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Asymptotic Safety Seminar 5.11.2012

IR Completion of Asymptotic Safety

and the flow of the graviton propagator

based on

arXiv:1209.4038 [hep-th]

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Outline

Introduction

- great success of the Asymptotic Safety Scenario in quantum gravity

higher dimensional operators

coupling to other fields (gauge/matter)

extra dimensions

phenomenology

→ strong support for the UV-fixed point scenario

Introduction

….. but there are still many things to do …....

e.g.

<u>background dependence:</u>
disentanglement of modes/parameters related to $\bar{g}$ and h respectively

- bi-metric calculations
[*Manrique,Reuter,Saueressig*]
- geometrical flows
[*Donkin,Pawlowski*]
- higher order vertex functions
[*NC,Litim,Pawlowski,Rodigast*]

<u>momentum dependence of vertex functions</u>

<u>infrared behaviour:</u>

Introduction

• flows hit a singular line (reg. dependent), or cross over to the region $\lambda > \frac{1}{2}$

unphysical

→ IR - incomplete

• but IR-completeness is of particular interest in late time cosmology

• progress in this direction

•postulation of IR-fixed point
[*Bonanno,Reuter*]

•finite net flows (geom. effective action)
[*Donkin,Pawlowski*]

•flow of the propagator
[*NC,Litim,Pawlowski,Rodigast*]

•screen singularity by reparameterization [*Nagy et al.*]

•IR-limit cycles
[*Litim,Satz*]

•projection from R^2 parameter space
[*Rechenberger,Saueressig*]

Background Dependence

- quantization of gravity

→ background dependence

- flow equation

$$\partial_t \Gamma_k[\bar{g};\phi] = \frac{1}{2}\,\mathrm{Tr}\,\frac{1}{\Gamma_k^{(2)}[\bar{g};\phi]+\mathcal{R}_k}\partial_t \mathcal{R}_k \quad , \quad \phi = (h, \bar{C}, C)$$

$$\left.\frac{\delta^2 \Gamma_k[\bar{g};h]}{\delta h^2}\right|_{h=0}$$

- gauge invariant effective action:

→ evaluate flow at $\bar{g} = g$

↓

flow is not closed in this setting:

$$\left.\frac{\delta^2 \Gamma_k[\bar{g};h]}{\delta h^2}\right|_{h=0} \neq \frac{\delta^2 \Gamma_k[\bar{g};h=0]}{\delta \bar{g}^2}$$

see [*Litim*,*Pawlowski*]

Background Dependence

- background approximation: $\left.\frac{\delta^2\Gamma_k[\bar{g};h]}{\delta h^2}\right|_{h=0} \approx \frac{\delta^2\Gamma_k[\bar{g};h=0]}{\delta\bar{g}^2}$

→ $\bar{g}$ and h are treated democratically

→ unphysical regulator contributions: $\mathcal{R}_k = \mathcal{R}_k[\bar{g}]$

$$\frac{\delta\dot{\Gamma}_k}{\delta\bar{g}} \sim \frac{\delta\mathcal{R}_k}{\delta\bar{g}}$$

- in the case of Yang-Mills theory:

↙ important for confinement

↘ important for asymptotic freedom

→ redefine the couplings by a different scheme!

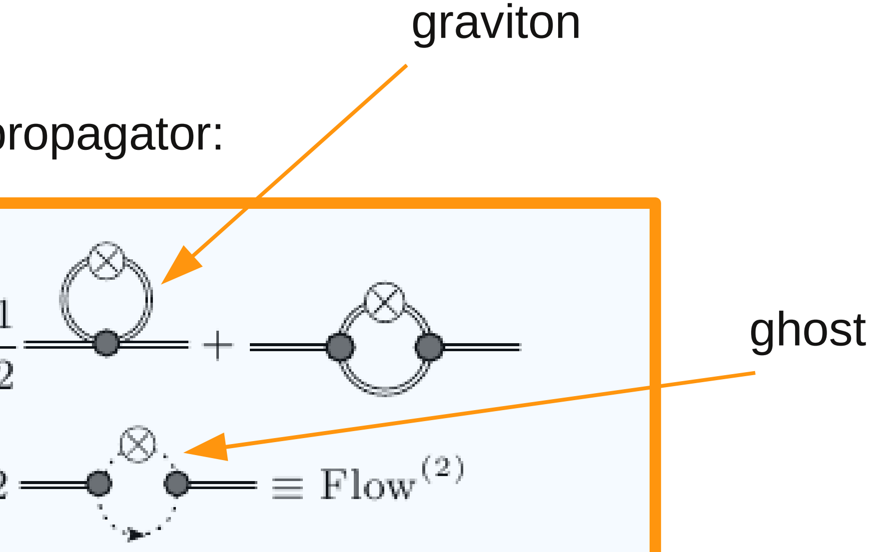
Flow of the Propagator

- the problem can be avoided if we take the derivatives before setting $\bar{g} = g$
- remember: expansion of the effective action

$$\Gamma[\bar{\phi}, \phi] = \sum_{n=0}^{\infty} \int \prod_{m=0}^{n} \mathrm{d}x_m \frac{1}{n!} \frac{\delta^n \Gamma[\bar{\phi}, \phi]}{\delta\phi(x_1)...\delta\phi(x_n)}\bigg|_{\phi=0} \circ\phi(x_1)\circ...\circ\phi(x_n)$$

- the corresponding hierachy of flow equations

for the inverse propagator:



$$\partial_t \frac{\delta^2 \Gamma_k[\bar{g}; h]}{\delta h^2}\bigg|_{h=0} = -\frac{1}{2}\,[\text{graviton tadpole}] + [\text{graviton loop}] - 2\,[\text{ghost loop}] \equiv \mathrm{Flow}^{(2)}$$

Flow of the Propagator

- **Approximation:**

scale dependent, gauge fixed Einstein Hilbert action

$$\Gamma_{\mathrm{EH},k}[\bar{g};h,\bar{C},C] = \frac{Z_{N,k}}{16\pi G_N}\int\sqrt{|g|}\;\left(-R(g)+2\Lambda_k\right) + \frac{Z_{\alpha,k}}{2\alpha}\int\sqrt{|\bar{g}|}\;\bar{g}^{\mu\nu}F_\mu(\bar{g},h)F_\nu(\bar{g},h) - \sqrt{2}\int\sqrt{|\bar{g}|}\;\bar{C}_\mu M^\mu{}_\nu(\bar{g},h)C^\nu$$

$$\left.\frac{\delta^n\Gamma_k[\bar{g},h,\bar{C},C]}{\delta h(p_1)...\delta h(p_n)}\right|_{h=0}(p_1,...,p_n) \equiv \left.\frac{\delta^n\Gamma_{\mathrm{EH},k}[\bar{g};h,\bar{C},C]}{\delta h(p_1)...\delta h(p_n)}\right|_{h=0}(p_1,...,p_n)\;,\qquad n=2,3,4$$

Flow of the Propagator

- we use Landau-deWitt gauge: $\alpha = 0$

→ circumvent the calculation of $Z_{\alpha,k}$

- we choose a regulator of the form

$$\mathcal{R}_k\left(q^2\right) = \Gamma_k^{(2)}\Big|_{\lambda=0} r\left(q^2/k^2\right)$$
$$\text{with} \qquad r(z) = (\tfrac{1}{z} - 1)\theta(1-z)\,.$$

- the physical spin 2 d.o.f of the graviton: transverse-traceless (TT) projection

→ $\dot{\Gamma}^{(2)}_{TT} = \mathcal{P}^{\alpha\beta\mu\nu}_{TT}\dot{\Gamma}^{(2)}_{\alpha\beta\mu\nu}$

- expand around flat backgrounds $\bar{g}_{\mu\nu} = \delta_{\mu\nu}$.

↓

$$\partial_t \Gamma^{(2)}_{k,\mathrm{TT}} = \frac{1}{32\pi\, G_N}\left(p^2 \partial_t Z_{N,k} - 2\partial_t\left(Z_{N,k}\Lambda_k\right)\right)$$

Momentum Dependence

• recall: the vertex functions $\left.\frac{\delta^n \Gamma_k[\bar{\phi},\phi]}{\delta\phi(x_1)...\delta\phi(x_n)}\right|_{\phi=0}$ depend on n external momenta

explicitly via the explicit derivative operators in the action

implicitly via momentum dependent couplings:

$$g_k(p_i^2/k^2)$$

→ full theory is in principle obtained from

$$\lim_{k\to 0} \left.\frac{\delta^n \Gamma_k[\bar{\phi},\phi]}{\delta\phi(p_1)...\delta\phi(p_n)}\right|_{\phi=0} (p_1, ..., p_n)$$

Momentum Dependence

• the implicit momentum dependence is ignored in all FRG-gravity calculations so far:

$$g_k(p_i^2/k^2) \longrightarrow g_k$$

→ for applications: scale identification !

• in our formalism:

→ resolve the explicit momentum dependence of the flowing propagator

↓

define the couplings from the vertex functions at some specified momentum scale

↓

first: analyze the momentum behaviour of the flow

Momentum Dependence

- investigate the RHS $\mathrm{Flow}^{(2)}_{\mathrm{TT}}(p)$ of the flow equation as a function of momentum

→ of particular importance for gravity: UV behaviour ($p \longrightarrow \infty$)

↓

from momentum counting we expect a divergence $\sim p^2$

→ in this limit the flow can be calculated analytically

↓

→ we find an exact cancellation of the terms $\sim p$, $\sim p^2$!!!

$\Longrightarrow$ finite UV limit !

$$\frac{\mathrm{Flow}^{(2)}_{\mathrm{TT}}(p)}{k^4} \xrightarrow[p\to\infty]{} \frac{-20+42\lambda-48\lambda^2+(1+\lambda)\eta_N}{192\pi^2(1-2\lambda)^2}$$

Momentum Dependence

- global momentum behaviour:

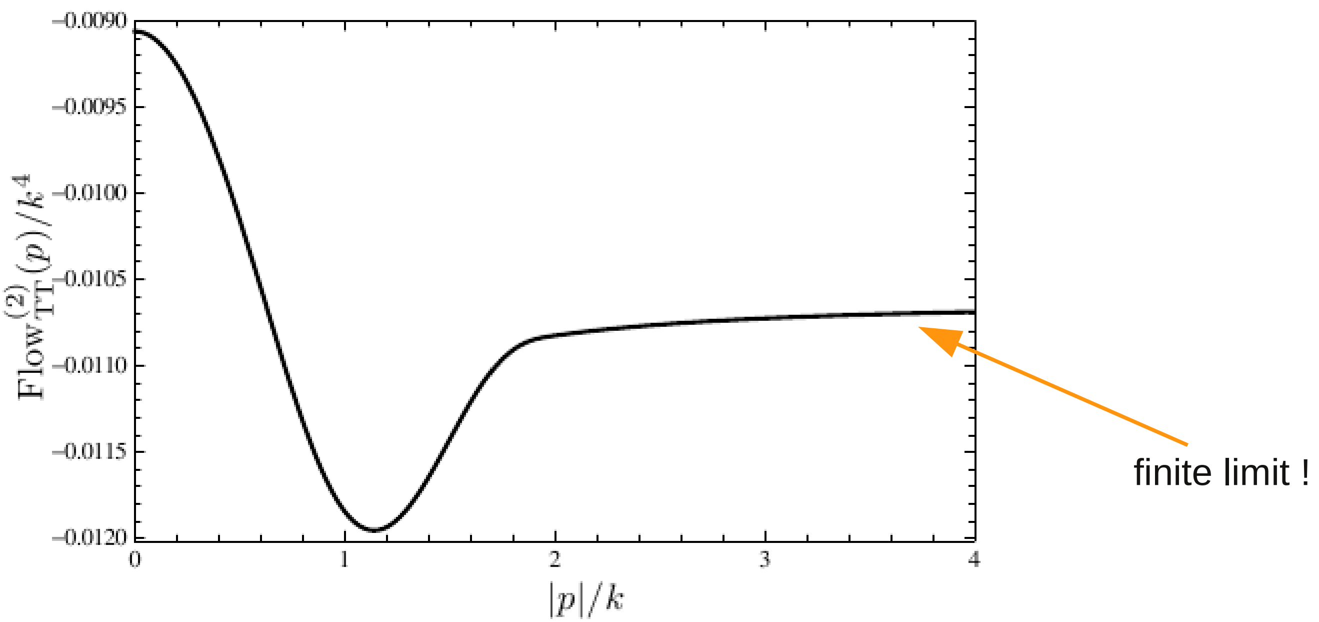


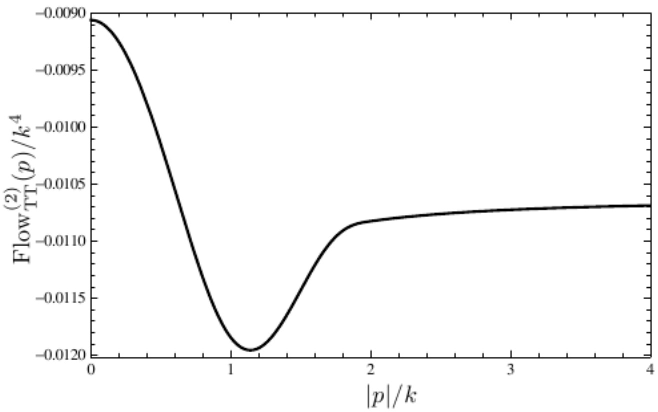
Fig.: The flow as a function of the external momentum with the parameter set $(\lambda = 0, \eta = 0)$

→ non-trivial momentum dependence

- What is a good definition of the couplings in terms of the vertex functions?

Momentum Dependence

→ simplest case: projection at $p = 0$



• but: integrate fluctuations $\approx k^2$

• $Z_{N,k}$ is sensitive to the curvature of the flow

↓

$p = k$ vs $p = 0$ → sign change in $Z_{N,k}$!!!

• comsological constant $\longleftrightarrow$ effective graviton mass

→ "direct" definition at $p = 0$

→ otherwise: quadratic interpolation

→ e.g. with $p = k$: offset too large

Definition of the Couplings

- we propose the following bi-local projection:

$$\partial_t (Z_{N,k}\Lambda_k) = -16\pi\, G_N\, \mathrm{Flow}^{(2)}_{\mathrm{TT}}(p)\Big|_{p=0}$$

$$\partial_t Z_{N,k} = 16\pi\, G_N\, \partial_p^2\, \mathrm{Flow}^{(2)}_{\mathrm{TT}}(p)\Big|_{p=k}$$

- by constructing a running $G_{N,k}$ from $Z_{N,k}$ we obtain the usual ODE for

$$g_N \equiv k^2 G_N / Z_{N,k} \text{ and } \lambda \equiv \Lambda_k / k^2$$

given by

$$\partial_t g_N = (2 + \eta_N)\, g_N \quad \text{and} \quad \partial_t \lambda = -2\lambda + \eta_\lambda$$

with

$$\eta_{N,k} \equiv -\partial_t Z_{N,k} / Z_{N,k} \qquad \eta_\lambda \equiv (\partial_t \Lambda_k)/k^2$$

Flow of the Couplings

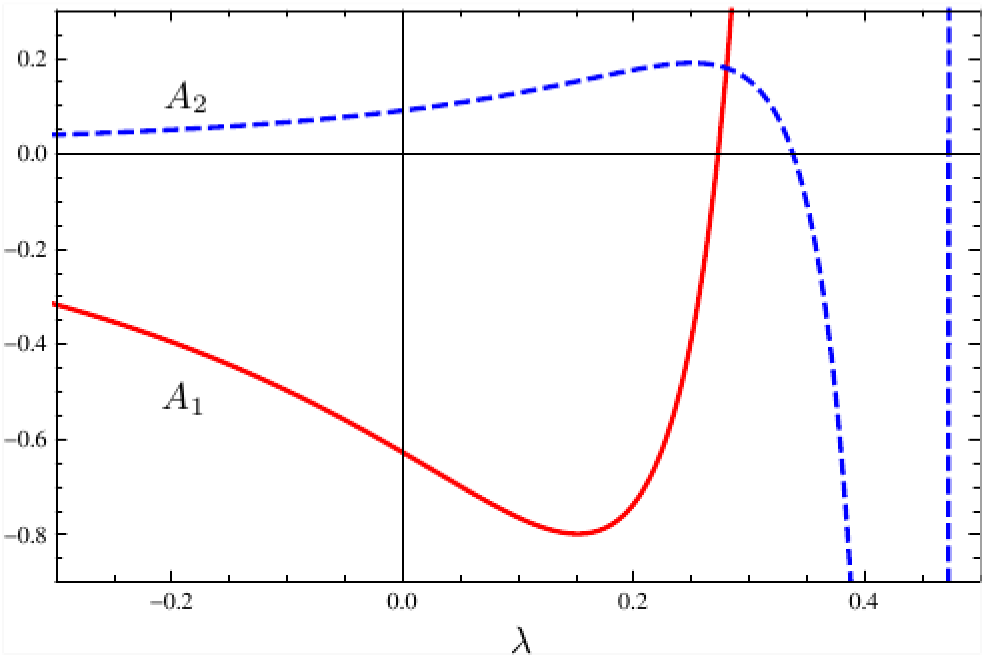
- in a $p = 0$ projection: → analytical solution of the momentum integrals
- in a $p = k$ projection: → partially numerical solution of the momentum integrals

$$\eta_\lambda = \eta_N \, \lambda + g \, A_3(\lambda, \eta)$$

$$A_3 = \frac{1 + 4(1-2\lambda)^2}{12\pi(1-2\lambda)^3} - \eta_N \frac{12 - 45\lambda - 40\lambda^2}{180\pi(1-2\lambda)^3} + \frac{1}{\pi}$$

$$\eta_N = \frac{A_1(\lambda) g_N}{1 - A_2(\lambda) g_N} - g_N \frac{237\sqrt{3} - 160\pi}{240\pi^2} \, .$$

ghost contributions



UV-Fixed Point Structure

- fixed point analysis: $(\partial_t g_N, \partial_t \lambda) = (0,0)$

$$g^{\mathrm{UV}}_{N\,*} = 2.03 \quad , \quad \lambda^{\mathrm{UV}}_{*} = 0.22$$

UV fixed point !

→ **Strong support for the Asymptotic Safety Scenario !**

- effect of ghosts?

(1) first approximation: ignore all ghost contributions
(2) add classical ghost fields

	$g^{\mathrm{UV}}_{N\,*}$	$\lambda^{\mathrm{UV}}_{*}$	$\lambda^{\mathrm{UV}}_{*} \times g^{\mathrm{UV}}_{N\,*}$
no ghosts	1.95	0.11	0.21
with ghosts	2.03	0.22	0.45

→ classical ghosts: preserve fixed point

- dynamical ghosts in this setting: very small effect (N.C., diploma thesis)

→ perfect agreement with (Eichhorn,Gies),(Groh,Saueressig)

UV-Fixed Point Structure

- flow properties in the FP regime

critical exponents = - eigenvalues of stablity matrix

stability matrix:

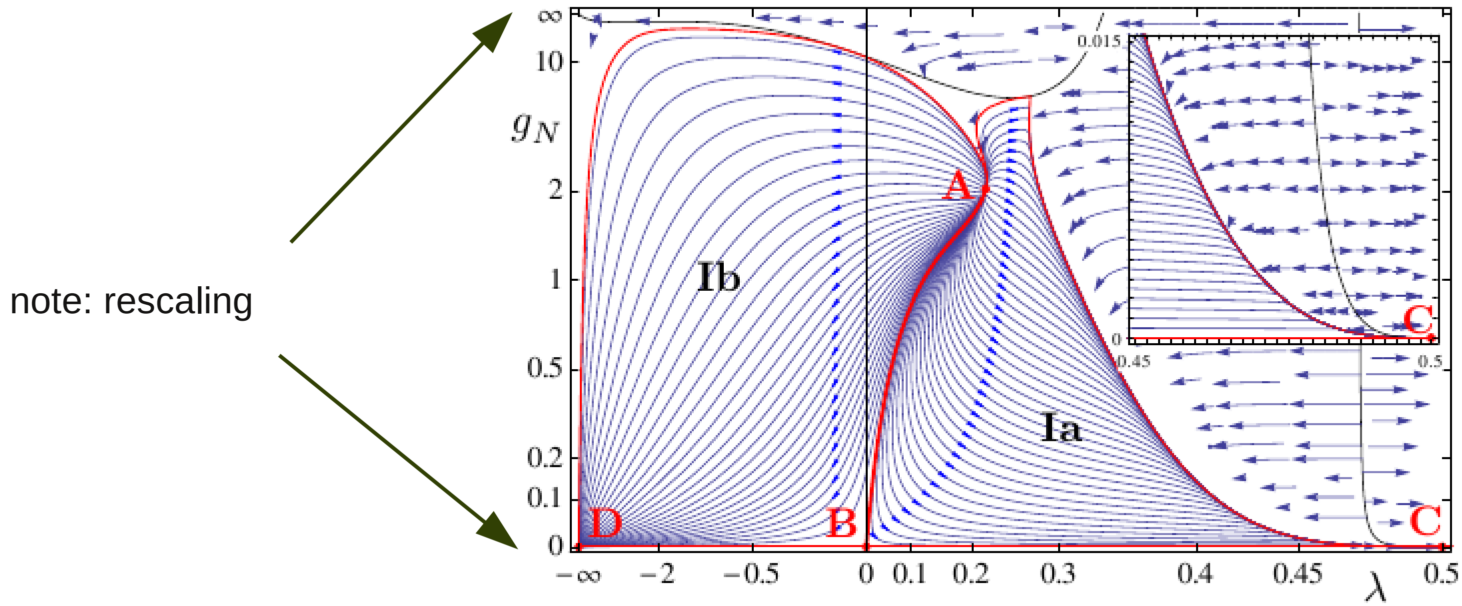
	g_{N*}^{UV}	λ_*^{UV}	$\lambda_*^{UV} \times g_{N*}^{UV}$	θ_1	θ_2
no ghosts	1.95	0.11	0.21	$3.09 + 2.00\,i$	$3.09 - 2.00\,i$
with ghosts	2.03	0.22	0.45	8.38	2.60

→ critical exponents turn real !!!

→ complex critical exponents $\overset{??}{\longleftrightarrow}$ unitarity violation

Phase Diagram & IR-Fixed Point

- global phase diagram:

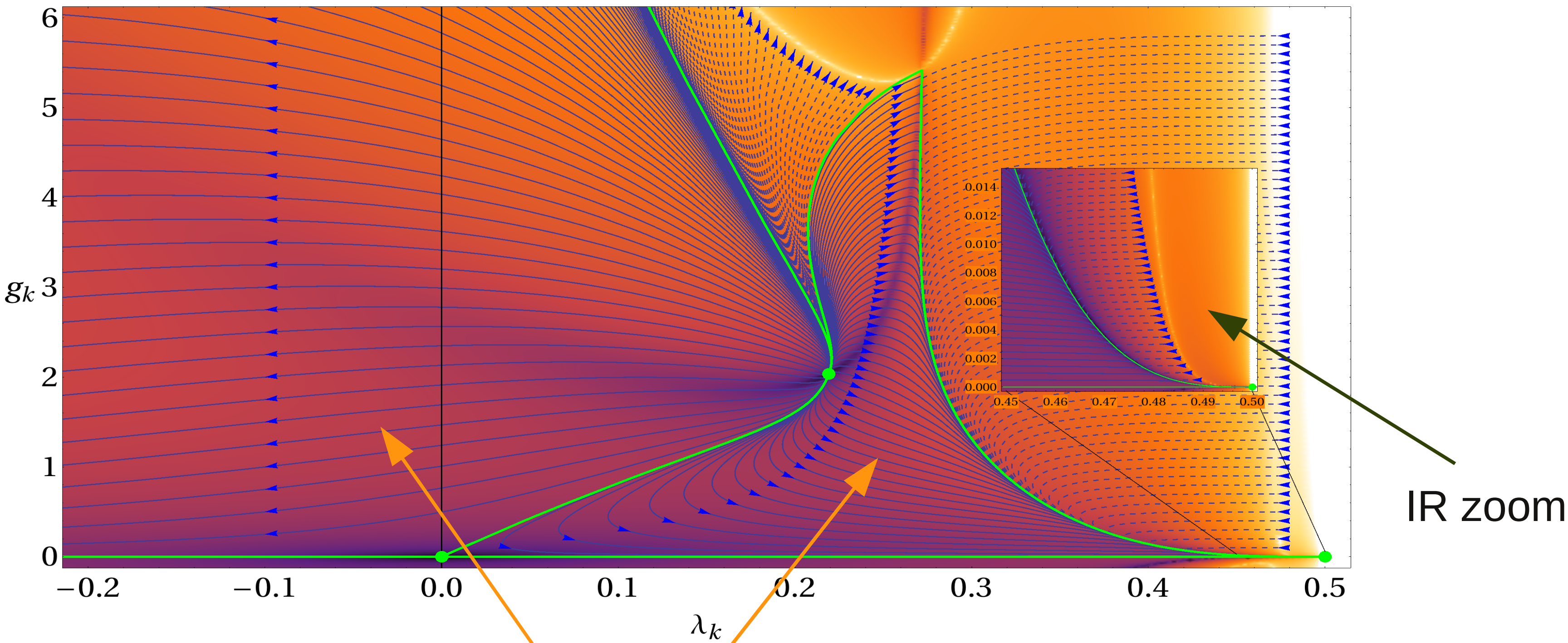


- non-Gaussian UV fixed point A: $(g_{N\,*}^{\mathrm{UV}}, \lambda_*^{\mathrm{UV}}) = (2.03, 0.22)$
- anti-de-Sitter IR-fixed point D: $(g_{N\,*}^{\mathrm{IR}}, \lambda_*^{\mathrm{IR}}) = (0, -\infty)$
- repulsive Gaussian IR fixed point B: $(g_{N\,*}^{\mathrm{IR}}, \lambda_*^{\mathrm{IR}}) = (0, 0)$

- attractive de-Sitter IR-fixed point C (new): $(g_{N\,*}^{\mathrm{IR}}, \lambda_*^{\mathrm{IR}}) = (0, 1/2)$

Phase Diagram & IR-Fixed Point

• new feature: **smooth infrared fixed point** $(g_{N\,*}^{\mathrm{IR}}, \lambda_*^{\mathrm{IR}}) = (0, 1/2)$ **!!!!!**



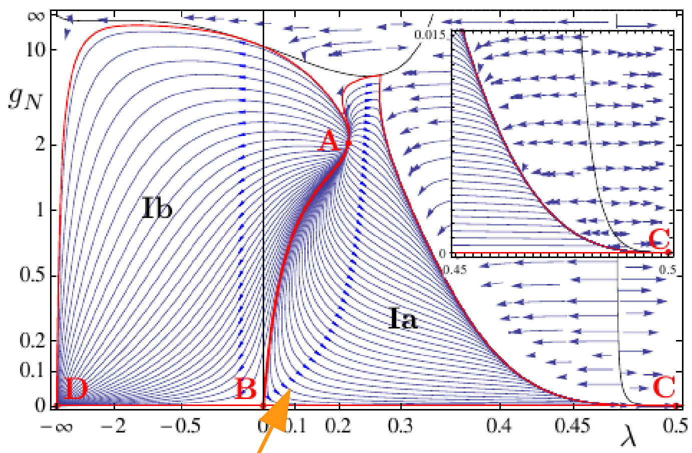
• the two UV and IR-safe regions with the separatrices

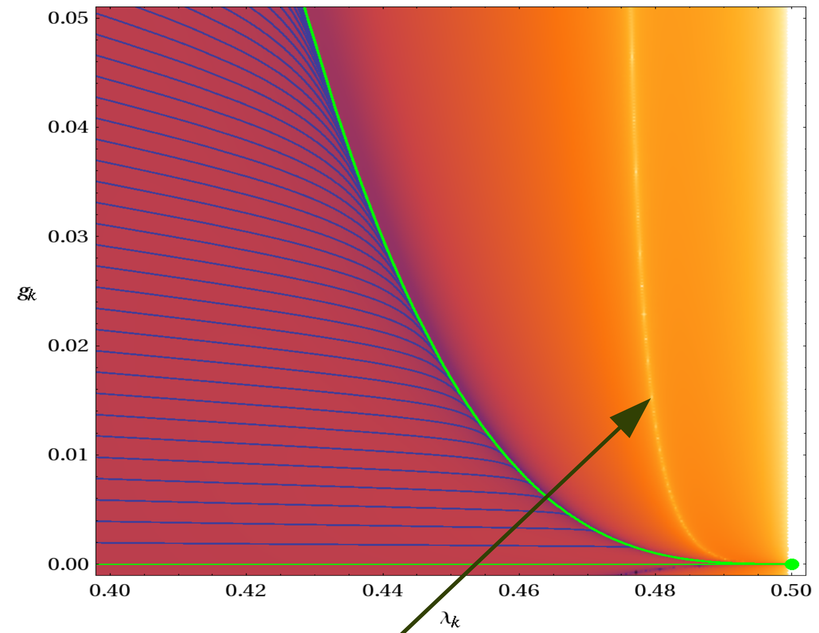
⟶ "physical" region of globally defined trajectories

$\lambda \leq \frac{1}{2}$!!

• real critical exponents $\Longrightarrow$ no spiralling

IR Analysis





semicalssical regime: $g \sim k^2$, $\lambda \sim k^{-2}$

the singular line is always to the right of the separatrix !
$\Longrightarrow$ no divergences !

→ <u>smooth UV-IR transition</u>

how is that possible for $\lim_{\lambda \to 1/2} (\partial_t g, \partial_t \lambda)$?

IR Analysis

- three propagators lead to divergence $\sim (1-2\lambda)^{-3}$ in $\mathrm{Flow}^{(2)}_{\mathrm{TT}}(p)$

→ $\partial_t \lambda \sim g_N A_3$

with $A_3 = \frac{1+4(1-2\lambda)^2}{12\pi(1-2\lambda)^3} - \eta_N \frac{12-45\lambda-40\lambda^2}{180\pi(1-2\lambda)^3} + \frac{1}{\pi}$

→ g_N must cancel this divergence in the IR limit !

- in the deep IR, $k < k_{\mathrm{IR}}$, we have a scaling

$$g_N = k^{\triangle_g} \quad , \quad \triangle_g > 0$$

$$(1-2\lambda) = k^{\triangle_{1-2\lambda}} \quad , \quad \triangle_{1-2\lambda} > 0$$

→ no singularity $\Longleftrightarrow$ $\triangle_g \geq 3\triangle_{1-2\lambda}$

IR Analysis

- scaling analysis of the flow with a fixed point assumption leads to

$$\triangle_g = 3\triangle_{1-2\lambda}$$

- a numerical study yields

$\triangle_g \approx 5.5$ and $\triangle_{1-2\lambda} \approx 1.8$

⟶ satisfies the exact relation ☑

⟶ **BUT : Non-classical scaling !**

physical relevance of the new fixed point ?

IR Analysis

- smooth, global trajectories Ia → (IR) - phenomenology

- two posssibilities:

IR modification of gravity due to strong RG effects?

→ in principle testable [*Reuter,Weyer*]

$g_N(k)$ and $\lambda(k)$ could be bad choices as observables:

- at best: $g_N(k) * \lambda(k)$ is observable
- the construction $G_N(k) = \frac{G_{N_0}}{Z_N(k)}$ is not valid
- relation graviton mass $\longleftrightarrow$ cosmological constant ?

Summary/Outlook

• distentangle fluctuation and background modes in the flow

→ flow of the graviton propagator

• momentum dependence: finite limit for large p

• definition of the couplings via bi-local projection

• phase diagram: UV fixed point + new IR fixed point: $(g^{\mathrm{IR}}_{N\,*}, \lambda^{\mathrm{IR}}_{*}) = (0, 1/2)$

smoothly connected!

Outlook:

• full momentum dependence: $Z_{N,k}(p^2)$

• three point function

• higher dimensional operators

• coupling to matter/gauge fields