PERIMETER INSTITUTE FOR THEORETICAL PHYSICS

Higgs As Curvaton in Asymptotic Safe Gravity Scenario

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Based on:

Y.-F. Cai, Y.-C. Chang, P. Chen, D. A. Easson, T. Qiu, arXiv:1304.6938 [hep-th].

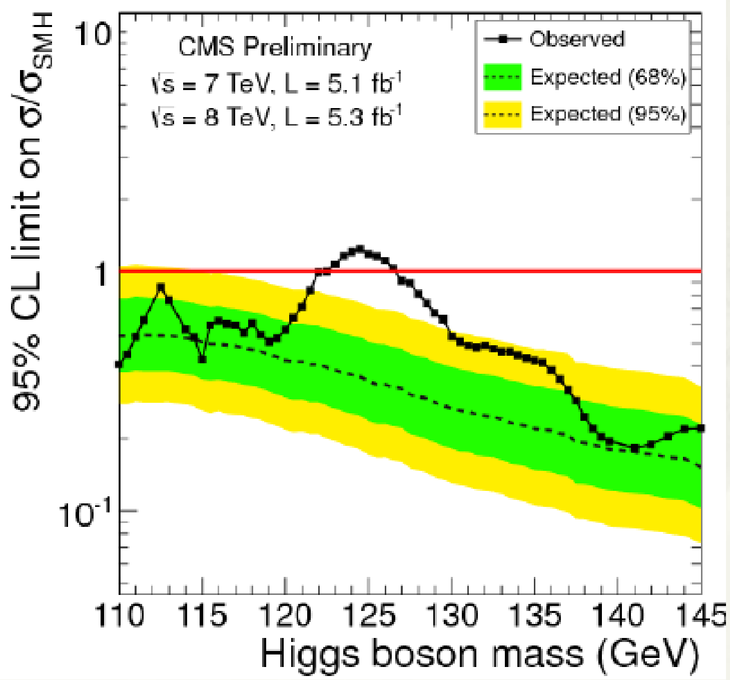
Outline

- Higgs vs. Cosmology
- Preliminary of Asyptotic Safe Gravity
- Our model of Higgs Curvaton in ASG
  - background
  - perturbation
  - non-Gaussianities
  - contraints form Planck data
- Conclusion

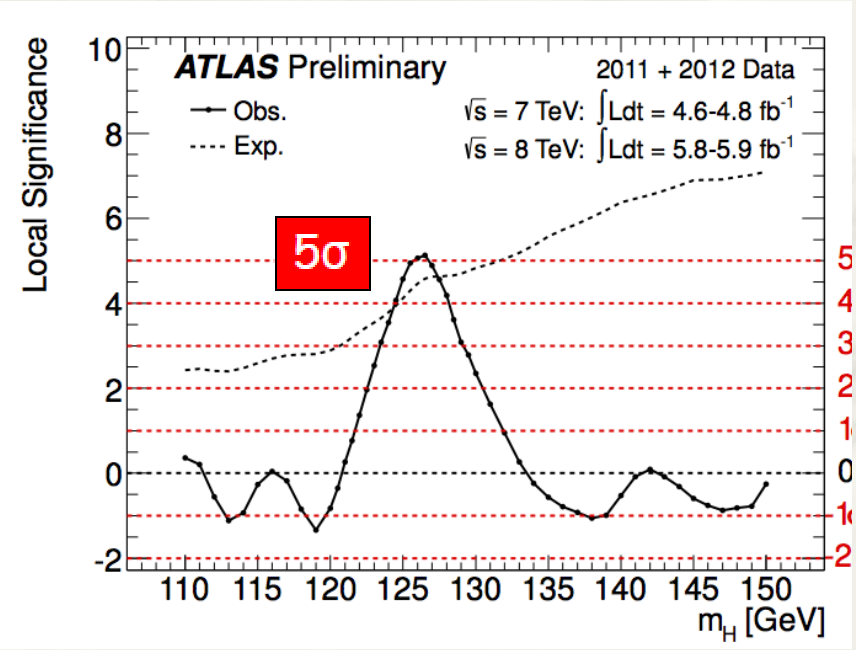
Higgs vs. Cosmology

The discovery of Higgs Boson

Recently, two groups (ATLAS and CMS) in LHC experiment have confirmed the existence of Higgs boson particles predicted in standard model particle physics with the mass value $m_H \simeq 125 - 126 GeV$



CMS Collaboration



ATLAS Collaboration

Can Higgs play a role in Cosmology?

Connecting cosmology with particle physics:

Scalar fields:

cosmology		Particle physics
inflaton, curvaton, quintessence……	$\Longleftrightarrow$	Higgs boson

Attempts & Problems:

Higgs inflation → unitarity problem

C. P. Burgess, H. M. Lee and M. Trott, JHEP 0909, 103 (2009).

Higgs curvaton → Too small energy density ratio cannot give rise to enouph perturbation.

T. Kunimitsu and J. 'i. Yokoyama, Phys. Rev. D 86, 083541 (2012);
K. -Y. Choi and Q. -G. Huang, arXiv:1209.2277 [hep-ph].

The problem of Higgs as a curvaton

The problem of Higgs curvaton:

When Higgs is to be curvaton, the value of Higgs field h remains constant till the curvaton decays, when the effective mass is close to Hubble parameter:

$$m_h^2 = 3\lambda h^2 \simeq H_{osc}^2 \Rightarrow \frac{\rho_h}{\rho} \simeq \frac{H^2}{M_p^2} \sim 10^{-10}$$

which induces

$$\frac{\delta\rho}{\rho} \simeq \frac{\rho_h}{\rho}\frac{\delta\rho_h}{\rho_h} \simeq \frac{\rho_h}{\rho}\frac{\delta h}{h}$$

$$\mathcal{O}(1) \because \delta h \simeq H/2\pi \ , \ h \simeq H$$

Too small to fit the observational data!

K. -Y. Choi and Q. -G. Huang, arXiv:1209.2277 [hep-ph].

Can the problem be solved in framework of Asymptotic Safe Gravity?

Preliminary of Asyptotic Safe Gravity

Problem: Quantization of Gravity

In quantum field theory, people often check the renormalizability of any terms in Lagrangian by its coupling constant!

Consider the operator in the lagrangian:

$\mathcal{L}[\phi, \partial\phi] \supset \mathcal{O}(\phi) = g_n \phi^n$ $\quad [g_n] = 4 - n \quad$ $\int g_n p^{n-5} dp \sim p^{n-4}$

$n = 4$	$[g_n] = 0$	$\int g_n p^{n-5} dp \sim \ln p$	renormalizable	$\lambda\phi^4$
$n < 4$	$[g_n] > 0$	$\int g_n p^{n-5} dp \sim p^{-2}$	Super-renormalizable	$\frac{1}{2}m^2\phi^2$
$n > 4$	$[g_n] < 0$	$\int g_n p^{n-5} dp \sim p^2$	non-renormalizable	$\frac{(\Box\phi)^2}{M^2}$

Another non-renormalizable example is the operator of Einstein-Hilbert Gravity $\frac{R}{8\pi G_N}$ whose coupling constant is G_N with $[G_N] = -2$

What is Asymptotic Safe Gravity (ASG)

Proposal 1970's by Steven Weinberg

S. Weinberg., Plenum Press, New York, 1977;
S. Weinberg., Cambridge University Press, 1979.

Action

$$S_p[g_{\mu\nu}] = \int d^4x \sqrt{-g}\Big[p^4 g_0(p) + p^2 g_1(p) R + g_{2a}(p) R^2 + g_{2b}(p) R_{\mu\nu}R^{\mu\nu} + g_{2c}(p) R_{\mu\nu\sigma\rho}R^{\mu\nu\sigma\rho} + O(p^{-2}R^3) + \cdots\Big],$$

Here the newly defined g's are all dimensionless, but may run with scale!

when $p \to \infty$ g_i diverge: unrenormalizable (UV incomplete)!
g_i finite: renormalizable (UV complete)!

The renormalization group equations which rule the behaviors of g's:

$$\frac{d}{d\ln p} g_i(p) = \beta_i[g(p)]$$

What is Asymptotic Safe Gravity (ASG)

- ✓ Condition for the theory to be renormalizable (UV complete):

  All g's are fixed at some point where $g_i = g_i^*$ when $p \to \infty$

  $\beta_i(g_i^*) = 0$

  ($g_i^* = 0$: Gaussian Fixed Point; $g_i^* \neq 0$: Non-Gaussian Fixed Point)

  The solution to the renormalization group equation:

$$g_i(p) = g_i^* + \sum_m e_i^n \left(\frac{p}{M_*}\right)^{v_n}$$

  e^n and v_n : eigenvectors and corresponding eigenvalues of the matrix $\mathcal{B}_{ij} \equiv \frac{\partial \beta_i[g]}{\partial g_j}^*$

- ✓ Condition for the fixed point to be attractor:

$$Re[v_n] < 0$$

How It Can Solve Quantum GR

The first order action of ASG:

$$\mathcal{L}_{\mathrm{AS}} = \frac{R - 2\Lambda(p)}{16\pi G(p)}$$

where $G(p)$ and $\Lambda(p)$ are Newtonian coupling constant and cosmological constant which vary with energy scale, and $G(p) = \frac{1}{g(p)p^2}$

The quantum correction:

$$\int_0^\infty G(p)p\,dp = \int_0^\infty \frac{1}{g(p)p}dp = \int_0^\infty \frac{1}{g(p)}d\ln p$$

If $g(p)$ reaches fixed point g^* when $p \to \infty$, we have:

$$\int_0^\infty \frac{1}{g(p)}d\ln p \simeq \int_0^{p_*} \frac{1}{g(p)}d\ln p + \frac{1}{g^*}\int_{p_*}^\infty d\ln p$$

Renormalizable!

Note that when $p \to 0$, $G(p)$ should also be able to return to G_N in order to recover General Relativity!

Further Investigations

- Calculate non-trivial ultraviolet fixed point in d dimension ($2 \simeq d \leq 4$) [hep-th/9907027]
- Answer Big-Bang Problems in framework of Asymptotic Safe Gravity [hep-th/0106133]
- Construct exact RG equations for effective action of quantum gravity [hep-th/0108040]
- Study fixed points in arbitrary dimensions [hep-th/0312114]
- Study fixed points in higher-derivative gravity [hep-th/0607128]
- …………

Our model of Higgs Curvaton in ASG

Model of Higgs in ASG

The action:

$$S = \int d^4 \sqrt{-g}\Big[\frac{R}{16\pi G(p)} - \frac{\Lambda(p)}{8\pi G(p)} + \mathcal{L}_m\Big]$$

where $\mathcal{L}_{\rm m}$ denotes matter Lagrangian, which is now Higgs:

$$\mathcal{L}_m \supset -\frac{1}{2}\partial_\mu h \partial^\mu h - \frac{\lambda}{4}(h^2 - v^2)^2 - V_{int}$$

Define dimensionless coupling constants:

$$g(p) \equiv \frac{p^2}{24\pi} G(p)\ , \quad \lambda(p) \equiv \frac{\Lambda(p)}{p^2}$$

Assuming renormalization group equations:

$$\begin{aligned} \beta_\lambda &\equiv p\partial_p \lambda = -2\lambda + 2\alpha g\ , \\ \beta_g &\equiv p\partial_p g = 2g - 2\beta^2 g^2/3\ , \end{aligned}$$

Model of Higgs in ASG

$g(p)$ and $\lambda(p)$ can be obtained by solving RG equations, which in turn gives:

$$G \simeq \frac{G_N}{1+\xi_G G_N p^2} ,$$

$$\Lambda \simeq \Lambda_{IR} + \xi_\Lambda p^2 - \frac{\xi_\Lambda}{\xi_G} G_N^{-1} \ln[1+\xi_G G_N p^2]$$

with two parameters $\xi_G = \frac{\beta^2}{72\pi}$ and $\xi_\Lambda = \frac{3\alpha}{\beta^2}$

$p \to \infty$ $\quad G \to \frac{1}{\xi_G p^2}$ $\quad \Lambda \to \xi_\Lambda p^2$

$p \to 0$ $\quad G \to G_N$ $\quad \Lambda \to \Lambda_{IR}$

Equivalence to Jordan-Brans-Dicke/f(R) theories

The action:

$$S = \int d^4 \sqrt{-g}\Big[\frac{R}{16\pi G(p)} - \frac{\Lambda(p)}{8\pi G(p)} + \mathcal{L}_m\Big]$$

Y.-F. Cai and D. A. Easson, Phys. Rev. D 84, 103502 (2011) .

Setting $\varphi = \frac{G_N}{G(p)}$ $U(\varphi) = \frac{\Lambda(p)}{8\pi G(p)}$ one gets:

$$S = \int d^4 \sqrt{-g}\Big[\frac{1}{16\pi G_N}\varphi R - U(\varphi) + \mathcal{L}_m\Big]$$

which corresponds to the Jordan-Brans-Dicke theory with $\omega_{BD} = 0$

EoM: $R - U_{,\varphi} = 0$ Solution: $\varphi = U^{-1}_{,\varphi}(R)$

As is well-known, it can be transformed to f(R) theory with

$f(R) = U^{-1}_{,\varphi}(R)R - U(\varphi(R))$

Via the conformal transformation

$\tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}$ $\Omega^2 = \varphi = \frac{G_N}{G(p)}$ $\phi = \sqrt{\frac{3}{8\pi G_N}}\ln\Omega$ $V(\phi) = \Omega^{-4}U(\varphi)$

The action can also be turned into form of canonical single scalar field in "Einstein frame":

$$S = \int d^4 \sqrt{-\tilde{g}}\Big[\frac{1}{16\pi G_N}\tilde{R} - \frac{1}{2}(\tilde{\nabla}\phi)^2 - V(\phi) + \Omega^{-4}\mathcal{L}_m\Big]$$

Equivalence to Jordan-Brans-Dicke/f(R) theories

Einstein equation for Asymptotic Safe Gravity:

$$R_{\mu\nu} - \frac{R}{2} g_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T^{(m)}_{\mu\nu} + G(\nabla_\mu \nabla_\nu - g_{\mu\nu}\Box) G^{-1}$$

Energy conservation equation (which comes from Bianchi Identity):

$$\frac{R - 2\Lambda(p)}{2G(p)} \nabla_\mu G(p) + \nabla_\mu \Lambda(p) = 0$$

With the parametrization of $G(p)$ and $\Lambda(p)$ one can get the f(R) form as:

$$f(R) \simeq \frac{R - 2\Lambda_{IR}}{16\pi G_N} + \frac{\xi_G}{32\pi\xi_\Lambda}(R - 2\Lambda_{IR})^2 \simeq \frac{R}{16\pi G_N} + \frac{\xi_G}{32\pi\xi_\Lambda} R^2$$

where we assume $\Lambda_{IR} \simeq 0$

“Effective” Double-field Model

Using conformal transformations, one can transform our action to a very neat form containing double field:

$$\mathcal{L} \supset \frac{\tilde{R}}{16\pi G_N} - \frac{(\tilde{\nabla}\phi)^2}{2} - \frac{(\tilde{\nabla}h)^2}{2\Omega^2(\phi)} - \tilde{V}(\phi, h)$$

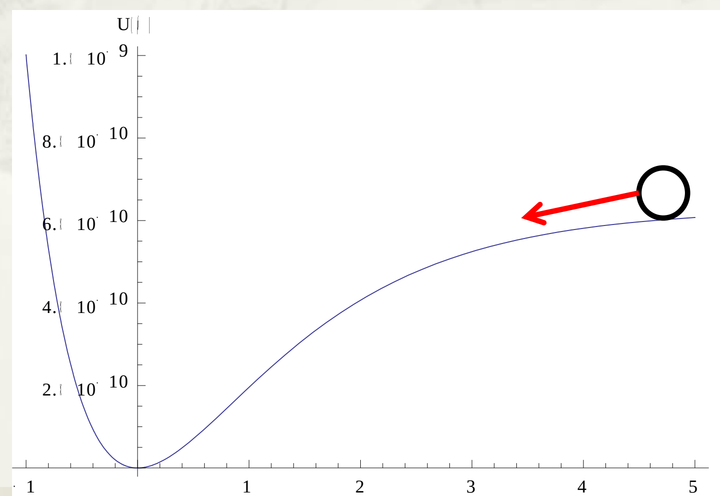
where the total potential looks like:

$$\tilde{V}(\phi, h) = U(\phi) + \frac{V(h)}{\Omega^4(\phi)} \qquad V(h) = \frac{\lambda}{4}(h^2 - v^2)^2$$

and the “effective” potential of phi (Asymptotic Safe Gravity) is:

plot of the potential:

$$U(\phi) = 2\pi M_p^4 \frac{\xi_\Lambda}{\xi_G}\left(1 - e^{-\frac{2\phi}{\sqrt{6}M_p}}\right)^2$$



“Effective” Double-field Model

From Friedmann Equation, we can easily find the “effective” energy density and pressure:

$$\rho = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2\Omega^2(\phi)}\dot{h}^2 + U(\phi) + \frac{V(h)}{\Omega^4(\phi)}$$
$$P = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2\Omega^2(\phi)}\dot{h}^2 - U(\phi) - \frac{V(h)}{\Omega^4(\phi)}$$

and the equation of motion for phi and h are:

$$\ddot{\phi} + 3H\dot{\phi} + U_{,\phi} - \frac{\Omega_{,\phi}}{\Omega^5}V + \frac{\Omega_{,\phi}}{\Omega^3}\dot{h}^2 = 0$$
$$\ddot{h} + 3H\dot{h} - 2\frac{\Omega_{,\phi}}{\Omega}\dot{\phi}\dot{h} + \frac{V_{,h}}{\Omega^2} = 0$$

Inflationary Solution

Define "slow-roll" parameters:

$$\epsilon \equiv -\frac{\dot{H}}{H^2} \ , \ \epsilon_\phi \equiv \frac{\dot{\phi}^2}{2M_p^2 H^2} \ , \ \epsilon_h \equiv \frac{\dot{h}^2}{2\Omega^2 M_p^2 H^2} \ , \ \eta_\phi \equiv \frac{\tilde{V}_{,\phi\phi}}{3H^2} \ , \ \eta_h \equiv \frac{\tilde{V}_{,hh}}{3H^2} \ , \ \eta_{\phi h} \equiv \frac{\tilde{V}_{,\phi h}}{3H^2}$$

One get the cosmological solution of phi and h:

$$\dot{\phi} \simeq -\frac{U_{,\phi}}{3H} \ , \ \dot{h} \simeq -\frac{V_{,h}}{3\Omega^2 H} \ , \ H^2 \simeq \frac{U}{3M_p^2}$$

$$\mathcal{N}(\phi) = -\int_i^f U d\phi / M_p^2 U_{,\phi}$$

e-fold number

$$H_I \simeq \sqrt{\frac{2\pi}{3}\frac{\xi_\Lambda}{\xi_G}} M_p$$

Hubble parameter

The initial and final values of phi:

$$\epsilon_\phi = 1 \Rightarrow \phi_f \simeq 0.94 M_p \qquad \mathcal{N} = 60 \Rightarrow \phi_i \simeq 5.46 M_p$$

Inflationary Solution

The value of h in inflationary era: from $\eta_h = V_{,hh}/3H^2\Omega^4 \lesssim 1$

$$h \simeq \frac{\Omega^2(\phi)}{\sqrt{\lambda}} H_I \qquad \Omega(\phi) = e^{\frac{\phi}{\sqrt{6}M_p}}$$

The initial and final values of h:

$$\begin{matrix} \phi_i \simeq 5.46 M_p \\ \phi_f \simeq 0.94 M_p \end{matrix} \Rightarrow \begin{matrix} \Omega_i \simeq \mathcal{O}(10) \\ \Omega_f \simeq \mathcal{O}(1) \end{matrix} \Rightarrow \begin{matrix} h_i \sim 100H \\ h_f \sim H_I \end{matrix}$$

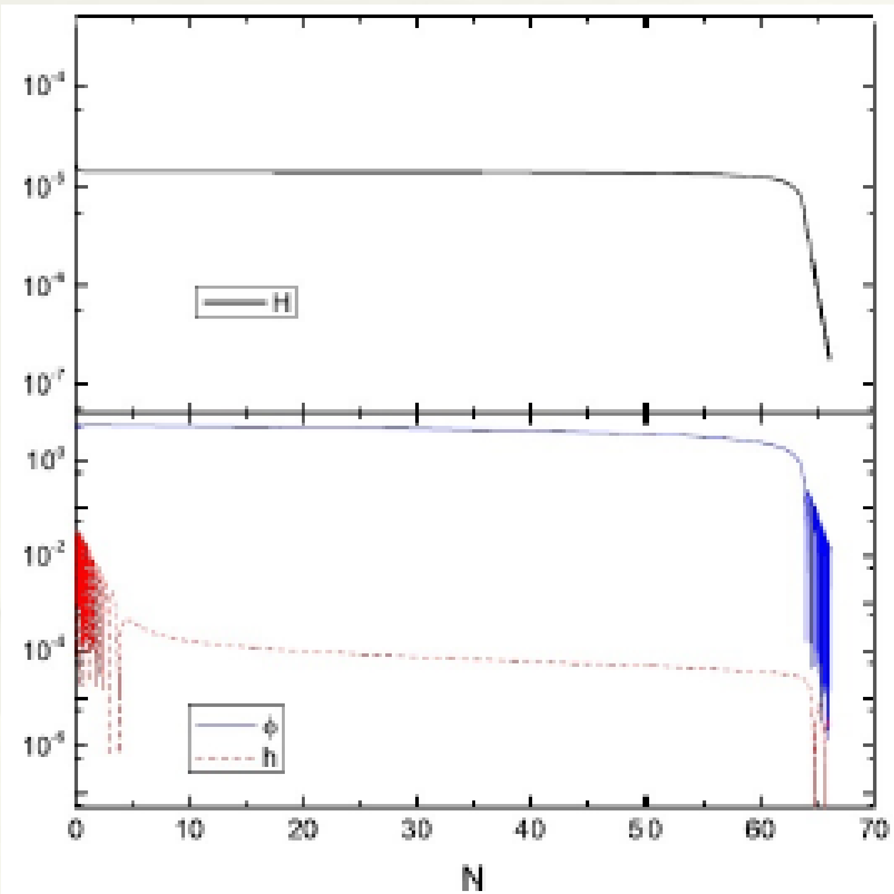
The intial value of density perturbation:

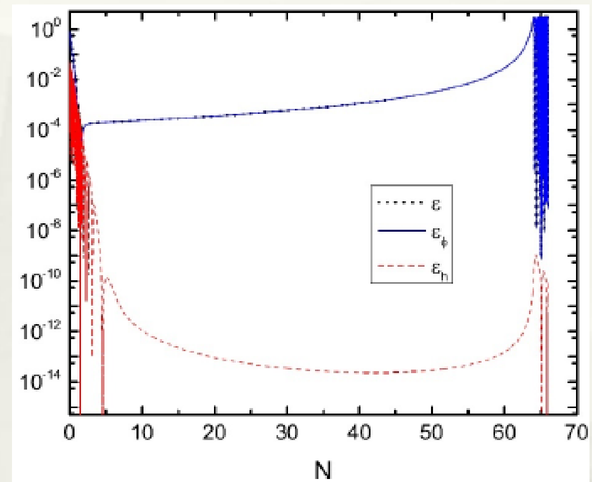
$$\frac{\rho_h}{\rho} \simeq \frac{\lambda h^4}{H^2 M_p^2} \sim 10^{-2}$$

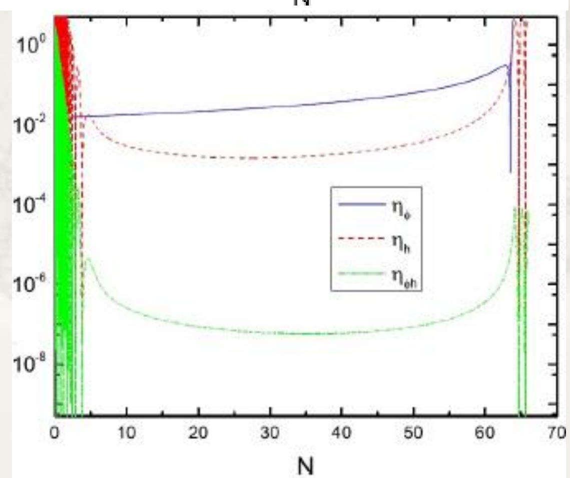
which is enhanced (by the conformal factor) by several orders of magnitude. The aforementioned problem of Higgs curvaton solved!

Inflationary Solution

Numerical results:





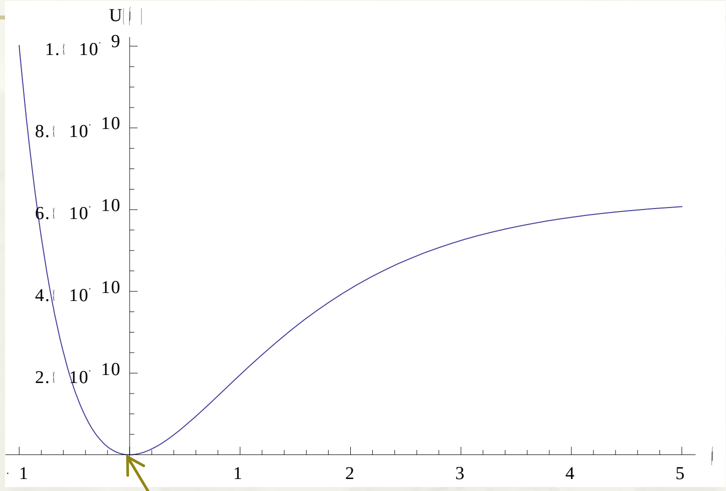


Numerical Inputs:

$\xi_G = 0.72$ $\xi_\Lambda = 10^{-10}\xi_G$ $\lambda = 0.13$ $\phi_i = 5.46 M_p$ $h_i = 10^{-2} M_p$

"Higgs Decay" After Inflation

phi will oscillate around phi=0 when dacay!
Since phi is coupled to h through the term of $\frac{V(h)}{\Omega^4(\phi)}$, it is natural to assume that the decaying is modulated by h.



minimum where phi oscillates

One example: $V_{int}^{\phi} \supset \frac{\kappa}{M_p} h \chi^2 \phi^2$

and the decay rate is: $\Gamma(h) \simeq \frac{\kappa^2 h^2}{16\pi M_\phi^3 M_p^2} \rho_\phi$

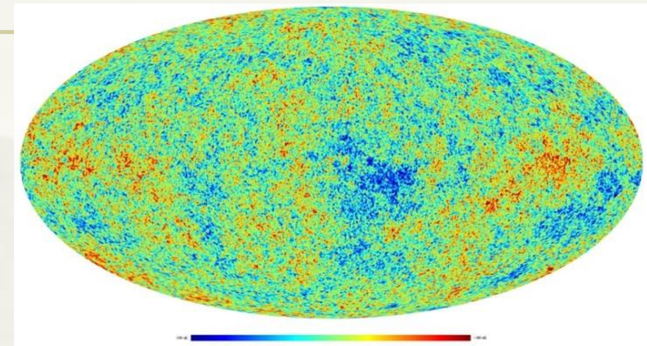
With the relation at the phase transition to radiation domination $H \simeq \Gamma$ one can estimate the value of phi at this transition surface:

$$\phi_D \simeq \frac{10\lambda}{\kappa^2} M_p$$

Perturbation Theory

Why perturbations?

Primordial perturbations provide seeds for structure formation and explains why our current universe is not complete isotropic.



$$\frac{\Delta T}{T} \sim 10^{-5}$$

Constraints for primordial perturbations by PLANCK:

For power spectrum:

$$n_s = 0.959 \pm 0.007 \quad (68\%;\ \textit{Planck}+\text{WP}+\text{highL})$$

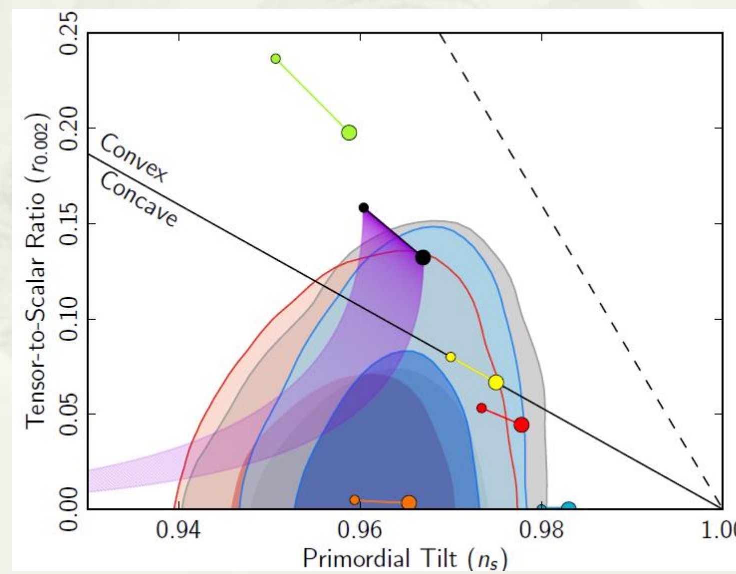
$$r_{0.002} < 0.11 \quad (95\%;\ \text{no running}),$$

$$r_{0.002} < 0.26 \quad (95\%;\ \text{including running}).$$

For Non-Gaussianities:

$$f_{\rm NL}^{\rm local} = 2.7 \pm 5.8,\ f_{\rm NL}^{\rm equil} = -42 \pm 75$$

(68% CL statistical)



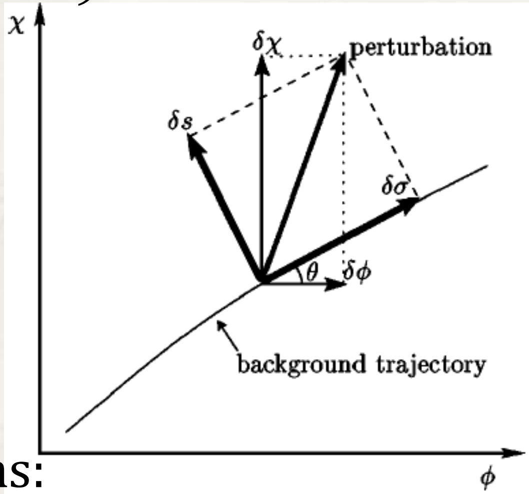
Planck Collaboration (P.A.R. Ade et al.), arXiv:1303.5076, 1303.5084.

Cosmological Perturbation

In spatial flat gauge: $\phi(t, x^i) = \phi(t) + \delta\phi(t, x^i)$ $h(t, x^i) = h(t) + \delta h(t, x^i)$

Decompose into adiabatic and isocurvature perturbations:

C. Gordon, D. Wands, B. Bassett, R. Maartens, Phys.Rev. D63 (2001) 023506



"adiabatic" direction: $\dot{\sigma} = \cos\theta\dot{\phi} + \Omega^{-1}\sin\theta\dot{h}$,

"isocurvature" direction: $\dot{s} = -\sin\theta\dot{\phi} + \Omega^{-1}\cos\theta\dot{h}$

where $\cos\theta = \dfrac{\dot{\phi}}{\sqrt{\dot{\phi}^2 + \Omega^{-2}\dot{h}^2}}$, $\sin\theta = \dfrac{\Omega^{-1}\dot{h}}{\sqrt{\dot{\phi}^2 + \Omega^{-2}\dot{h}^2}}$

and adiabatic/isocurvature perturbations are defined as:

$$\delta\sigma = \cos\theta\delta\phi + \Omega^{-1}\sin\theta\delta h\ , \quad \delta s = -\sin\theta\delta\phi + \Omega^{-1}\cos\theta\delta h$$

Curvaton scenario: $\delta s \gg \delta\sigma$

$$\delta s_* = H_*/2\pi \qquad \delta h_* = \Omega_*\delta s_* = \Omega_*\frac{H_*}{2\pi}$$

*: horizon-crossing point

Scalar Perturbation

The curvature perturbation generated after horizon-crossing:

delta-N formalism:

$$\zeta_h(x) \simeq -\Theta_1 \frac{\delta h}{h} - \frac{1}{2}\Theta_2 \left(\frac{\delta h}{h}\right)^2 - \frac{1}{6}\Theta_3 \left(\frac{\delta h}{h}\right)^3 \Big|_D$$

where

$$\begin{aligned}
\Theta_1 &= \frac{h}{6}\frac{\Gamma_{,h}}{\Gamma} , \\
\Theta_2 &= \frac{h^2}{6}\left(\frac{\Gamma_{,hh}}{\Gamma} - \frac{\Gamma_{,h}^2}{\Gamma^2}\right) , \\
\Theta_3 &= \frac{h^3}{6}\left(\frac{\Gamma_{,hhh}}{\Gamma} - 3\frac{\Gamma_{,h}\Gamma_{,hh}}{\Gamma^2} + 2\frac{\Gamma_{,h}^3}{\Gamma^3}\right)
\end{aligned}$$

Considering the leading order:

$$\delta h_D = \frac{\Omega_D}{\Omega_*}\delta h_* = \frac{\Omega_D H_*}{2\pi}$$

perturbation of h at decaying time

$$P_{\zeta_h} = \Theta_1^2 \frac{\delta h_D^2}{h_D^2} \simeq \Theta_1^2 \Omega_D^2 \frac{H_*^2}{4\pi^2 h_*^2}$$

h-induced curvature perturbation

Scalar Perturbation

Power spectrum:

$$P_{\zeta_h} \simeq \frac{\lambda}{8\pi^2}\Theta_1^2\frac{\Omega_D^2}{\Omega_I^4}$$

"h-induced"

$$P_{\zeta_\phi} = \frac{H_I^2}{8\pi^2\epsilon M_p^2}$$

"phi-induced"

$$\boxed{P_{total} = P_{\zeta_h} + P_{\zeta_\phi}}$$

Define the ratio: $$q_h \equiv \frac{P_{\zeta_h}}{P_{\zeta_h} + P_{\zeta_\phi}} = \frac{\epsilon\lambda\Theta_1^2\Omega_D^2 M_p^2}{\epsilon\lambda\Theta_1^2\Omega_D^2 M_p^2 + \Omega_I^4 H_I^2}$$

Spectral index:

$$n_{\zeta_\phi} - 1 \ = \ -6\epsilon + 2\eta_\phi = -6\epsilon + \frac{2U_{,\phi\phi}}{3H^2} \ ,$$

$$n_{\zeta_h} - 1 \ = \ -2\epsilon + 2\eta_h = -2\epsilon + \frac{2V_{,hh}}{3\Omega_I^4 H^2} \ ,$$

(both are scale-invariant!)

Tensor Perturbation

Tensor modes, though not directly detected, is also important because an upper bound of tensor-scalar ratio is already given, and B-mode CMB polarization detection will be next step mission of PLANCK!

Perturbed metric:

$$ds^2 = -a^2(\eta)[d\eta^2 - (\delta_{ij} + \gamma_{ij})dx^i dx^j]$$

Perturbed action (up to second order):

$$\mathcal{S}_2^T = \frac{1}{2}\int d\eta d^3x a^2[\gamma_{ij}'^2 - (\partial_k \gamma_{ij})^2]$$

Expand the tensor perturbation:

$$\gamma_{ij} = \sum_{\lambda=1}^{2}\int \frac{d^3k}{(2\pi)^{3/2}}\gamma_{k,\lambda}e_{ij}^{\lambda}e^{ikx}$$

Equation of motion:

$$v_g^\lambda(k)'' + (k^2 - \frac{a''}{a})v_g^\lambda(k) = 0 \qquad v_g^\lambda(k) \equiv a\gamma_{k,\lambda}$$

Tensor Perturbation

The spectrum of tensor perturbation is defined by $P_T \equiv \frac{k^3}{2\pi^2}|\gamma_{ij}|^2$ and in our model it becomes

$$P_T = \frac{2H^2}{\pi^2 M_p^2}$$

Spectrum index:

$$n_T = -2\epsilon$$

Tensor-scalar ratio:

$$r_T \equiv \frac{P_T}{P_{\zeta_h} + P_{\zeta_\phi}} = 16\epsilon(1 - q_h)$$

(small tensor-scalar ratio requires both $|\epsilon| \ll 1$ and $q_h \simeq 1$)

Non-Gaussianities

Expansion of curvature perturbation to 4th order:

$$\zeta(x) \;=\; \zeta_1(x) + \frac{3}{5} f_{\rm NL}\zeta_1^2(x) + \frac{9}{25} g_{\rm NL}\zeta_1^3(x) + \mathcal{O}(\zeta_1^4)$$

Gaussian part

Definition of 2, 3 and 4-point correlation functions:

$$\langle\zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\rangle = (2\pi)^3 P(k_1)\delta^3(\sum_{n=1}^{2}\mathbf{k}_n)\ ,$$

$$\langle\zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_3)\rangle = (2\pi)^3 B(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)\delta^3(\sum_{n=1}^{3}\mathbf{k}_n)\ ,$$

$$\langle\zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_3)\zeta(\mathbf{k}_4)\rangle = (2\pi)^3 T(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3,\mathbf{k}_4)\delta^3(\sum_{n=1}^{4}\mathbf{k}_n)\ ,$$

power spectrum:

$$\mathcal{P}_\zeta(k_1) \equiv \frac{k^3}{2\pi^2} P(k_1)$$

Express bi- and tri-spectrum in terms of power spectrum:

$$B(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3) \;=\; \frac{6}{5} f_{\rm NL}[P(k_1)P(k_2) + 2\ \text{perm.}]\ ,$$

$$T(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3,\mathbf{k}_4) \;=\; \frac{54}{25} g_{\rm NL}[P(k_1)P(k_2)P(k_3) + 3\ \text{perm.}] + \tau_{\rm NL}[P(k_1)P(k_2)P(|\mathbf{k}_1+\mathbf{k}_3|) + \mathbf{11}\ \text{perm.}]$$

Non-Gaussianities of Higgs Curvaton

Two kinds of origins of $\langle \zeta_1 \cdots \zeta_n \rangle$:

universal: originated from non-linear relation between ζ and h, even if h is gaussian (super-Hubble)

$$B_\zeta^{\rm un}(\vec{k}_1, \vec{k}_2, \vec{k}_3) = N_I N_{JK} N_L \left(P_{\vec{k}_1}^{IK} P_{\vec{k}_2}^{JL} + 2 \text{ permutations} \right)$$

$$T_\zeta^{\rm un}(\vec{k}_1, \vec{k}_2, \vec{k}_3, \vec{k}_4) = N_{IJ} N_{KL} N_M N_N \left(P_{\vec{k}_{12}}^{IL} P_{\vec{k}_1}^{JM} P_{\vec{k}_3}^{KN} + 11 \text{ permutations} \right) + N_{IJK} N_L N_M N_N \left(P_{\vec{k}_1}^{IL} P_{\vec{k}_2}^{JM} P_{\vec{k}_3}^{KN} + 3 \text{ permutations} \right)$$

non-universal: originated from intrinsic non-Gaussianities of h, caused by the non-quadratic potential of h (during Hubble-crossing)

$$\langle \zeta_1 \cdots \zeta_n \rangle |^{n-un} = N_1^{a_1} \cdots N_n^{a_n} \langle \delta h_{a_1} \cdots \delta h_{a_n} \rangle$$

where $N_i^{a_i} \equiv \frac{\partial N}{\partial h_{a_i}}$

Andrea De Simone, Hideki Perrier, Antonio Riotto, JCAP 1301 (2013) 037.

Non-Gaussianities of Higgs Curvaton

Universal: using delta-N formalism

The local type:

$$f_{\mathrm{NL,un}}^{\mathrm{local}} = 5q_h^2\left(1-\frac{\Gamma\Gamma_{,hh}}{\Gamma_{,h}^2}\right),$$

$$g_{\mathrm{NL,un}}^{\mathrm{local}} = \frac{50}{3}q_h^3\left(2-3\frac{\Gamma\Gamma_{,hh}}{\Gamma_{,h}^2}+\frac{\Gamma^2\Gamma_{,hhh}}{\Gamma_{,h}^3}\right)$$

according to our parameter choice:

$$f_{\mathrm{NL,un}}^{\mathrm{local}} \simeq 5/2 \qquad g_{\mathrm{NL,un}}^{\mathrm{local}} \simeq 25/3$$

consistent with the constraints on local NG from PLANCK!
($f_{\mathrm{NL}}^{\mathrm{local}} = 2.7 \pm 5.8$ at 68% C.L.)

Non-Gaussianities of Higgs Curvaton

Non-universal: important to know the n-point correlation function of δh

$$\langle \delta h_{\mathbf{k}_1}(\tau)\delta h_{\mathbf{k}_2}(\tau)\cdots\delta h_{\mathbf{k}_n}(\tau)\rangle = -i\langle| \int_{-\infty}^{\tau} a d\tau' [\delta h_{\mathbf{k}_1}(\tau)\delta h_{\mathbf{k}_2}(\tau)\cdots\delta h_{\mathbf{k}_n}(\tau),\ H_{int}^{(n)}(h(\tau'))]|\rangle$$

where $\delta h_{\mathbf{k}} = \frac{i\Omega H}{\sqrt{2k^3}}(1+ik\tau)e^{-ik\tau}$ $\quad H_{int}^{(n)}(\tau) = \int d^3 x a^3 \Omega^{-4}(\phi)\frac{1}{n!}V^{(n)}(h)\delta h^n$

The equilateral type:

$$f_{\rm NL,int}^{\rm equil} \simeq -\frac{5\lambda h_*^2}{3\Theta_1\Omega_*^2 H_*^2} q_h^2 \left(N_K + \gamma_E - 3\right),$$

$$g_{\rm NL,int}^{\rm equil} \simeq -\frac{25\lambda h_*^2}{27\Theta_1^2\Omega_*^2 H_*^2} q_h^3 \left(N_K + \gamma_E - \frac{169}{48}\right).$$

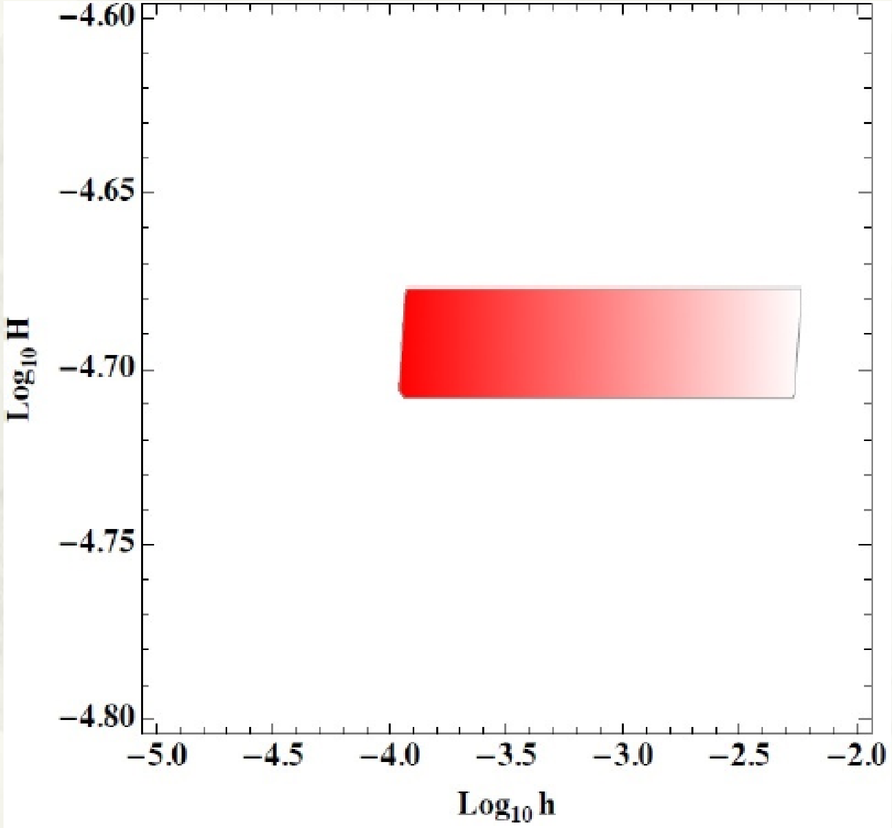
The local type:

$$f_{\rm NL,int}^{\rm local} \simeq -\frac{5\lambda h_*^2}{3\Theta_1\Omega_*^2 H_*^2} q_h^2 \left(N_K + \gamma_E - \frac{7}{3}\right),$$

$$g_{\rm NL,int}^{\rm local} \simeq -\frac{25\lambda h_*^2}{27\Theta_1^2\Omega_*^2 H_*^2} q_h^3 \left(N_K + \gamma_E - 3\right).$$

Constraints from PLANCK data

Conditions: $\frac{\Omega_f^2}{\sqrt{\lambda}} H_I < h < \frac{\Omega_i^2}{\sqrt{\lambda}} H_I$ $\quad 2.07 < 10^9 P_\zeta < 2.39$



Parameter choice: $\epsilon \sim 10^{-4}$ $\lambda \sim 0.13$

Conclusion

- It is appealing to have Higgs boson connected with cosmology, but problems will appear.
- The interesting idea of ASG is introduced, which, by the running effect of G, may help solve the dilemma.
- Our model of Higgs Curvaton in ASG
  - background: analogy of two-field inflation model
  - perturbation: consistent with observation
  - non-Gaussianity: using modulated reheating approach/two different types
  - constraints placed from PLANCK data

Thank you for your attention!