Structural Aspects of Asymptotically Safe Black Holes

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Asymptotic Safety Seminar
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Outline

- I Introduction
- Ia Functional Renormalization Group for gravity (FRG)
- Ib FRG improvement and scale setting for black holes
- II Results
- IIa FRG improved black holes, in the UV
- IIb FRG improved black holes, global structure
- IIc Thermodynamics and state counting
- III Summary

I
Introduction

Renormalizable?

Introduction I

Special seminar on Asymptotic Safety (AS)[∗]

You are experts in AS

Skip typical renormalization-AS introduction slides

[∗] S. Weinberg, "General Relativity" Cambridge University Press

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"Asymptotic Safety"
Wetterichs realization in terms of functional equation[**]

$$\partial_t \Gamma[\psi] = \frac{1}{2} Tr\left[\partial_t R_k \cdot (\Gamma^{(2)}[\psi] + R_k)^{-1}\right] \tag{1}$$

Flow equation where ψ are fields, $\Gamma^{(2)} = \delta^2(\Gamma/\delta\psi^2)$, $t = \ln(k)$, and R_k cut-off function.

$\Rightarrow$ running couplings

[**] M. Reuter, C. Wetterich, Nucl.Phys. B417, 181 (1994)

Define dimensionless couplings

$$g_k = k^2 G_k \qquad \lambda_k = \frac{\Lambda_k}{k^2} \tag{2}$$

G_0: Newtons constant, Λ_0: Cosmological constant

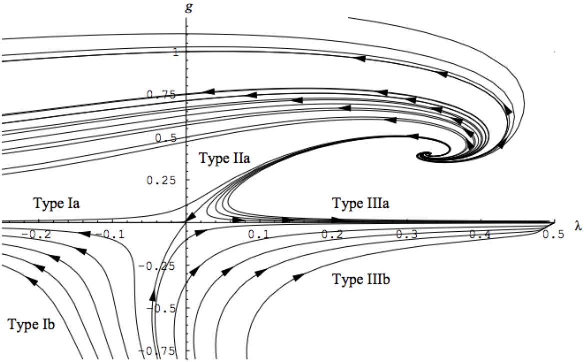
With Wetterichs equation one can get running gravitational couplings [*]

$$\begin{aligned} \beta_\lambda(g,\lambda) = &\quad (\eta_N - 2)\lambda + \tfrac{1}{2\pi} g \left[10\Phi^1_2(-2\lambda) - 8\Phi^1_2(0) - 5\eta_N \tilde{\Phi}^1_2(-2\lambda)\right], \\ \beta_g(g,\lambda) = &\quad (2+\eta_N) g \,. \end{aligned} \tag{3}$$

$$\lambda_* = 0{,}193\,, \qquad g_* = 0{,}707\,, \qquad g_*\lambda_* = 0{,}137 \quad . \tag{4}$$

[*]M. Reuter and F. Saueressig, Phys. Rev. D 65, 065016 (2002); hep-th/0110054

Solve numerically:



Numerical solution of (3)

Has UV fixed points → "Asymptotic Safety"

Would be nice to have analytical expression to work with ...

FRG Flow

Introduction Ia

Expand beta functions for small couplings $g, \lambda \ll 1$:

$$\beta_g = g(k)(2 - 24g(k)) \tag{5}$$

$$\beta_\lambda = 12g(k) - 2\lambda(k) \tag{6}$$

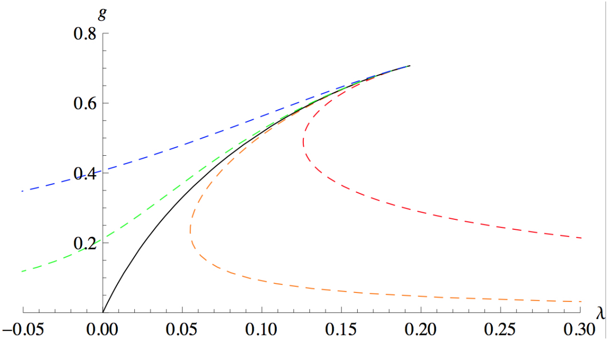
Solve

$$g_{FRG}(k) = \frac{k^2 G_0 g_U^*}{g_U^* + G_0 k^2} \tag{7}$$

$$\lambda(k)_{FRG} = \lambda_U^* + \frac{1}{k^2}\Lambda_0 - \frac{g_U^* \lambda_U^*}{G_0 k^2} \mathrm{Log}\left[\left(1 + G_0 \frac{k^2}{g_U^*}\right)\right] \tag{8}$$

with fixed points g_U^* and λ_U^* used as free parameters.

Analytically approximated flow [*]



Analytical approximation (7, 8)

[*]B. K. and I. Ramirez, Class. Quant. Grav. 28, 055008 (2011); arXiv:1010.2799

Black holes in the AS context

Improving black hole solutions
Introduction Ib

Work pioneered by Bonanno and Reuter [*],
but now do not neglect cosmological term Λ_k

Take classical Schwarzschild (A)dS-solution

$$ds^2 = f(r)dt^2 - f^{-1}(r)dr^2 - r^2 d\Omega_{d+2} \tag{9}$$

$$\text{with} \quad f(r) = 1 - \frac{2G_k M}{r^1} - \frac{1}{3}\Lambda_k r^2 \; .$$

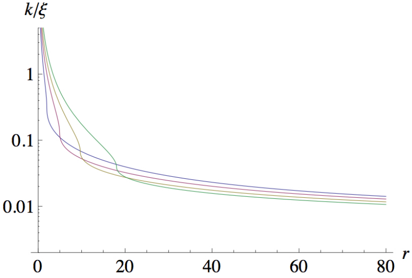
But: Take couplings scale dependent $G = G_k$ and $\Lambda = \Lambda_k$

[*] Phys. Rev. D **62**, 043008 (2000);

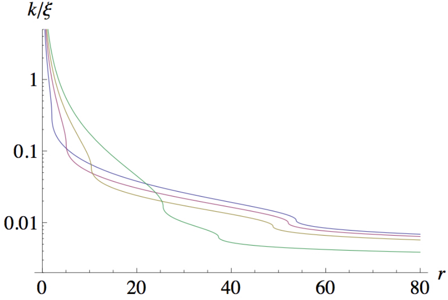
Sensible scale setting $k \leftrightarrow r$

$$k(r) = \frac{\xi}{d(r)} \quad \text{with} \quad d(r) = \int_{\mathcal{C}_r} \sqrt{|ds^2|} \tag{10}$$

along purely radial curve $\mathcal{C}_r$ with parameter ξ



$k(r)$ for AdS



$k(r)$ for dS

II Results

B. K. and F. Saueressig, arXiv:1306.1546 [hep-th].

FRG improved (A)dS black holes, in the UV

Close to the UV-NGFP

$$lim_{k\to\infty} G_k = g_* \, k^{-2}\,, \qquad lim_{k\to\infty}\Lambda_k = \lambda_* \, k^2 \quad . \tag{11}$$

with

$$k(r) \simeq \frac{3}{2}\sqrt{2G_0M}\,\xi\, r^{-3/2}\,. \tag{12}$$

one finds

$$f_*(r) = 1 - \frac{2\,G_0\,M}{r}\left(\frac{3}{4}\lambda_*\,\xi^2\right) - \frac{1}{3}\,\left(\frac{4\,g_*}{3\,G_0\,\xi^2}\right)\,r^2\,. \tag{13}$$

$$f_*(r) = 1 - \frac{2\,G_0\,M}{r}\left(\frac{3}{4}\,\lambda_*\,\xi^2\right) - \frac{1}{3}\,\left(\frac{4\,g_*}{3\,G_0\,\xi^2}\right)\,r^2\,.$$

Selfsimilar for

$$\xi_{sc}^2 = \frac{4}{3\lambda_*}\,. \tag{14}$$

with

$$f_{*,sc}(r) = 1 - \frac{2G_0M}{r} - \frac{1}{3}\Lambda_{eff}\,r^2\,, \tag{15}$$

and effective cosmological constant

$$\Lambda_{eff} = \frac{g_*\,\lambda_*}{G_0}\,. \tag{16}$$

Note:

- Λ_{eff} governed by fixed point product $g_* \lambda_*$
- This is solution of the improved equations of motion
- Does not resolve singularity:

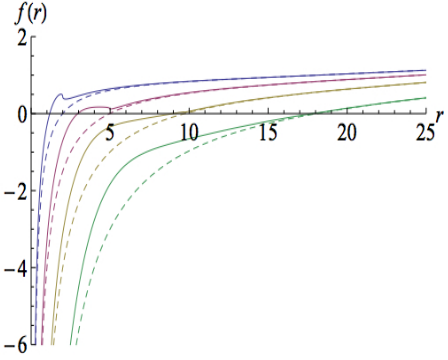
$$R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} = \frac{48 G_0^2 M^2}{r^6} \left(\frac{3}{4} \lambda_* \xi^2 \right)^2 + \frac{8}{3} \left(\frac{4 g_*}{3 G_0 \xi^2} \right)^2 \quad (17)$$

FRG improved black holes, global structure

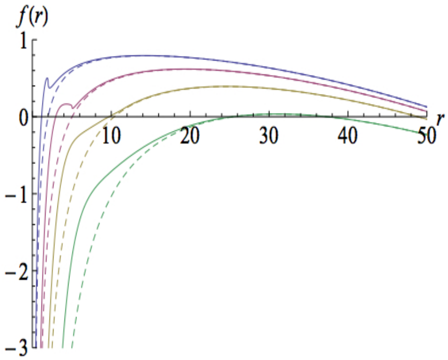
Results II

FRG improved black holes, global structure

For global structure of $f(r)_{improved}$ (numerical)

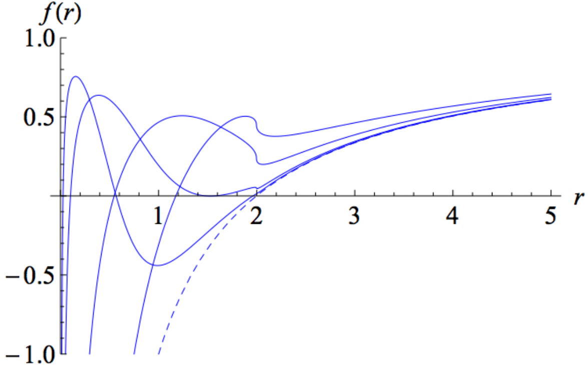


$f(r)$ for AdS



$f(r)$ for dS

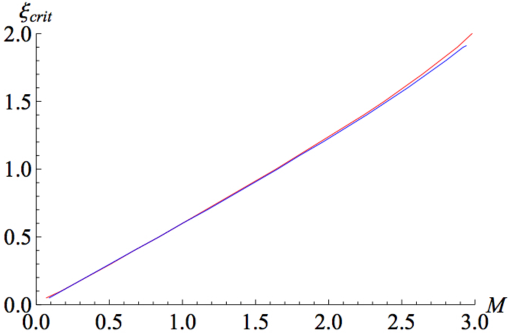
Parameter ξ dependence



$f(r)$ for $M = 1$, $\Lambda_0 = -0{,}001$, $G_0 = 1$. From top to bottom $\xi = \{\xi_{sc}, 1{,}5, 0{,}6, 0{,}3\}$

Note: For $\xi < \xi_{cr}$ can have additional pair of horizons

Critical value of ξ is mass dependent



$\xi(M)$

For $M > M_{crit}$ no additional pair of horizons whatsoever.

$$\text{AdS}: \quad \Lambda_0 = -0{,}001 \quad M_{\text{crit}} = 2{,}94 \tag{18}$$

$$\text{dS}: \quad \Lambda_0 = 0{,}001 \quad M_{\text{crit}} = 2{,}98\,. \tag{19}$$

Thermodynamics

Thermodynamics (inner horizon)

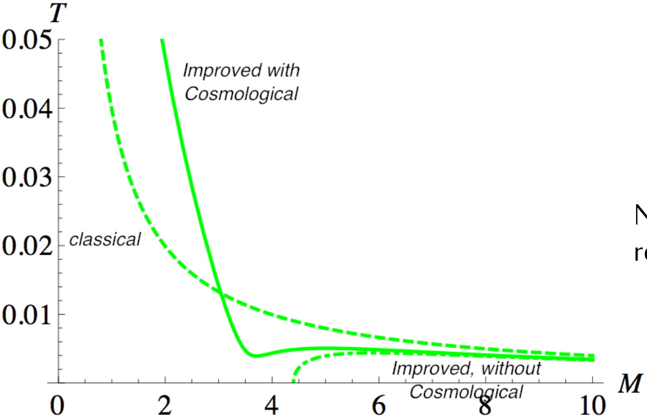
Hawking temperature

$$T_H = \left. \frac{\partial_r f(r)}{4\pi} \right|_{r=r_{\mathrm{H}}} \tag{20}$$

Beckenstein-Hawking entropy

$$S = \frac{\pi r_{\mathrm{H}}^2}{G} \,. \tag{21}$$

Thermodynamics (inner horizon)



No more black hole remnant!

$T(M)$

UV micro black holes, standard thermodynamics at r_-

$$\tilde{S}_* = \frac{4\pi}{g_*\lambda_*} \cos^2\left(\frac{1}{3}(\pi+\theta)\right) . \tag{22}$$

with $\theta \equiv \arctan\left(\sqrt{\frac{4}{27\tilde{m}^2}-1}\right)$ and rescaled mass $\tilde{m} = M\xi\sqrt{G_0}\sqrt{g_*}\,\lambda_*$.
Limited by $0 \leq \tilde{m}^2 \leq 4/27 \leftarrow$ **Nariai black hole** with critical mass M_{crit}.
Thus:

$$\tilde{S}_{max} = \frac{\pi}{g_*\lambda_*} . \tag{23}$$

State counting

Thermodynamics and state counting

Results II

Compare to state counting property [*]

$$\ln \mathbb{Z}_k \equiv -\Gamma_k[g]|_{g=g_{\mathrm{sol}}} \, , \tag{24}$$

($\mathbb{Z}$: partition function, $\Gamma_k[g]$: effective average action)
since in UV (13) is solution, can use

$$g_{sol} = g(f_{sc}) \tag{25}$$

Actually calculation already done for extremal Nariai black hole and corresponding Einstein spaces with $S^2 \times S^2$ topology [**]

$$\ln \mathbb{Z}_* = \frac{2\pi}{g_* \, \lambda_*} \, . \tag{26}$$

[*]D. Becker and M. Reuter, JHEP 1207, 172 (2012), arXiv:1205.3583. D. Becker and M. Reuter, arXiv:1212.4274.

[**] P. H. Ginsparg and M. J. Perry, Nucl. Phys. B 222, 245 (1983); R. Bousso and S. W. Hawking, Phys. Rev. D 54, 6312 (1996); M. S. Volkov and A. Wipf, Nucl. Phys. B 582, 313 (2000).

Thus:

$$\ln \mathbb{Z}_* = 2\tilde{S}_{\mathrm{max}} \tag{27}$$

First explicit example where state counting agrees with thermodynamics

II Summary

- Cosmological term makes huge difference
- Universal short distance behavior independent of G_0 and Λ_0
- UV: Singularity persists
- UV: limit same form as classical solution, but g and λ change role
- UV: Improved solution also solution of improved equations of motion, (but not for all scales k)
- Example for analytic micro state counting
- No stable remnant

Thank you

Saludos from Chile

Backups

Scale setting
intuition → something like 1/ distance

$$k(r) = \frac{\xi}{d(r)} \tag{28}$$

Something with r, M, G_0, usually [*]

$$d_{(2)}(r) = \int_{\mathcal{C}_r} \sqrt{|ds^2|} \approx |_{UV} \frac{1}{R_H^{\frac{1}{2}}} \frac{2}{3} r^{\frac{3}{2}} \tag{29}$$

Put this into $f(r)$

[*] A. Bonanno and M. Reuter, Phys. Rev. D **62**, 043008 (2000) [arXiv:hep-th/0002196];

No (global) solution

Plug improved solution $f(r)$ into Einstein equations

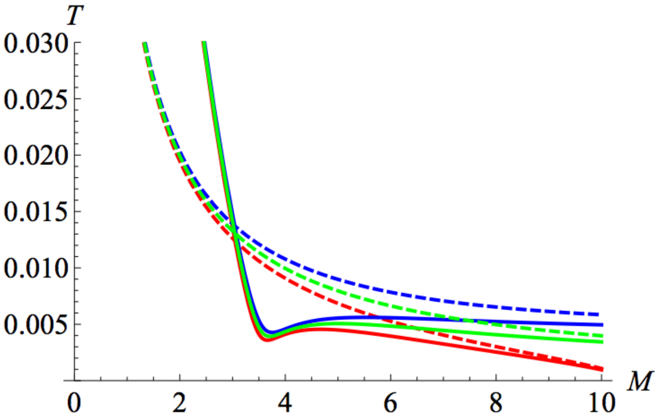
$$G_{\mu\nu} \neq -g_{\mu\nu}\Lambda_k + 8\pi G_k T_{\mu\nu} \tag{30}$$

Why?
Because $G \to G_k \to G(r)$
Need take into account variable $G(r)$ when deriving the eoms,

$$\begin{aligned} G_{\mu\nu} &\neq -g_{\mu\nu}\Lambda_k + 8\pi G_k T_{\mu\nu} - \Delta t_{\mu\nu} \quad \text{with,} \\ \Delta t_{\mu\nu} &= G_k \left(g_{\mu\nu}\Box - \nabla_\mu \nabla_\nu \right) \frac{1}{G_k} \end{aligned} \tag{31}$$

Still no global solution

Thermodynamics



Classical Nariai black hole

For increasing M, the black hole horizon grows while the cosmological horizon $r_c \equiv r_+$ shrinks. For the critical mass

$$M_{\text{max}} \equiv \frac{1}{3\,G\,\sqrt{\Lambda}} \tag{32}$$

the two horizons coincide and one obtains the Nariai black hole as the maximal black hole in dS space

Cardy Verlinde thermodyniamics

The evaluation of the Temperature and the entropy at the black hole horizon $\tilde{r}$ gives

$$\tilde{T} = \frac{1}{4\pi\tilde{r}}\left(1 - \Lambda\,\tilde{r}^2\right)\,, \qquad \tilde{S} = \frac{\pi\tilde{r}^2}{G}\,. \tag{33}$$

The same formulas hold for the cosmological horizon r_+