type IIa cutoff
   - Type IIb cutoff
4. Discussion

A puzzling ambiguity

The modified effective action for a fermion field is:

$$\Gamma = -\mathrm{Tr}\log\left(\not{D}\right) = -\frac{1}{2}\mathrm{Tr}\log\left(\not{D}^2\right) \longrightarrow \Gamma_k = -\frac{1}{2}\mathrm{Tr}\log\left(-\nabla^2 + \frac{R}{4} + R_k\right)$$

We will consider:

$$\text{Type I: } R_k\left(-\nabla^2\right) \qquad \text{or} \qquad \text{Type II: } R_k\left(-\nabla^2 + \tfrac{R}{4}\right)$$

In the literature[1]:

$$\text{Type I}: \frac{d\Gamma_k}{dt} = -\frac{1}{8\pi^2}\int d^dx\,\sqrt{g}\left[Q_2\left(\frac{\partial_t R_k}{P_k}\right) + \left(\frac{1}{6}Q_1\left(\frac{\partial_t R_k}{P_k}\right) - \frac{1}{4}Q_2\left(\frac{\partial_t R_k}{P_k^2}\right)\right)R\right]$$

$$\text{Type II}: \frac{d\Gamma_k}{dt} = -\frac{1}{8\pi^2}\int d^dx\,\sqrt{g}\left[Q_2\left(\frac{\partial_t R_k}{P_k}\right) - \frac{1}{12}R\,Q_1\left(\frac{\partial_t R_k}{P_k}\right)\right]$$

[1] A. Codello, R. Percacci, C. Rahmede, Ann. Phys. **324** (2009) 44, [arXiv:0805.2909(hep-th)]

Optimized cutoff

Cutoff function shape:

$$R_k(z) = (k^2 - z)\theta(k^2 - z)$$

In the two schemes:

$$\text{Type I}: \quad = \frac{1}{48\pi^2}\int d^4x\,\sqrt{g}\,\left(-6k^4 - \frac{1}{2}k^2 R\right)$$

$$\text{Type II}: \quad = \frac{1}{48\pi^2}\int d^4x\,\sqrt{g}\,\left(-6k^4 + k^2 R\right)$$

Also with exponential cutoff

Cutoff function shape:

$$R_k(z) = \frac{z e^{-az/k^2}}{1 - e^{-az/k^2}}$$

In the two schemes:

$$\text{Type I}: \frac{n_D}{8\pi^2} \int d^4x \sqrt{g} \left(-\frac{4\zeta(3)}{a^2} k^4 - \frac{\pi^2 - 9}{24a} k^2 R \right)$$

$$\text{Type II}: \frac{n_D}{8\pi^2} \int d^4x \sqrt{g} \left(-\frac{4\zeta(3)}{a^2} k^4 + \frac{\pi^2}{3a} k^2 R \right)$$

Also with Kähler fermions

$$\{\phi, \phi_\mu, \phi_{\mu\nu}, \phi_{\mu\nu\rho}, \phi_{\mu\nu\rho\sigma}\} \qquad \Longleftrightarrow \qquad 4 \times \psi$$

FRGE:

$$\frac{d\Gamma_k}{dt} = -2\,\frac{1}{2}\mathrm{Tr}_{(0)}\left(\frac{\partial_t R_k(\Delta^{(0)})}{P_k(\Delta^{(0)})}\right) - 2\,\frac{1}{2}\mathrm{Tr}_{(1)}\left(\frac{\partial_t R_k(\Delta^{(1)})}{P_k(\Delta^{(1)})}\right) - \frac{1}{2}\mathrm{Tr}_{(2)}\left(\frac{\partial_t R_k(\Delta^{(2)})}{P_k(\Delta^{(2)})}\right)$$

with a general cutoff

$$\text{Type I}: \; = -4 \times \frac{1}{8\pi^2}\int d^d x\,\sqrt{g}\left[Q_2\left(\frac{\partial_t R_k}{P_k}\right) + \left(\frac{1}{6}Q_1\left(\frac{\partial_t R_k}{P_k}\right) - \frac{1}{4}Q_2\left(\frac{\partial_t R_k}{P_k^2}\right)\right)R\right]$$

$$\text{Type II}: \; = -4 \times \frac{1}{8\pi^2}\int d^d x\,\sqrt{g}\left[Q_2\left(\frac{\partial_t R_k}{P_k}\right) - \frac{1}{12}R\,Q_1\left(\frac{\partial_t R_k}{P_k}\right)\right]$$

Spectral Sum

$$\Gamma = -\mathrm{Tr}\log\left(\not{D}\right) = -\frac{1}{2}\mathrm{Tr}\log\left(-\nabla^2 + \frac{R}{4}\right)$$

$$\Gamma = -\mathrm{Tr}\log\left(\not{D}\right) \longrightarrow \Gamma_k = -\mathrm{Tr}\log\left(|\not{D}| + R_k^D(|\not{D}|)\right)$$

On a spherical background

$$\frac{d\Gamma_k}{dt} = -\,\mathrm{Tr}\left[\frac{\partial_t R_k^D(|\not{D}|)}{P_k^D(|\not{D}|)}\right] = -\sum_n m_n \frac{\partial_t R_k^D(|\lambda_n|)}{P_k^D(|\lambda_n|)}$$

$$= -\sum_{\pm}\sum_n m_n \theta(k - |\lambda_n|) = \frac{V(4)}{48\pi^2}\left(-6k^4 + k^2 R\right)$$

Defining a good cutoff

$$R_k \overset{?}{\longleftrightarrow} R_k^D$$

Formally

$$\Gamma_k = -\frac{1}{2}\mathrm{Tr}\log\left(|\not{D}| + R_k^D(|\not{D}|)\right)^2 = -\frac{1}{2}\mathrm{Tr}\log\left(-\nabla^2 + \frac{R}{4} + \underbrace{2|\not{D}|R_k^D(|\not{D}|) + R_k^D(|\not{D}|)^2}_{=R_k}\right)$$

Inverting

$$R_k^D(z) = -z + \sqrt{z^2 + R_k}$$

- must be continuous and monotonically decreasing in z^2 and k,
- it must go rapidly to zero for $z^2 > k^2$,
- it must tend to a positive value for $z^2 \to 0$
- it must tend to zero for $k \to 0$.

Type I vs Type II

Type II cutoff: $R_k(-\nabla^2 + R/4) = R_k(\not{D}^2) \equiv R_k(z^2)$

$$R_k^D(z) = -z + \sqrt{z^2 + R_k(z^2)} \quad \text{is always good}$$

With R_k optimized cutoff shape

$$R_k^D(z) = (k - z)\theta(k - z)$$

Type I cutoff $R_k(-\nabla^2) = R_k(\not{D}^2 - R/4) \equiv R_k(z^2 - R/4)$.

With R_k optimized cutoff shape

$$R_k^D(z) = \left(\sqrt{k^2 + \frac{R}{4}} - z\right)\theta\left(\sqrt{k^2 + \frac{R}{4}} - z\right)$$

With R_k exponential cutoff shape

$$R_k^D(z) = -z + \sqrt{z^2 + \left(z^2 - \frac{R}{4}\right)\frac{e^{-a\left(z^2 - \frac{R}{4}\right)/k^2}}{1 - e^{-a\left(z^2 - \frac{R}{4}\right)/k^2}}}$$

for both cases

$$\lim_{k\to 0} R_k^D(z) = \left(\sqrt{\frac{R}{4}} - z\right)\theta\left(\sqrt{\frac{R}{4}} - z\right)$$

Constraints on Matter from Asymptotic Safety

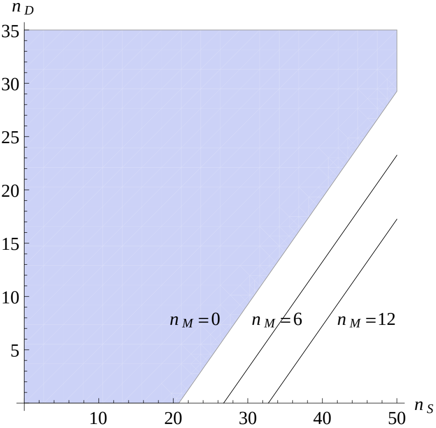
One loop approximation:

$$\partial_t \tilde{G} = 2\tilde{G} - \frac{1}{2\pi}\left(\frac{13}{6}Q_1\left(\frac{\partial_t R_k}{P}\right) + \frac{57}{6}Q_2\left(\frac{\partial_t R_k}{P^2}\right)\right)\tilde{G}^2 + \frac{1}{6\pi}\left(n_S + 2n_D - 4n_M\right)Q_1\left(\frac{\partial_t R_k}{P}\right)\tilde{G}^2$$

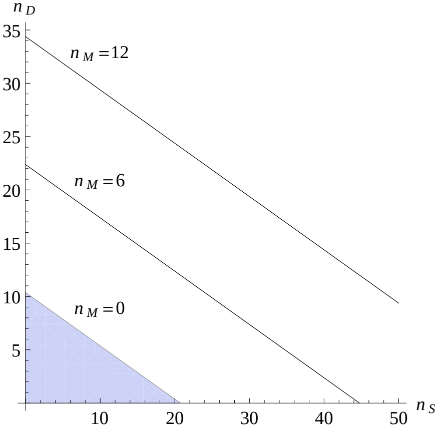
Requiring a positive FP:

$$n_S + 2n_D - 4n_M - \left[\frac{13}{2} + \frac{57}{2}Q_2\left(\frac{\partial_t R_k}{P_k^2}\right)\Big/ Q_1\left(\frac{\partial_t R_k}{P_k}\right)\right] < 0$$

Type I:



Type II:



Tetrad Gravity

- This has been discussed recently by Harst and Reuter [3] (Type Ia cutoff, $\alpha = 1$)
- Einstein-Hilbert truncation with the usual diffeomorphism ghosts and gauge fixing (and symmetric vielbein)
- $O(d)$ ghost contribution depending on a free ghost mass parameter μ

$$S_{\text{gh}} = -\int dx\bar{e}\left(\bar{C}^{\mu}, \bar{\Sigma}'^{\mu\nu}\right)\begin{pmatrix} \sqrt{2}\xi^{-1}\left(\delta^{\mu}{}_{\rho}\bar{D}^{2} + \bar{R}^{\mu}{}_{\rho}\right) & 0 \\ 2\,\xi^{-\frac{1}{2}}\mu\,\delta^{\mu}{}_{\rho}\bar{D}^{\nu} & 2\xi^{-\frac{1}{2}}\mu^{2}\delta^{[\mu}_{\rho}\delta^{\nu]}_{\sigma} \end{pmatrix}\begin{pmatrix} C^{\rho} \\ \Sigma'^{\rho\sigma} \end{pmatrix}$$

- Substituting the relation $h_{\mu\nu} = 2\varepsilon_{(\mu\nu)} + \varepsilon_{(\mu}{}^{\rho}\varepsilon_{\nu)\rho}$ gives

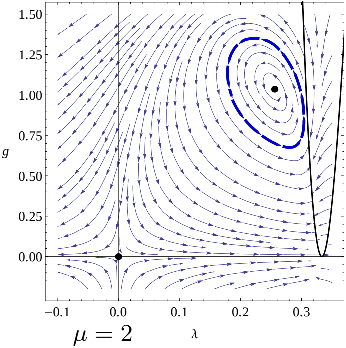
$$\Gamma_{k}^{(2)}[e, \bar{e}] = 2\frac{\delta\Gamma^{EH}}{\delta g_{\mu\nu}\delta g_{\rho\sigma}}\varepsilon_{\mu\nu}\varepsilon_{\rho\sigma} + \zeta\frac{\delta\Gamma^{EH}}{\delta g_{\mu\nu}}\varepsilon_{\mu}{}^{\rho}\varepsilon_{\nu\rho}$$

- We used a Type Ib and IIb cutoff

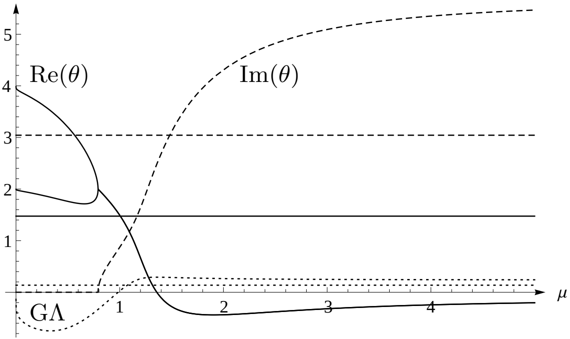
[3] U. Harst and M. Reuter, JHEP **1205** (2012) 005 [arXiv:1203.2158 [hep-th]]

Type Ia cutoff [3] $\left(h^T_{\mu\nu}, h\right)$ $R_k(-\nabla^2)$

Gauge $\alpha = 1$, dependence on μ:



$\mu = 2$

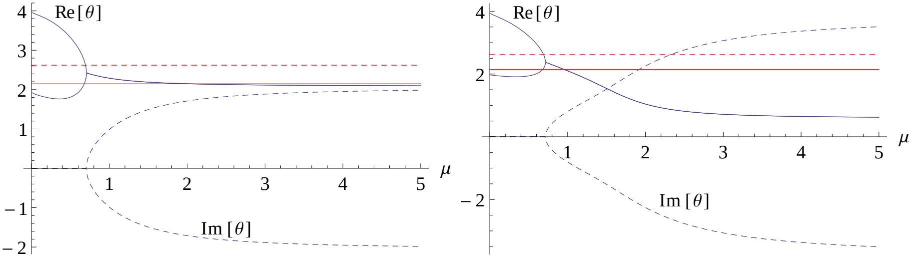


[3] U. Harst and M. Reuter, JHEP **1205** (2012) 005 [arXiv:1203.2158 [hep-th]]

Type Ib cutoff $\left(h_{\mu\nu}^{TT}, \xi, \sigma, h\right)$ $R_k(-\nabla^2)$

Dependence on μ:

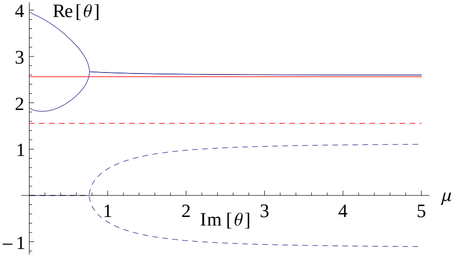
$\alpha = 0$ $\alpha = 1$



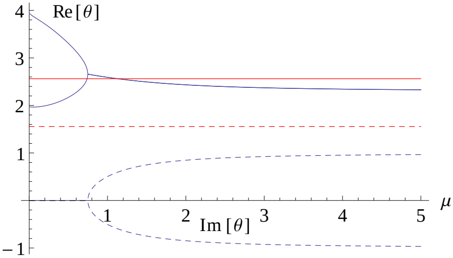
Type IIb cutoff $\left(h_{\mu\nu}^{TT}, \xi, \sigma, h\right)$ $R_k(-\nabla^2 + CR)$

Dependence on μ:

$\alpha = 0$

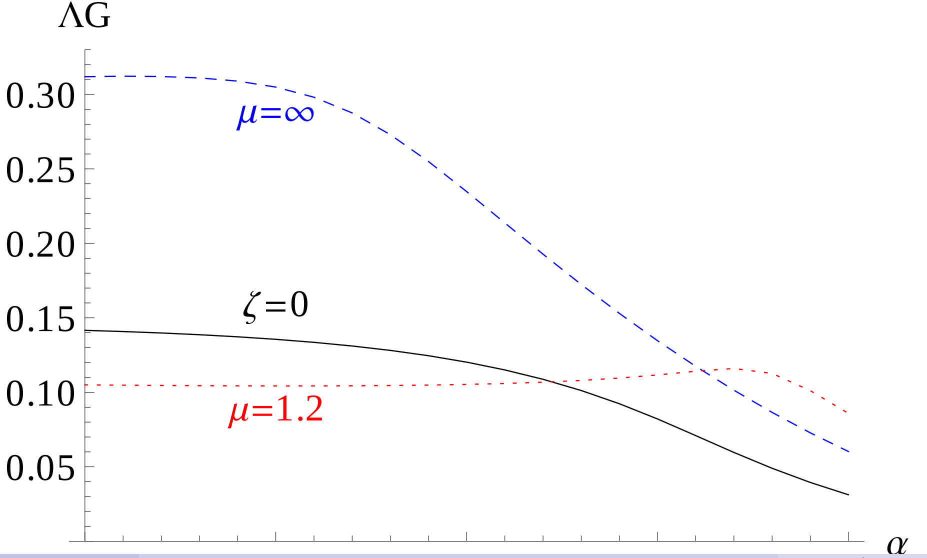


$\alpha = 1$



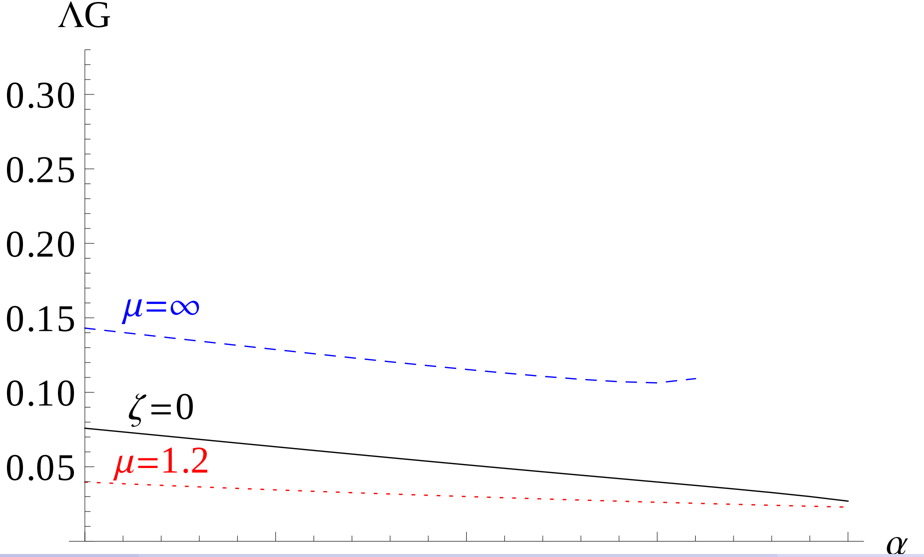
Type Ib cutoff $\left(h_{\mu\nu}^{TT}, \xi, \sigma, h\right)$ $R_k(-\nabla^2)$

Dependence on α:



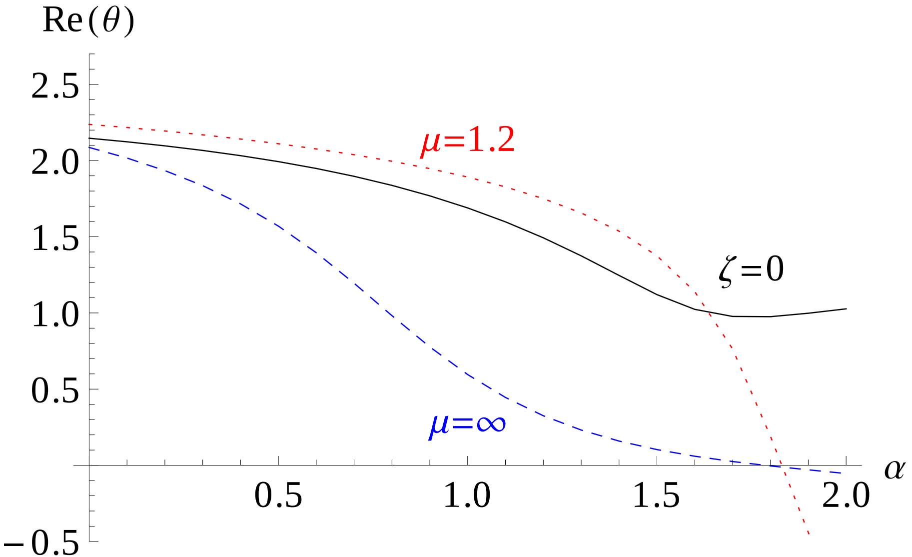
Type IIb cutoff $\left(h_{\mu\nu}^{TT}, \xi, \sigma, h\right)$ $R_k(-\nabla^2 + CR)$

Dependence on α:



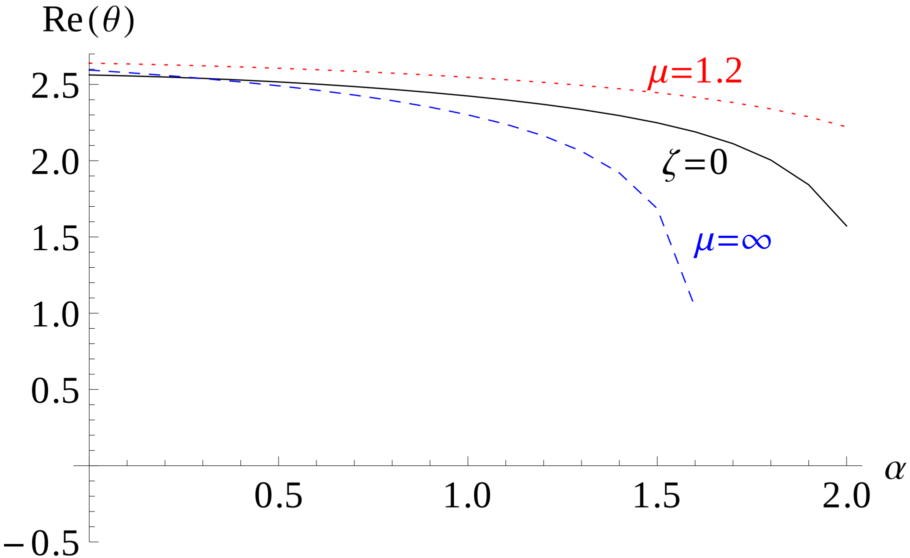
Type Ib cutoff $\left(h_{\mu\nu}^{TT}, \xi, \sigma, h\right)$ $R_k(-\nabla^2)$

Dependence on α:



Type IIb cutoff $\left(h_{\mu\nu}^{TT}, \xi, \sigma, h\right)$ $R_k(-\nabla^2 + CR)$

Dependence on α:



Type IIa cutoff $\left(h_{\mu\nu}^{TT}, h\right)$ $R_k(-\nabla^2 + CR)$

This scheme coincides with IIb for $\alpha = 1$ [4].
Type IIa:

$$\frac{d\Gamma_k}{dt} = \sum_{\lambda_n} \frac{\partial_t R_k(\lambda_n) - \eta R_k(\lambda_n)}{P_k(\lambda_n) - 2\Lambda}$$

Type IIb:

$$\frac{d\Gamma_k}{dt} = \sum_n \frac{\partial_t R_k(\lambda_n^{TT}) - \eta R_k(\lambda_n^{TT})}{P_k(\lambda_n^{TT}) - 2\Lambda} + \sum_n \frac{\partial_t R_k(\lambda_n^{\xi}) - \eta R_k(\lambda_n^{\xi})}{P_k(\lambda_n^{\xi}) - 2\Lambda} + \sum_n \frac{\partial_t R_k(\lambda_n^{\sigma}) - \eta R_k(\lambda_n^{\sigma})}{P_k(\lambda_n^{\sigma}) - 2\Lambda} + \sum_n \frac{\partial_t R_k(\lambda_n^{h}) - \eta R_k(\lambda_n^{h})}{P_k(\lambda_n^{h}) - 2\Lambda}$$

[4] C. Rahmede, PhD thesis, SISSA (2008).

Discussion

1. Problems with bad cutoffs
   - "squaring" is OK but must use Type II
2. Constraints on matter content from asymptotic safety
   - in one loop approximation bounds on n_S,n_F, SM is OK
3. Tetrad formalism
   - extended the results of Harst and Reuter to more general cutoff schemes and gauges
   - cutoffs of Type b are less sensitive to μ for $\alpha = 0, 1$
   - pathologies arise again for $\alpha \gtrsim 2$

Thank you