A coarse-grained model of black hole thermodynamics

Kevin Falls, University of Sussex

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Outline

Main Ideas [1]:

- Understanding black hole (BH) thermodynamics as arising from an averaging of degrees of freedom via the renormalisation group.
- Go beyond the semi-classical approximation using a systematic coarse graining idea, interpolating from largest to smallest BH masses.

Outline:

- Black hole thermodynamics.
- Coarse grained model.
- Implications for asymptotic safety.
- Conclusion.

[1] based on KF and D Litim arXiv:1212.1821

Black holes in general relativity

- Solutions to Einsteins equations
- Stationary solutions are parametrized by just M, J and q.
- End point of gravitational collapse
- Uniqueness $M, J, q \to$ "No hair"

$$A = A(M, J, q)$$

- First law [2]

$$\frac{\kappa}{8\pi G_N}\delta A = \delta M - \Omega\delta J - \Phi\delta q$$

[2] Bardeen, Carter and Hawking '73

Black holes and thermodynamics

- Four laws of black hole mechanics
- Analogous to the laws of thermodynamics

Thermodynamics	Black holes
$T\delta S = \delta Q$	$\frac{\kappa}{8\pi G}\delta A = \delta Q$
T	κ
$\delta S \geq 0$	$\delta A \geq 0$

- Generalised second law [3] $\delta S + \delta S_{BH} \geq 0$

[3] Bekenstein '73

Beyond the analogy

- Hawking radiation ('75) QFT on curved space-time

$$T = \hbar \frac{\kappa}{2\pi}, \qquad S_{BH} = \frac{A}{\hbar 4G} \, .$$

- Jacobson(95')

$$\frac{\delta Q}{T} = \delta S \quad \Longleftrightarrow \quad G_{\mu\nu} + g_{\mu\nu}\Lambda = 8\pi G \; T_{\mu\nu}$$

for all local Rindler causal horizons.

- Suggests a deep connection between classical gravity, thermodynamics and quantum mechanics due to the presence of causal horizons.
- Is gravity/space-time fundamental or emergent?

Challenges for quantum gravity

- Black hole thermodynamics appears when quantum fields propagate on a background black hole space-time.
- A thermal bath of particles, as seen by observers far from the horizon, seems to contain no information of the matter that initially collapsed to form the black hole
- What happens when the black hole evaporates away completely? Information loss?
- Black hole thermodynamics seems to suggest that there exists an underlying microstructure of space-time
  $\Longrightarrow$ What are the fundamental building blocks of space-time?
- Strings, branes ? Spin foam?...

Quantum gravity

- Saddle point approximation to the Euclidean path integral

$$e^{-\Gamma} = \int \mathcal{D}g_{\mu\nu}\, e^{-I_{EH}[g_{\mu\nu}]}, \qquad \frac{\delta I_{EH}}{\delta g_{\mu\nu}}[\bar{g}_{\mu\nu}] = 0$$
$$\approx e^{-I_{EH}[\bar{g}_{\mu\nu}]}$$

- Hawking and Gibbons found that in this approximation

$$F \equiv T\Gamma = M - TS$$

- New idea: Use the coarse grained effective action $\Gamma \to \Gamma_k$ which provides a set of scale dependent models where fluctuations $p^2 > k^2$ have been included.

Wilsonian black hole thermodynamics

- Here we utilise the coarse grained effective action Γ_k which provides a set of scale dependent models where fluctuations $p^2 > k^2$ have been included.
- Jacobson and Satz arXiv:1212.6824 [hep-th]. Can the long wavelength modes $p^2 < k^2$ be interpreted as accounting for the entanglement entropy?
- Becker and Reuter arXiv:1205.3583 [hep-th]. Running boundary terms.

Wilsonian black hole thermodynamics

- Implement quantum corrections to the physics of black hole thermodynamics using the renormalisation group.
- We consider $4d$ gravity coupled to $U(1)$ gauge fields.
- The action

$$\Gamma_k[g_{\mu\nu}, A_\mu] = \int d^4x \sqrt{-\det g_{\mu\nu}} \left[\frac{1}{8\pi G_k}\mathcal{R} + \frac{1}{4\alpha_k} F^{\mu\nu} F_{\mu\nu} \right] + S_m \, .$$

- Scale dependent couplings G_k and $\alpha_k \equiv \frac{e_k}{4\pi}$ which run under the renormalisation group flow of the theory

QFT à la Wilson

- For intermediate scales k has the interpretation as the inverse "resolution"
$$\ell \approx 1/k \tag{1}$$
of the RG microscope through which the physics is observed.
- The equations of motion for the effective average action $\Gamma_k[\phi]$ give the coarse grained version of the microscopic dynamics where only the modes $p^2 > k^2$ have been averaged over
$$\frac{\delta \Gamma_k}{\delta \phi}[\phi] = \left\langle \frac{\delta I}{\delta \varphi}[\varphi] \right\rangle_k$$

Scale dependent black holes

- At fixed k, and by varying Γ_k with respect to the $g_{\mu\nu}$ and A_μ we recover Einstein-Maxwell theory.
- Solutions include a family of Kerr–Newman-type black holes additionally parameterised by the RG scale k via the running couplings.

$$A = A(M, J, q; k)$$

- Semi-classical limit ($k \rightarrow 0$) we have $G \approx 6.674 \times 10^{-11}\ \mathrm{N}\,(\mathrm{m}/\mathrm{kg})^2$ and $\alpha \approx \frac{1}{137}$.

Flowing entropy

- Consider the entropy off-shell

$$S_k = \frac{A}{4G_k}$$

  Depending independently on the metric via A and the coarse graining scale k.

- Decreasing k for fixed A we go from fine grained to coarse grained entropy. RG flow given by

$$\frac{\partial}{\partial \ln k} S_k = -S_k \frac{\partial \ln G_k}{\partial \ln k}$$

Variation of the entropy

- Vary the entropy at fixed coarse graining scale k to obtain thermodynamics
$$\delta S = \frac{\delta A}{4G_k}$$
- In analogy to the coarse grained dynamics obtained from varying Γ_k
$$\frac{\delta \Gamma_k}{\delta g_{\mu\nu}} = 0$$

Scale identification

- We will assume that there exists a scale $k = k_{\mathrm{opt}}$, associated to the macroscopic spacetime geometry,

$$k_{\mathrm{opt}} = k_{\mathrm{opt}}(M, J, q)$$

  such that $\Gamma_{k_{\mathrm{opt}}}$ gives a approximation to the full partition function.

- Under this identification we have a new set of RG improved Kerr-Newman-type black holes defined by a new state function

$$A(M, J, q) = A(M, J, q; k_{\mathrm{opt}})$$

- Again parameterised by three quantities.

Thermal equilibrium

- What about the thermodynamics? Reply Bekenstein's thought experiment.
- We now imagine that a small amount of matter to falls into a black hole of mass M, angular momentum J and charge q.
- While assuming the relation

$$\frac{\delta Q}{T} = \delta S_{k_{\mathrm{opt}}}$$

holds with $\delta S_{k_{\mathrm{opt}}} = \frac{\delta A}{4G_{k_{\mathrm{opt}}}}$.

- Heat crossing the horizon of a black hole

$$\delta Q = \delta M - \Omega \delta J - \Phi e_{k_{\mathrm{opt}}} \delta q \, .$$

Thermal equilibrium

- The black hole settles down into a new state with an area $A + \delta A$

$$\delta A = 4 G_{k_{\rm opt}} \frac{2\pi}{\kappa} \delta Q + \left. \frac{\partial A(M, J, q; k)}{\partial \ln k} \right|_{k = k_{\rm opt}} \frac{\delta k_{\rm opt}}{k_{\rm opt}} \, .$$

- First term follows from the classical first law of black hole mechanics.
- Second term is proportional to the RG β-functions and takes into account quantum corrections.

Thermal equilibrium

- The black hole settles down into a new state with an area $A + \delta A$

$$\delta A = 4 G_{k_{\rm opt}} \frac{2\pi}{\kappa} \delta Q + \left. \frac{\partial A(M, J, q; k)}{\partial \ln k} \right|_{k = k_{\rm opt}} \frac{\delta k_{\rm opt}}{k_{\rm opt}} \, .$$

- Rearranging this equation for the heat δQ and inserting it into the LHS of $\frac{\delta Q}{T} = \frac{\delta A}{4 G_{k_{\rm opt}}}$ we obtain

$$\left(1 - \frac{2\pi}{\kappa} T \right) \delta A = \left. \frac{\partial A(M, J, q; k)}{\partial \ln k} \right|_{k = k_{\rm opt}} \frac{\delta k_{\rm opt}}{k_{\rm opt}} \, .$$

- In the presence of RG corrections describes quantum corrections to the temperature.

Thermal equilibrium

- Thermodynamics implies that $\delta k_{\rm opt}$ must be proportional to δA independently of the heat δQ. Under our assumptions

$$\delta k_{\rm opt} \propto \delta A, \qquad \text{thus} \quad k_{\rm opt}(M, J, q) \equiv k_{\rm opt}(A(M, J, q))$$

- Dimensional analysis then dictates that this relation reads

$$k_{\rm opt}^2 = \frac{4\pi}{A}\,\xi^2$$

where ξ is an undetermined dimensionless positive constant of order unity.

- This constant depends on the RG-scheme used $\xi = \xi(R_k)$.
- Will set $\xi = 1$ for simplicity.

Mass function

- Modified relation between the area A and the parameters M, J and q.

$$M^2 \equiv \frac{4\pi}{A}\left[\left(\frac{A + 4\pi G_{\rm opt}(A) e^2_{\rm opt}(A) q^2}{8\pi G_{\rm opt}(A)}\right)^2 + J^2\right]$$

- Temperature

$$T = 4 G_{\rm opt}(A) \frac{\partial M}{\partial A}$$

- Semi-classical limit $A \to \infty$

Asymptotic safety

- Our reasoning so far has been independent of the actual form of the RG running couplings and therefore the UV completion of gravity
- Asymptotic safety is a possible UV completion of gravity
- Dimensionless coupling constants reach a non-Gaussian fixed point at high energies.
  $\longrightarrow$ Ensures that the continuum limit may be taken.
- Finite number of relevant directions flowing away from the fixed point to the IR.
  $\longrightarrow$ Theory is predictive.

Asymptotic safety and black hole thermodynamics

- Go beyond the semiclassical approximation assuming

$$\frac{1}{G_k} = \frac{1}{G_N} + \frac{k^2}{g^*}$$

- Cross over between classical $G \approx G_N$ and fixed point $G \propto k^{-2}$ scaling governed by the characteristic energy scale

$$E_c^2 = g^* M_P^2 = \frac{g^*}{G_N}$$

Asymptotic safety and black hole thermodynamics

- Inserting the running Newton's constant into the mass function with $q = 0$ and resolve for A

$$A_{\pm} = 4\pi G_N \left(2 G_N M^2 - G_N M_c^2 \pm 2\sqrt{G_N^2 M^4 - J^2 - G_N^2 M_c^2 M^2} \right) .$$

- Characteristic mass scale

$$M_c^2 = \frac{1}{g^*} M_P^2 .$$

  - Smallest black hole mass.

- Semi-classical limit $1/g^* \to 0$ leads to $M_c \to 0$ and $E_c \to \infty$.

Temperature

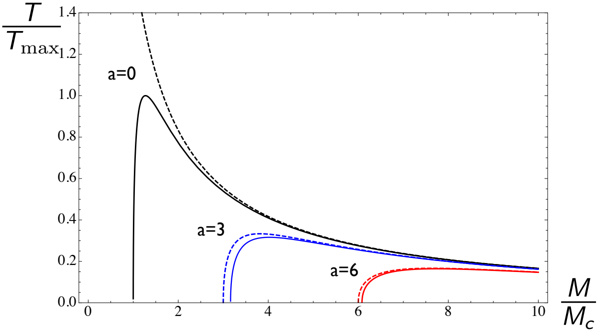

Figure: Horizon temperature as a function of the black hole mass, comparing classical gravity with asymptotically safe gravity with $g_* = 1$ for several angular momenta $a = J/M$, with a given in units of $1/M_c$.

Specific heat

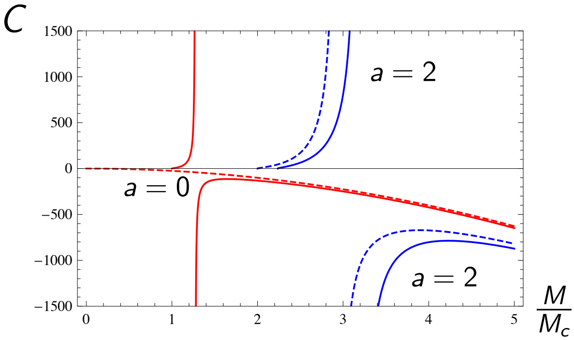

Figure: Specific heat as a function of the black hole mass, comparing classical gravity with asymptotically safe gravity ($g_* = 1$, solid lines) for several angular momenta a, given in units of $1/M_c$.

Conformal scaling

- Suggested by O. Aharony and T. Banks and by A. Shomer that gravity cannot exist as a local QFT
- Consider the case for general d with $J = q = 0$
- At a UV fixed point we expect that a theory behaves as a CFT where by the entropy and energy should scale as $S \sim (RT)^{d-1}$ and $E \sim R^{d-1}T^d$.
- For black holes the radius R depends on the energy $E = M$.
- Therefore we consider the relation

$$\frac{S}{R^{d-1}} \sim \left(\frac{E}{R^{d-1}}\right)^{\nu} .$$

With $\nu_{\mathrm{CFT}} = \frac{d-1}{d}$. However for classical black holes $\nu_{\mathrm{BH}} = \frac{1}{2}$

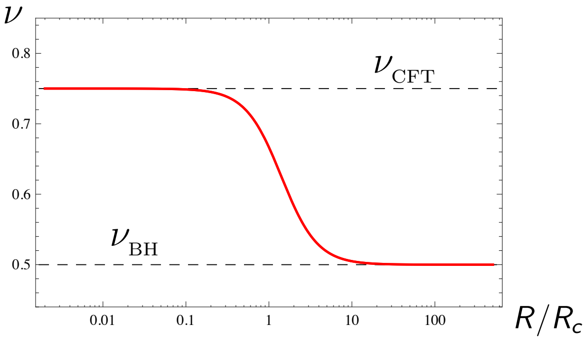

Figure: Scaling index for an asymptotically safe Schwarzschild black hole in four dimensions interpolating between the classical value $\nu_{\rm BH}$ for large horizon radii and the conformal limit $\nu_{\rm CFT}$ for small radii.

Relation to RG improved space-times

- RG improved space-times put $G(k)$ directly into the classical metric
- Using a scale identification $k(r)$ (Reuter and Bonnano '00, K.F., Litim, Raghuraman '10, Cai, Easson '10,Reuter, Tuiran, '10)

$$
\begin{aligned}
ds^2 &= -\left(1-\frac{2G(r)Mr}{\rho^2}\right)dt^2 - \frac{2G(r)Mar\sin^2\theta}{\rho^2}(dtd\phi + d\phi dt) \\
&+ \frac{\rho^2}{\Delta}dr^2 + \rho^2 d\theta^2 + \frac{\sin^2\theta}{\rho^2}\left[(r^2+a^2)^2 - a^2\Delta sin^2\theta\right]d\phi^2
\end{aligned}
$$

where $a = \frac{J}{M}$, $\Delta(r) = r^2 - 2G(r)Mr + a^2$, $\rho^2 = r^2 + a^2\cos^2\theta$

- Can these metrics carry the same thermodynamics?

Relation to RG improved space-times

- Horizons at

$$\Delta(r_s) \equiv r_\pm^2 - 2G(r_\pm)Mr_\pm + a^2 = 0$$

$A_\pm = 4\pi(r_\pm^2 + a^2)$

- Can be shown to be equivalent for

$$k^2(r) = \frac{\xi^2}{(r^2 + a^2)}$$

- Reproducing

$$\delta M = \frac{T}{4G(r)}\delta A + \Omega\delta J + e^2(r)\Phi\delta q$$

where $T = \frac{\kappa}{2\pi}$.

RG improved Schwarzschild entropy

- Reuter and Bonnano 00': Put the RG improved Schwarzschild metric into the classical Euclidean action to obtain the free energy

$$F = \frac{r_+}{2G_N} - \frac{A}{4G_N} T$$

- If instead the action with $G_N \to G_{\rm opt}(A)$ is used and identify k^2 with the inverse area one obtains

$$F = M - ST$$

where $S = \frac{A}{4G_{\rm opt}(A)}$ and M is RG improved mass function.

Conclusions

Black hole thermodynamics + RG

- is compatible with RG running of couplings assuming a scale dependent entropy
- requires that RG cutoff is set by BH area
- has physical interpretation that sub-horizon modes are integrated out - consistent picture.
- Implies thermodynamics is consistent even away from the semi-classical limit.

Conclusions

Asymptotic Safety + Black hole thermodynamics

- Existence of smallest black hole mass M_c.
- Maximum temperature $T_{\max}$.
- Generic existence of inner horizon.
- Potential existence of BH remnant.
- Conformal scaling.