Dilaton Quantum Gravity

A Functional Renormalization Group Approach

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based on arXiv:1304.7743

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Outline

Renormalizibility and Quantum Gravity

- Einstein-Hilbert action

$$\Gamma\left[g_{\mu\nu}\right] = \frac{1}{16\pi G_N} \int \mathrm{d}^d x \sqrt{g}\, (2\Lambda - R[g_{\mu\nu}])$$

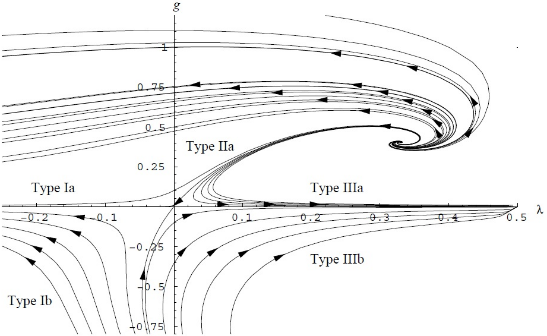
- mass dimensions of the couplings

$$dim(\Lambda) = 2$$

$$dim(G_N) = 2 - d$$

$\Longrightarrow$ Perturbatively not renormalizable if $d > 2$.

The Flow Diagram of Quantum Einstein Gravity

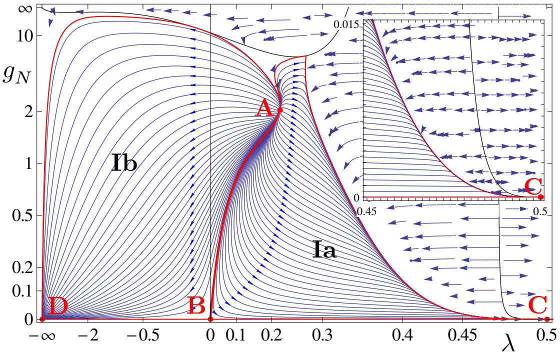


Reuter, 1998

Coupling Constants approach a nontrivial UV Fixed Point

$\Longrightarrow$ Prospect of Gravity being Asymptotically Safe

The Flow Diagram of Quantum Einstein Gravity



Christiansen, Litim, Pawlowski, Rodigast, 2012

Stable Infrared Scenarios

$\Longrightarrow$ Prospect of UV and IR consistent theory

Asymptotically Safe Quantum Gravity

FRG Technicalities

- Regulator Dependence
- Background Dependence
- Truncation Stability

Coupling to SM

- Yang Mills Theory
- Background Dependence
- Asymptotic Freedom

Infrared Limit

- Trajectory UV $\rightarrow$ IR
- GR as limiting case?

Hierarchy Problem

- $M_{SM} \approx 10^2$ GeV
- $M_{Planck} \approx 10^{19}$ GeV

Setup & Action

$$\Gamma_k\left[g_{\mu\nu}\right] = \frac{1}{16\pi G_{N,k}} \int \mathrm{d}^d x \sqrt{g}\left(2\Lambda_k - R[g_{\mu\nu}]\right)$$

$$\Downarrow \qquad \Downarrow \qquad \Downarrow$$

$$\Gamma_k\left[g_{\mu\nu},\chi\right] = \int \mathrm{d}^d x \sqrt{g}\left(V_k[\chi] - \frac{1}{2}F_k[\chi]R[g_{\mu\nu}] + \frac{1}{2}g_{\mu\nu}\partial^\mu\chi\partial^\nu\chi\right)$$

First investigated by Narain & Percacci, 2010

Extensions:

- investigation of IR limit
- trajectory UV $\leftrightarrow$ IR
- improved UV analysis
- all investigations in $d = 4$ & deDonder gauge

Dilatation Symmetry

- Dilatations $\leftrightarrow$ Conformal Transformations
  $g_{\mu\nu}(x) \mapsto \Omega(x) g_{\mu\nu}(x)$
  with $\Omega = const.$
- Dilatation $\leftrightarrow$ global resetting of the physical scale
- Dilatation Symmetry $\leftrightarrow$ Physical Scale is introduced only by expectation value of the scalar field
- Arising Goldstone Boson: Dilaton

Dilatation Symmetric Actions

Γ is invariant under dilatations

$\Updownarrow$

all couplings have scaling dimension 0.
$(d = 4 : F \propto \chi^2,\ V \propto \chi^4)$

Physical Motivation

Infrared Limit	Hierarchy Problem
• Trajectory UV $\rightarrow$ IR • GR as limiting case?	• $M_{SM} \approx 10^2$ GeV • $M_{Planck} \approx 10^{19}$ GeV

a scale invariant IR limit for Einstein-Hilbert Quatum Gravity

b generation of Planck mass via breaking of dilatation symmetry

c quintessence cosmology scenarios with vanishing cosmological constant

Fixed Point Action

Let $y = \frac{\chi^2}{k^2}$, $\quad V_k(\chi^2) = k^4 y^2\, v_k(y), \quad F_k(\chi^2) = k^2 y\, f_k(y)$

$\Rightarrow$ Dilatation Symmetric parts factored out

proposed fixed point for large y

$$\lim_{y\to\infty} f(y) = \xi \qquad \lim_{y\to\infty} v(y) = 0.$$

$$\Rightarrow \quad \Gamma = \int d^4x \sqrt{g} \left(\tfrac{1}{2} g^{\mu\nu} \partial_\mu \chi\, \partial_\nu \chi - \tfrac{1}{2} \xi \chi^2 R \right) \qquad \text{(Jordan)}$$

$$\Rightarrow \quad \Gamma = \int d^4x \sqrt{g} \left(\tfrac{1}{2} g^{\mu\nu} \partial_\mu \phi\, \partial_\nu \phi - \tfrac{1}{2} M^2 R \right) \qquad \text{(Einstein)}$$

Properties of the Fixed Point

- Let $f_0 = \lim_{y\to\infty} f_k(y)$

$\Rightarrow$ Gravitational Interaction scales like $f_0^{-1}\chi^{-2}$

$\Rightarrow$ Gravity induced flow from $f_0^{-1}y^{-1}$ with $\lim_{y\to\infty} f_0^{-1}y^{-1} = 0$

$\Rightarrow$ For $\lim_{y\to\infty} v_k(y) = 0$ free scalar theory

- Flow of free scalar theory can at most produce a constant term $\propto v_{-2}$ in V and
- For $g_N = 0$ no flow of gravitational interaction

$\Rightarrow$ Asymptotically

$$\lim_{y\to\infty} v(y) = v_{-2}\, y^{-2} + \dots$$

$$\lim_{y\to\infty} f(y) = \xi + f_{-1}\, y^{-1} + \dots$$

Flow Equation

- $\mathcal{O}(R)$ truncation
- no wave function renormalization
- curved background spacetime
- optimized cut-offs with

$$\begin{aligned} \Gamma_k^2(p^2) + \mathcal{R}_k(p^2) &= \Gamma_k^2\left(p^2 + r_k(p^2)\right) \\ r_k(p^2) &= (k^2 - p^2) \times \Theta(k^2 - p^2) \end{aligned}$$

- set of flow equations at fixed y

$$\begin{aligned} \partial_t v_k(y) &= 2\,y\,v_k'(y) + \frac{1}{y^2}\,\zeta_V, \\ \partial_t f_k(y) &= 2\,y\,f_k'(y) + \frac{1}{y}\,\zeta_F. \end{aligned}$$

- deDonder gauge fixing and mostly $d = 4$

Flow Equation II

$$\zeta_V = \frac{1}{192\pi^2}\left\{6 + \frac{30\,\tilde{V}}{\Sigma_0} + \frac{3(2\Sigma_0 + 24\,y\,\tilde{F}'\,\Sigma_0' + \tilde{F}\Sigma_1)}{\Delta} + \delta_V(\partial_t \tilde{F})\right\}$$

$$\begin{aligned}
\zeta_F = \frac{1}{1152\pi^2}\Bigg\{ & 150 + \frac{30\,\tilde{F}\,(3\,\tilde{F} - 2\tilde{V})}{\Sigma_0^2} \\
& - \frac{12}{\Delta}\left(24\,y\,\tilde{F}'\,\Sigma_0' + 2\Sigma_0 + \tilde{F}\Sigma_1\right) - 6y\,(3\,\tilde{F}'^2 + 2\Sigma_0'^2) \\
& - \frac{36}{\Delta^2}\Big[2y\,\Sigma_0\,\Sigma_0'\,(7\,\tilde{F}' - 2\tilde{V}')\,(\Sigma_1 - 1) + 2\,\Sigma_0^2\,\Sigma_2 \\
& + 2\,y\Sigma_1\,(7\,\tilde{F}' - 2\tilde{V}')\,(2\,\Sigma_0\,\tilde{V}' - \tilde{V}\,\Sigma_0') \\
& + 24\,y\,\tilde{F}'\,\Sigma_0\,\Sigma_0'\,\Sigma_2 - 12\,y\,\tilde{F}\,\Sigma_0'^2\,\Sigma_2\Big] + \delta_F(\partial_t \tilde{F})\Bigg\}
\end{aligned}$$

$$\begin{aligned}
\tilde{V} = y^2\,v_k(y)\;,\;\; \tilde{F} &= y\,f_k(y), \\
\Sigma_0 = \frac{1}{2}\tilde{F} - \tilde{V}\;,\;\; \Delta &= \left(12\,y\,\Sigma_0'^2 + \Sigma_0\,\Sigma_1\right), \\
\Sigma_1 = 1 + 2\,\tilde{V}' + 4\,y\,\tilde{V}''\;,\;\; \Sigma_2 &= \tilde{F}' + 2\,y\,\tilde{F}''
\end{aligned}$$

Fixed Point Solutions

Aim: Find Fixed Point Solutions

a for $y \to 0$ or ∞

- Taylor Expansions in y or y^{-1}
- free parameter $\xi = \lim_{y\to\infty} f_0$

Ultraviolet: $y \to 0$

- $\chi \to 0$ or $k \to \infty$
- Percacci at al, 2010

Infrared: $y \to \infty$

- $\chi \to \infty$ or $k \to 0$
- current project, 2013

b globally

- connecting UV & IR
- Padé ressumation
- exponential improvement

The Limit $y \to \infty$

- Expand

$$v(y) = v_0 + v_{-1} y^{-1} + v_{-2} y^{-2} + \dots,$$
$$f(y) = f_0 + f_{-1} y^{-1} + \dots$$

- Finite limits for ζ_V and ζ_F

$\Rightarrow$ Flow of v_0, v_{-1} and f_0 vanishes

- For $v_0 = v_{-1} = 0$

$$\bar{\zeta}_V = \lim_{y\to\infty} \zeta_V = \frac{3}{32\pi^2} + \frac{5 + 33 f_0}{96\pi^2 (1 + 6 f_0)} \frac{\partial_t f_0}{f_0}$$

$$\bar{\zeta}_F = \lim_{y\to\infty} \zeta_F = \frac{77 + 534 f_0}{192\pi^2 (1 + 6 f_0)} + \frac{17 + 186 f_0 + 720 f_0^2}{576\pi^2 (1 + 6 f_0)^2} \frac{\partial_t f_0}{f_0}$$

- closed set of flow equations guaranteed by derivative structure

$\Rightarrow$ fixed point solutions

$$\lim_{y\to\infty} f^*(y) = \xi + \frac{\bar{\zeta}_F}{2y}, \quad \lim_{y\to\infty} v^*(y) = \frac{\bar{\zeta}_V}{4y^2}.$$

The Limit $y \to 0$

- Expand

$$\tilde{F} = yf = F_0 + F_1 y + F_2 y^2 + \dots$$
$$\tilde{V} = y^2 v = V_0 + V_1 y + V_2 y^2 + \dots$$

- system of flow equations not closed

$\Rightarrow$ Treat V_0 and F_0 as “free” parameters

- extension from Narain, Percacci 2010

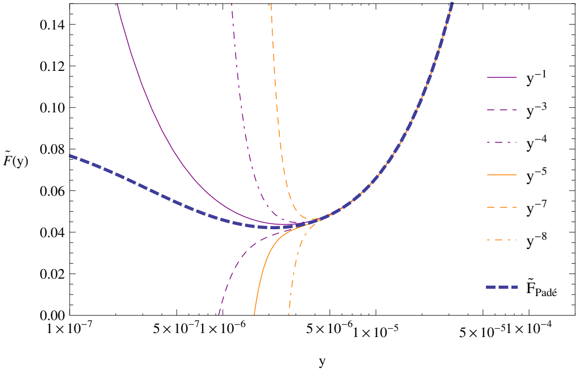
- Apparent good convergence of the series

Closing the Gap: Possible Global Solutions

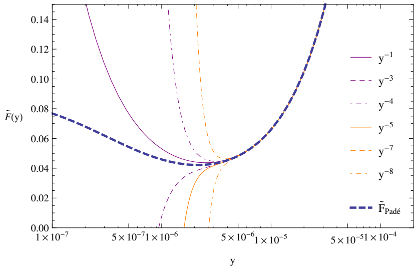
a Einstein-Hilbert Gravity + Scalar Field

$\Leftrightarrow \tilde{V} = y^2 v(y) = 0.008620 \quad \text{and} \quad \tilde{F} = yf(y) = 0.04751$

b Continuations of Taylor Expansions around $y = \infty$



Scaling Solution



Define an error via

$$
\begin{aligned}
\tilde{V}''(y) &= \mathcal{G}_{\tilde{V}}(y, \tilde{F}, \tilde{F}', \tilde{V}, \tilde{V}'), \\
\tilde{F}''(y) &= \mathcal{G}_{\tilde{F}}(y, \tilde{F}, \tilde{F}', \tilde{V}, \tilde{V}') \\
\varepsilon &= \frac{1}{2}\frac{\left(\mathcal{G}_{\tilde{V}} - \tilde{V}''\right)^2}{\mathcal{G}_{\tilde{V}}^2 + \tilde{V}''^2} + \frac{1}{2}\frac{\left(\mathcal{G}_{\tilde{F}} - \tilde{F}''\right)^2}{\mathcal{G}_{\tilde{F}}^2 + \tilde{F}''^2} \\
\mathcal{E} &= \int_0^{y_0} \varepsilon(y)dy
\end{aligned}
$$

- Taylor expansions around $y = \infty$ have $\epsilon \leq 10^{-2}$ for $y \leq y_0 \approx 1/(100\xi)$
- Brakedown of y^{-1} expansions expected for $y \to 0$

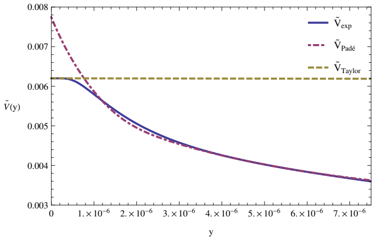
Improvement I: Padé ressummation

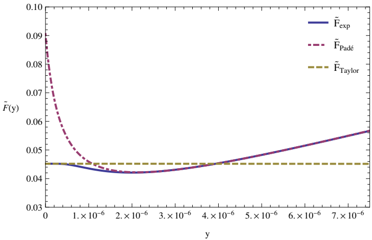
Strategy:

- stick with Taylor Solution for as long as possible
- smoothen solution for $y \to 0$
- connect to known solution for small y

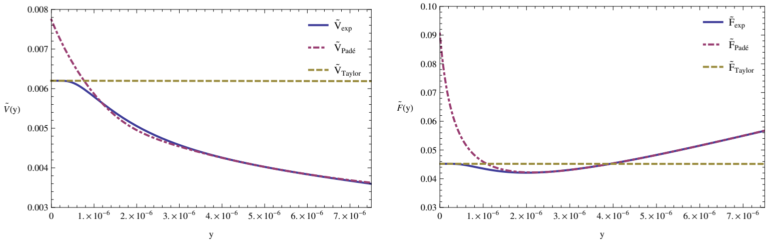
Padé resummation:

- Expand both numerator and denominator in powers of y^{-1}
- dependency on parameter ξ remains





Improvement II: Exponential Functions



- Propagators Σ_0, Δ produce singularities in intermediate region
- See y as a coupling & use exponential parts $\propto \exp(-c/y)$

$$\tilde{F}_{\text{exp}}(y) = \tilde{F}_{\text{Taylor}}(y) + e^{-c/y}\tilde{F}_e(y); \quad \tilde{V}_{\text{exp}}(y) = \tilde{V}_{\text{Taylor}}(y) + e^{-c/y}\tilde{V}_e(y)$$

- parameter optimisation to connect solutions & minimise integrated error $\mathcal{E}$

Physical Implications

- Dilatation symmetric infrared limit due to
$$\lim_{y\to\infty} V = 0, \qquad \lim_{y\to\infty} F = \xi\chi^2$$
- Planck mass is generated by spontaneous breaking of Dilatation Symmetry via $\langle\chi\rangle = 0$

$\Rightarrow$ Realistic gravity possible on a pure fixed point trajectory

- quintessence cosmology for small deviations from fixed point and $y \gg 1$

$$\begin{aligned}
\Gamma &= \int d^4x\sqrt{g}\left\{-\frac{M^2}{2}R + \frac{1}{2}k^2(\varphi)\partial^\mu\varphi\partial_\mu\varphi + V(\varphi)\right\} \\
k^2(\varphi) &= \frac{1}{4\xi}\left[1 + 6\xi + \left(\frac{M^2}{\bar{F}}\exp\left(\frac{\varphi}{M}\right) - 1\right)^{-1}\right] \\
V(\varphi) &= \bar{V}\exp\left(-\frac{2\varphi}{M}\right)
\end{aligned}$$

$\Rightarrow$ cosmological constant vanishes for $\varphi \to \infty$

Summary and Outlook

- prospect of scaling solution with scale invariant IR limit
- cosmological constant vanishes on and away from FP in the IR
- for small deviation from the fixed point trajectory admits quintessence cosmology

Thank you!