On the physical mechanism underlying Asymptotic Safety

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Based on: A.N. and M. Reuter, arXiv:1208.0031

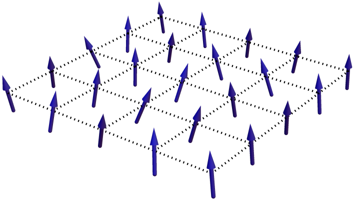
OUTLINE

- MOTIVATION
- FOUNDATIONS: GENERALIZED MAGNETIC INTERACTIONS
- PARAMAGNETIC DOMINANCE IN QEG
- QEG SPACETIMES AS A POLARIZABLE MEDIUM
- CONCLUSION

MOTIVATION: ASYMPTOTIC FREEDOM IN QCD

QCD analogs of elementary magnetic systems

- Color electric and color magnetic fields
- Interactions and effects similar to standard magnetism
- Alignment of gluon spins in external color magnetic field



Consequence

Asymptotic freedom in QCD can be considered
a paramagnetic effect of the gluon spins

N. K. Nielsen, Am. J. Phys. **49** (1981) 1171

MOTIVATION: ASYMPTOTIC FREEDOM IN QCD

- External color magnetic field B → vacuum polarization
  → change of vacuum energy density $\mathcal{E} = -\frac{1}{2}\chi_{\mathsf{mag}} B^2$

Magnetic susceptibility χ_{mag}

$\chi_{\mathsf{mag}} < 0 \quad (\mu < 1)$

diamagnetic medium

$\chi_{\mathsf{mag}} > 0 \quad (\mu > 1)$

paramagnetic medium

- With $\varepsilon\mu = 1$ and
  $\beta_{g^2} \equiv \beta_0 g^4 + \mathcal{O}(g^6)$:

$$\chi_{\mathsf{mag}}(B) = -\frac{1}{2}\,\beta_0\, g^2 \ln\left(\frac{\Lambda^2}{gB}\right)$$

$$\boxed{\beta_0 = -\frac{(-1)^{2S}}{4\pi^2}\left[(2S)^2 - \frac{1}{3}\right]}$$

In QCD: $\beta_0 < 0$

⇒ asymptotic freedom

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GENERALIZATION OF MAGNETIC INTERACTIONS

- Nonrelativistic electrons in an external field

$$H_{\mathsf{P}} = \frac{1}{2m}\left(\mathbf{p} - e\mathbf{A}\right)^2 + \mu_{\mathsf{B}}\,\mathbf{B}\cdot\boldsymbol{\sigma}$$

orbital motion: Landau diamagnetism ($\chi_{\mathsf{d}} < 0$)

spin alignment: Pauli paramagnetism ($\chi_{\mathsf{p}} > 0$)

- Paramagnetic dominance: $\chi_{\mathsf{d}} = -\frac{1}{3}\,\chi_{\mathsf{p}}$
- How can this be generalized?
  In position space 2 contributions to nonminimal operator:

$$-\boldsymbol{D}^2 = -(\boldsymbol{\nabla} - ie\mathbf{A})^2 \quad \text{vs.} \quad \mathbf{B}\cdot\boldsymbol{\sigma}$$

- Relativistic analog → applicable to QED

$$-\not{D}^2 = -D^2 + \frac{i}{2}\,e\,\gamma^\mu\gamma^\nu F_{\mu\nu}$$

GENERALIZATION OF MAGNETIC INTERACTIONS

- In Yang-Mills theory:
  - Perform background split of the gauge field $A_\mu^a = \bar{A}_\mu^a + a_\mu^a$
  - Fluctuations described by $\int a_\mu^a \left(\Delta^{ab\,\mu}{}_\nu\right) a^{b\,\nu}$ with

$$\Delta^{ab\,\mu}{}_\nu = -\left(\bar{D}^2\right)^{ab} \delta^\mu{}_\nu + 2ig\bar{F}^{ab\,\mu}{}_\nu$$

- In QEG (Einstein-Hilbert truncation):
  - Background split $\gamma_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$, expand EH action
  - Fluctuations described by $\int h_{\mu\nu} \left(\Delta^{\mu\nu}{}_{\rho\sigma}\right) h^{\rho\sigma}$ with

$$\Delta^{\mu\nu}{}_{\rho\sigma} = -\bar{K}^{\mu\nu}{}_{\rho\sigma} \bar{D}^2 + \bar{U}^{\mu\nu}{}_{\rho\sigma}$$

- Paramagnetic dominance? Consequences?

PARAMAGNETIC DOMINANCE IN QED

Lowest order of the β-function with functional RG methods

- FRGE: $\partial_t \Gamma_k = \frac{1}{2}\,\mathrm{STr}\left[\left(\Gamma_k^{(2)} + \mathcal{R}_k\right)^{-1} \partial_t \mathcal{R}_k\right]$
- Ansatz: $\Gamma_k[A, \bar{\psi}, \psi] = \int \mathrm{d}^4x \left[Z_{F,k}\,\frac{1}{4}\,F_{\mu\nu}F^{\mu\nu} + \bar{\psi}\,\not{D}\,\psi\right]$
- Reexpress $\Gamma^{(2)}$: $\not{D}$ appears as $-\not{D}^2 = -D^2 + \frac{i}{2}\,\bar{e}\,\gamma^\mu\gamma^\nu F_{\mu\nu}$

$$\boxed{\partial_t e^2 = \beta_{e^2} = \frac{1}{4\pi^2}\left[\Big\{1\Big\}_{\text{para}} + \Big\{-\frac{1}{3}\Big\}_{\text{dia}}\right] e^4}$$

- Result: $\beta_{e^2} > 0$ due to paramagnetic dominance
  $\Rightarrow$ charge screening, no asymptotic freedom

PARAMAGNETIC DOMINANCE IN YANG-MILLS THEORY

- Background formalism for the gauge field: $A_\mu^a = \bar{A}_\mu^a + a_\mu^a$
- Notion: $\bar{A}_\mu^a$ corresponds to external color magnetic field
- Ansatz: $\Gamma_k[A, \bar{A}\,] = \frac{1}{4} \int \mathrm{d}^d x \, Z_{F,k} \, F_{\mu\nu}^a[A] F_a^{\mu\nu}[A] + \Gamma_k^{\mathsf{g.f.}} + \Gamma_k^{\mathsf{gh}}$
- FRGE: $$\partial_t \Gamma_k = \frac{1}{2} \mathrm{Tr} \left(\frac{\partial_t \mathcal{R}_k}{\Gamma_k^{(2)} + \mathcal{R}_k} \right) - \mathrm{Tr} \left(\frac{\partial_t \mathcal{R}_k^{\mathsf{gh}}}{\underset{\substack{\downarrow \\ \text{ghost-dia}}}{-\bar{D}^2} + \mathcal{R}_k^{\mathsf{gh}}} \right)$$
- Separation into the various magnetic contributions:

$$\Gamma_k^{(2)} \equiv \frac{\delta^2}{\delta A^2} \Gamma_k[A, \bar{A}\,]\Big|_{A=\bar{A}} = Z_{F,k} \left(\underset{\substack{\downarrow \\ \text{dia}}}{-\bar{D}^2} + \underset{\substack{\downarrow \\ \text{para}}}{2i \, \bar{g} \, \bar{F}} \right)$$

PARAMAGNETIC DOMINANCE IN YANG-MILLS THEORY

- Expand traces in FRGE as heat kernel series, keeping magnetic contributions separated ⇒ running of $Z_{F,k}$ ⇒ anomalous dimension $\eta_F \equiv -\partial_t \ln Z_{F,k}$
- In terms of the renormalized charge $g^2 \equiv k^{d-4} Z_{F,k}^{-1}\,\bar{g}^2$:

$$\boxed{\eta_F = -\frac{N\,\Phi^1_{d/2-2}(0)}{3\,(4\pi)^{d/2}}\Big[\{-d\}_{\text{dia}} + \{24\}_{\text{para}} + \{2\}_{\text{ghost-dia}}\Big]g^2}$$

- For $d < 26$: $\eta_F < 0$
- For $d < 24$: paramagnetic dominance

PARAMAGNETIC DOMINANCE IN YANG-MILLS THEORY

- Running of the renormalized charge:

$$\partial_t g^2 = \beta_{g^2} = (d - 4 + \eta_F) g^2$$

- For $d = 4$:

$$\beta_{g^2} = -\frac{N}{24\pi^2}\left[\{-2\}_{\text{dia}} + \{12\}_{\text{para}} + \{1\}_{\text{ghost-dia}}\right] g^4$$

$\Rightarrow$ para term causes $\beta_{g^2} < 0$, dia drives in opposite direction

- Result: Color anti-screening and asymptotic freedom only due to paramagnetic dominance!

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MAGNETIC INTERACTIONS IN QEG

- Background formalism: $\gamma_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$,
  or $g_{\mu\nu} = \bar{g}_{\mu\nu} + \bar{h}_{\mu\nu}$ with expectation value $\bar{h}_{\mu\nu} = \langle h_{\mu\nu} \rangle$
- Interaction between metric fluctuations and background
- Background "=" external "magnetic" field
  → polarizes quantum vacuum of $h_{\mu\nu}$-particles
- Different kinds of interaction: dia vs. para?
- Ansatz: $\Gamma_k = \Gamma_k^{\mathsf{EH}} + \Gamma_k^{\mathsf{g.f.}} + \Gamma_k^{\mathsf{gh}} \equiv \breve{\Gamma}_k + \Gamma_k^{\mathsf{gh}}$, where

$$\breve{\Gamma}_k[g, \bar{g}] = \frac{1}{16\pi G_k} \int \mathrm{d}^d x \sqrt{g} \left(-R[g] + 2\Lambda_k\right) + \frac{1}{32\pi G_k} \int \mathrm{d}^d x \sqrt{\bar{g}}\, \bar{g}^{\mu\nu} \left(\mathcal{F}_\mu^{\alpha\beta} g_{\alpha\beta}\right)\left(\mathcal{F}_\nu^{\rho\sigma} g_{\rho\sigma}\right)$$

MAGNETIC INTERACTIONS IN QEG

- Γ_k^{gh} = classical ghost action $\quad\Rightarrow\quad$ decomposed FRGE

$$\partial_t \Gamma_k = \frac{1}{2}\,\mathrm{Tr}\left(\frac{\partial_t \mathcal{R}_k^{\mathsf{grav}}}{\breve{\Gamma}_k^{(2)} + \mathcal{R}_k^{\mathsf{grav}}}\right) - \mathrm{Tr}\left(\frac{\partial_t \mathcal{R}_k^{\mathsf{gh}}}{-\mathcal{M} + \mathcal{R}_k^{\mathsf{gh}}}\right)$$

- Grav.: $\left(\breve{\Gamma}_k^{(2)}[g,\bar{g}]\right)^{\mu\nu}{}_{\rho\sigma}\Big|_{g=\bar{g}} = \dfrac{1}{32\pi G_k}\Big(\underset{\substack{\downarrow\\ \text{dia}}}{-\bar{K}^{\mu\nu}{}_{\rho\sigma}\bar{D}^2} + \underset{\substack{\downarrow\\ \text{para}}}{\bar{U}^{\mu\nu}{}_{\rho\sigma}}\Big)$

  where $\bar{K}^{\mu\nu}{}_{\rho\sigma} \equiv \bar{K}^{\mu\nu}{}_{\rho\sigma}(\bar{g})$

  and $\bar{U}^{\mu\nu}{}_{\rho\sigma} = \bar{U}^{\mu\nu}{}_{\rho\sigma}(\bar{g}, \bar{R}^{\lambda}{}_{\tau\alpha\beta})$

- Ghosts: $-\mathcal{M}[g,\bar{g}]^{\mu}{}_{\nu}\Big|_{g=\bar{g}} = \delta^{\mu}_{\nu}\Big(\underset{\substack{\swarrow\\ \text{ghost-dia}}}{-\bar{D}^2}\underset{\substack{\searrow\\ \text{ghost-para}}}{-\frac{1}{d}\,\bar{R}}\Big)$

MAGNETIC INTERACTIONS IN QEG

- Separation procedure: $\frac{1}{-D^2+\mathcal{R}_k+R} = \underset{\text{dia}}{\frac{1}{-D^2+\mathcal{R}_k}} - \underset{\text{para}}{\frac{1}{(-D^2+\mathcal{R}_k)^2}\,R} + \mathcal{O}(R^2)$
- Heat kernel series to evaluate traces in FRGE
  $\Rightarrow$ RG equations for $g_k \equiv k^{d-2} G_k$ and $\lambda_k \equiv k^{-2}\Lambda_k$

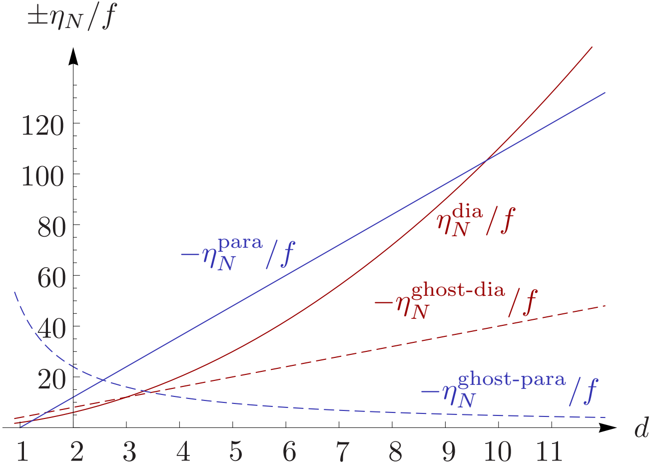
$$\partial_t \lambda_k = \beta_\lambda \qquad \text{and} \qquad \partial_t g_k = \beta_g \equiv (d-2+\eta_N)\, g_k$$

- Magnetic contributions to anomalous dimension $\eta_N \equiv \partial_t \ln G_k$

$$\boxed{\begin{aligned} \eta_N = \frac{(4\pi)^{1-\frac{d}{2}}}{3} \Bigg[& \Big\{ d(d+1)\,\Phi^1_{d/2-1}(-2\lambda) \Big\}_{\text{dia}} + \Big\{ -\,4d\,\Phi^1_{d/2-1}(0) \Big\}_{\text{ghost-dia}} \\ & + \Big\{ -\,6d(d-1)\,\Phi^2_{d/2}(-2\lambda) \Big\}_{\text{para}} + \Big\{ -\,24\,\Phi^2_{d/2}(0) \Big\}_{\text{ghost-para}} \Bigg] g + \mathcal{O}(g^2) \end{aligned}}$$

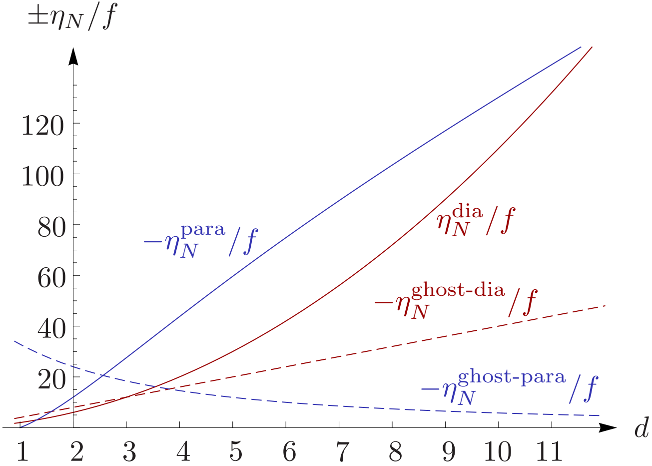
CONTRIBUTIONS TO η_N

- For the optimized shape function: $R^{(0)}(z) = (1-z)\Theta(1-z)$



CONTRIBUTIONS TO η_N

- For an exponential cutoff: $R_s^{(0)}(z) = \frac{sz}{e^{sz}-1}$, here with $s = 1$



EXISTENCE OF FIXED POINTS

Observation: sign of η_N determined by paramagnetic contributions,
independent of the cutoff chosen

Why is that sign important?

1.) Gravitational anti-screening $\quad \Leftrightarrow \quad \eta_N = k\partial_k \ln G_k < 0$

2.) Consider condition for non-Gaussian fixed point

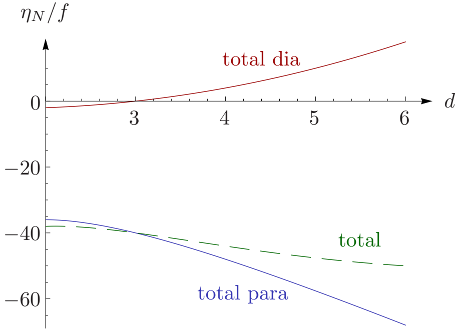
$$\beta_g = (d - 2 + \eta_n) g \overset{!}{=} 0 \quad \text{(for } g \neq 0) \quad \Rightarrow \quad \eta_N \overset{!}{=} 2 - d$$

If $d > 2$: fundamental requirement for fixed point: $\boxed{\eta_N \overset{!}{<} 0}$

Guess: No fixed point without paramagnetic terms

EXISTENCE OF FIXED POINTS

- Combine dia and ghost-dia, and para and ghost-para



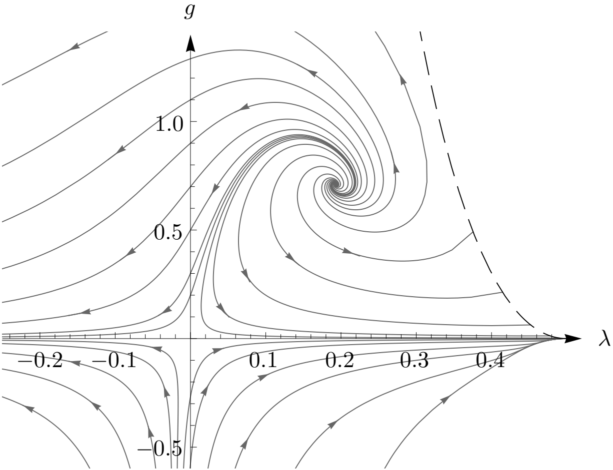
$\eta_N < 0$ due to total paramagnetic contribution ($d \geq 3$)

E.g. in $d = 4$: para 12 times stronger than dia

Fixed point only thanks to paramagnetic interactions!

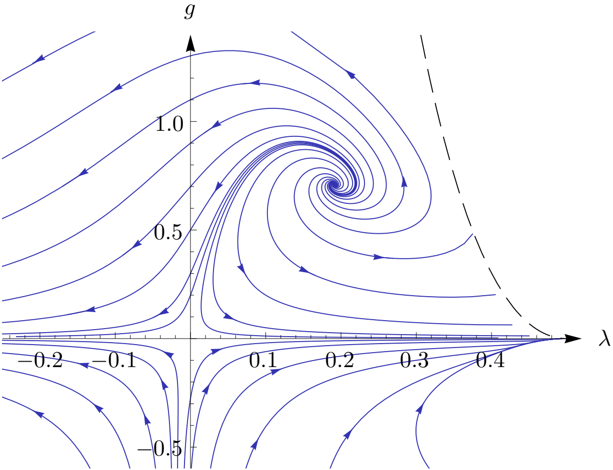
EXAMPLE: PHASE PORTRAIT IN $d = 4$

Recall flow diagram for Einstein-Hilbert truncation (full η_N)



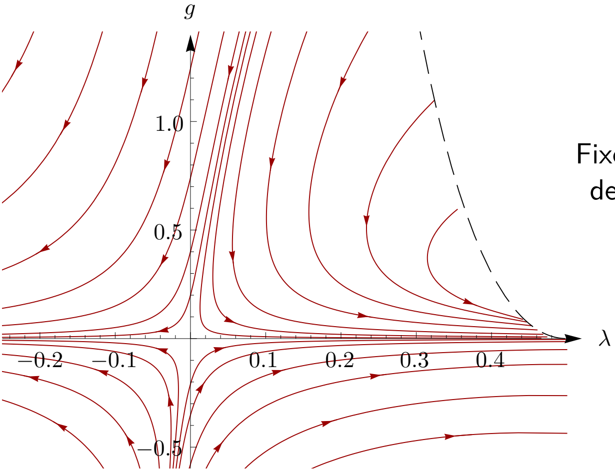
EXAMPLE: PHASE PORTRAIT IN $d = 4$

Retaining only paramagnetic terms in η_N



EXAMPLE: PHASE PORTRAIT IN $d = 4$

Retaining only diamagnetic terms in η_N



Fixed point destroyed!

THE β-FUNCTION OF g IN $2+\epsilon$ DIMENSIONS

- Near $d=2$: at leading order Φ's in η_N independent of cutoff

$$\eta_N = -b\,g + \mathcal{O}(g^2)$$

and NGFP $g_* = \epsilon/b + \mathcal{O}(\epsilon^2)$, with the *universal* coefficient

$$b = \frac{2}{3}\Big[\{1\}_{\text{total dia}} + \{18\}_{\text{total para}}\Big] = \frac{38}{3}$$

- Since $[G_k]=0$ in $d=2$, perturbative results are available
- In the literature: confusion about value of b

$$b = \frac{2}{3} \qquad \text{vs.} \qquad b = \frac{38}{3}$$

Gastmans et al.;
Christensen and Duff

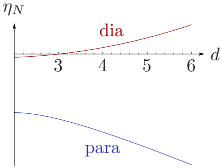
Tsao; Kawai and Ninomiya;
Jack and Jones

THE β-FUNCTION OF g IN $2+\epsilon$ DIMENSIONS

- The $\frac{2}{3}$-camp computed η_N on the boundary, $\eta_N = \eta_N^\partial$, and did not take into account the paramagnetic interactions
- FRGE calculations on the boundary confirm

$$\eta_N^\partial = -b^\partial\, g^\partial + \cdots \qquad \text{with } b^\partial = \frac{2}{3}$$

- Without paramagnetic terms, NGFP possible only below $d = 3$:



- FP with $b = \frac{38}{3}$ can be seen as the dimensional continuation of the NGFP from $d = 4$ to $d = 2+\epsilon$, the one with $b = \frac{2}{3}$ can not!

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PROPERTIES OF THE QEG VACUUM

Recall for QED: $\Gamma_k = -\int \frac{1}{4g_k^2} F_{\mu\nu}F^{\mu\nu} = \int \frac{1}{2g_\Lambda^2}\left(\varepsilon_k \mathbf{E}^2 - \frac{1}{\mu_k}\mathbf{B}^2\right)$

Is there a QEG analog?

- Consider metric at lowest post-Newtonian order

  $$g_{\mu\nu}\, dx^\mu\, dx^\nu = -(1 + 2\Phi_{\mathsf{grav}})\, dt^2 + 2\boldsymbol{\zeta} \cdot d\mathbf{x}\, dt + (1 - 2\Phi_{\mathsf{grav}})\, d\mathbf{x}^2$$

  $\Phi_{\mathsf{grav}}/\boldsymbol{\zeta}$: time independent gravitational scalar/vector potential
- Define gravitational electric/magnetic field

  $$\mathbf{E}_{\mathsf{grav}} = -\boldsymbol{\nabla}\Phi_{\mathsf{grav}}\,, \qquad \mathbf{B}_{\mathsf{grav}} \equiv -\tfrac{1}{2}\boldsymbol{\nabla} \times \boldsymbol{\zeta}$$

- Expand Lorentzian EAA $\Gamma_k^{\mathsf{Lor}}[g] = \frac{1}{16\pi G_k}\int \mathrm{d}^4x\sqrt{-g}\, R[g]$

  $$\Gamma_k^{\mathsf{Lor}}[g] = -\frac{1}{4\pi}\int \mathrm{d}^4x\, \frac{1}{2\,G_k}\left(\mathbf{E}_{\mathsf{grav}}^2 - \mathbf{B}_{\mathsf{grav}}^2\right)$$

PROPERTIES OF THE QEG VACUUM

- Define "gravi-dielectric constant" and "gravimagnetic permeability"

$$\varepsilon_k^{\text{grav}} = \frac{1}{\mu_k^{\text{grav}}} = \frac{G_\Lambda}{G_k}$$

$$\boxed{\Gamma_k^{\text{Lor}}[g] = -\frac{1}{4\pi}\int \mathrm{d}^4x\,\frac{1}{2\,G_\Lambda}\left(\varepsilon_k^{\text{grav}}\mathbf{E}_{\text{grav}}^2 - \frac{1}{\mu_k^{\text{grav}}}\mathbf{B}_{\text{grav}}^2\right)}$$

- As G_k increases if k decreases, we conclude

$$\varepsilon_k^{\text{grav}} \leq 1 \quad \text{and} \quad \mu_k^{\text{grav}} \geq 1 \qquad\qquad (\text{for } k \leq \Lambda)$$

$$\Downarrow \qquad\qquad\qquad \Downarrow$$

charge (mass) anti-screening medium is paramagnetic

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- In many systems fluctuations described by inverse propagator

$$-D_{\mathcal{A}}^2 + \mathbf{U}(F_{\mathcal{A}})$$

- Terms give rise to diamagnetic and paramagnetic interactions
- Antagonistic effect, but paramagnetic dominance
- Outlook: approximation scheme → omit dia contributions

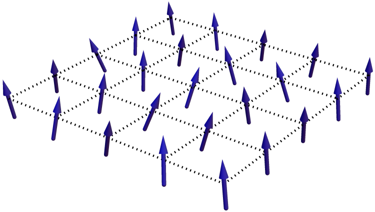
- In QEG:
  - Orbital motion effects caused by $\int h\bar{D}^2 h$ small
  - Spin orientation effects dominate: induced by ultralocal term $\int h(x)\,\bar{U}(x) h(x)$ analogous to $\int \bar{\psi}(x)\,\boldsymbol{\sigma}\cdot\mathbf{B}(x)\,\psi(x)$

- Asymptotic Safety in QEG due to paramagnetic interactions!

CONCLUSION

- Visualization of the QEG vacuum

Magnetic moments located
at fixed lattice points



rather than

gas of itinerant
electrons