An asymptotic safety scenario for gauged chiral Higgs-Yukawa models

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in collaboration with

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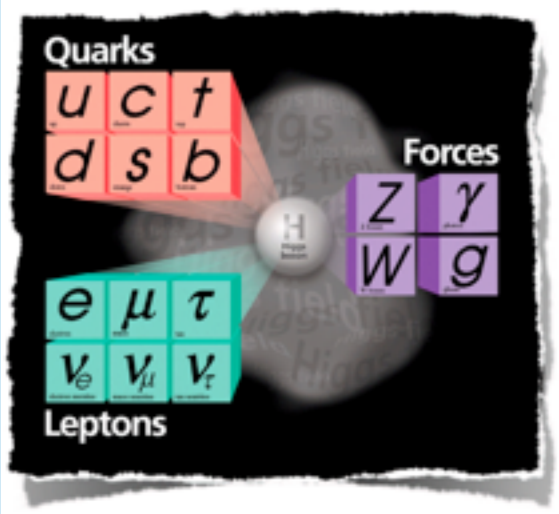
Outline

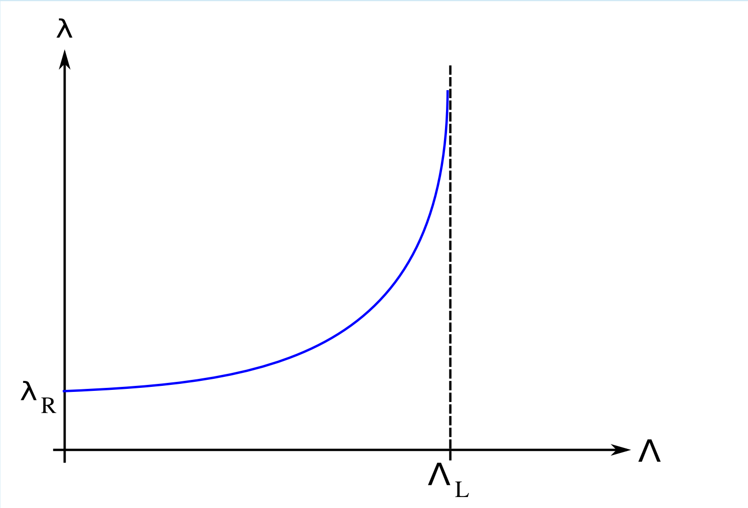
Introduction

Standard model of particle physics



today: no gravity

Landau pole of the Higgs self-interaction



Perturbation theory:

- Higgs suffers from *triviality problem* → Higgs sector not a fundamental QFT
- Running of Higgs requires *unnaturally fine-tuned* initial conditions to separate EW scale from Planck/GUT scale

[K. G. Wilson & J. B. Kogut, 1974]
[A, Hasenfratz et al., 1987]
[M. Luscher & P. Weisz, 1988]

Possible solutions:

- new d.o.f.
- new symmetries
- new quantization rules
- *Really cool nonperturbative QFT!* → *Asymptotic Safety*

[S. Weinberg, 1976]

FRG flow equation:

$$\partial_t \Gamma_k[\Phi] = \frac{1}{2}\mathrm{STr}\{[\Gamma_k^{(2)}[\Phi] + R_k]^{-1}(\partial_t R_k)\}\,.$$

[C. Wetterich, 1993]

Model:

- *Scalar Higgs-sector* chirally coupled to *top-bottom fermion sector* and *left-handed $SU(N_L)$ gauge group*

$$S_{\rm cl} = \int d^dx \Big[\frac{1}{4}F^i_{\mu\nu}F^{i\mu\nu} + (D^\mu\phi)^\dagger(D_\mu\phi) + \bar{m}^2\phi^{a\dagger}\phi^a + \frac{\bar{\lambda}}{2}\left(\phi^{a\dagger}\phi^a\right)^2 + i\left(\bar{\psi}^a_{\rm L} \not{D}^{ab}\psi^b_{\rm L} + \bar{\psi}_{\rm R}\not{\partial}\psi_{\rm R}\right) + \bar{h}\bar{\psi}_{\rm R}\phi^{\dagger a}\psi^a_{\rm L} - \bar{h}\bar{\psi}^a_{\rm L}\phi^a\psi_{\rm R}\Big]$$

details on slide 6

- Reduced model with the same structural deficits as Higgs sector of the standard model
- Perturbation theory of this model → *triviality & hierarchy problem*
- Model is motivated from simpler Higgs-Yukawa systems with UV fixed points for unrealistic parameters

[H. Gies & MMS, 2010]
[H. Gies, S. Rechenberger, MMS, 2010]

Introduction

Nonperturbative FRG:

- Includes perturbative physics

- Features *nonperturbative threshold phenomena*

  - decoupling of massive modes

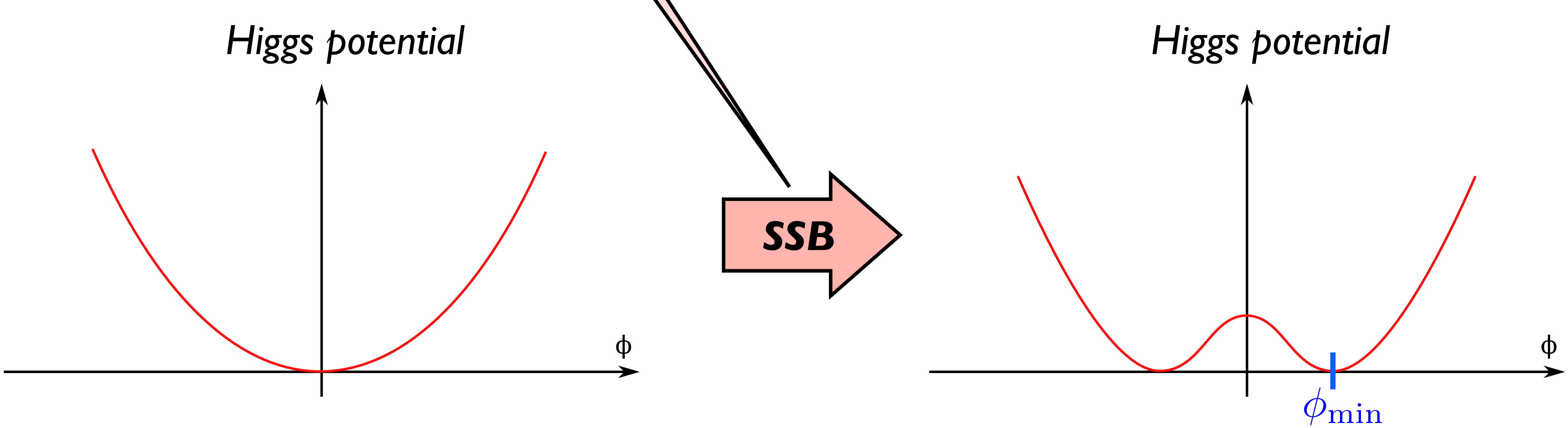
  - *dynamically generated masses* can be proportional to coupling λ

[J. Berges, N. Tetradis, C. Wetterich, 2002]

typical loop contribution:

$$\propto \frac{1}{(k^2 + \underbrace{2\,\lambda\,\rho_{\min}}_{m_\phi^2})^n}$$

- *no naive perturbative expansion even in weak-coupling regime*



Gauged chiral Higgs-Yukawa model

$$S_{\mathrm{cl}} = \int d^dx \Big[\frac{1}{4}F^i_{\mu\nu}F^{i\mu\nu} + (D^\mu\phi)^\dagger (D_\mu\phi) + \bar{m}^2\phi^{a\dagger}\phi^a + \frac{\bar{\lambda}}{2}\left(\phi^{a\dagger}\phi^a\right)^2 + i\left(\bar{\psi}^a_{\mathrm{L}} \not{D}^{ab}\psi^b_{\mathrm{L}} + \bar{\psi}_{\mathrm{R}}\not{\partial}\psi_{\mathrm{R}}\right) + \bar{h}\bar{\psi}_{\mathrm{R}}\phi^{\dagger a}\psi^a_{\mathrm{L}} - \bar{h}\bar{\psi}^a_{\mathrm{L}}\phi^a\psi_{\mathrm{R}}\Big]$$

- Boson mass $\bar{m}$, scalar self-interaction $\bar{\lambda}$, Yukawa coupling $\bar{h}$
- Complex scalar field (2N$_{\mathrm{L}}$ real scalar fields): $\phi^a = (\phi^a_1 + i\phi^a_2)/\sqrt{2}$, with invariant: $\rho := \phi^{a\dagger}\phi^a$
- Covariant derivatives D_μ in fundamental representation of gauge group $D^{ab}_\nu = \partial_\nu - i\bar{g}W^i_\nu(T^i)^{ab}$
- Indices $a, b, \ldots = 1, \ldots, N_{\mathrm{L}}$
- Gauge coupling $\bar{g}$,Yang-Mills vector potential W^i_ν
- Gauge group generators: $[T^i, T^j] = if^{ijk}T^k$
- Non-abelian field strength: $F^i_{\mu\nu} = \partial_\mu W^i_\nu - \partial_\nu W^i_\mu + \bar{g}f^{ijl}W^j_\mu W^l_\nu$ with adjoint indices $i, j, k, \ldots$

Gauged chiral Higgs-Yukawa model

$$S_{\rm cl} = \int d^dx \Big[\frac{1}{4} F^i_{\mu\nu} F^{i\mu\nu} + \left(D^\mu \phi\right)^\dagger \left(D_\mu \phi\right) + \bar{m}^2 \phi^{a\dagger}\phi^a + \frac{\bar{\lambda}}{2}\left(\phi^{a\dagger}\phi^a\right)^2 + i\left(\bar{\psi}^a_{\rm L} \not{D}^{ab} \psi^b_{\rm L} + \bar{\psi}_{\rm R} \not{\partial} \psi_{\rm R}\right) + \bar{h}\bar{\psi}_{\rm R}\phi^{\dagger a}\psi^a_{\rm L} - \bar{h}\bar{\psi}^a_{\rm L}\phi^a\psi_{\rm R}\Big]$$

- Fermion field (N_g generations*):
    - Left-handed N_L-plet → top-bottom doublet for SU(N_L =2)
    - One right-handed fermion (top-quark component)
    - Right-handed bottom-type components not considered
    - Only top can become massive upon SB

*global Witten anomaly for odd N_g with N_L=2 **[E. Witten, 1982]**

Flow equation & truncation

- FRG flow equation: $\partial_t \Gamma_k[\Phi] = \frac{1}{2}\mathrm{STr}\{[\Gamma_k^{(2)}[\Phi] + R_k]^{-1}(\partial_t R_k)\}\,.$

- Truncation:

$$\Gamma_k = \int d^d x \Big[\underbrace{U(\rho)}_{\text{Higgs effective potential}} + \underbrace{Z_\phi (D^\mu \phi)^\dagger (D_\mu \phi)}_{\text{Higgs}} + i(\overbrace{Z_{\mathrm{L}} \bar{\psi}_{\mathrm{L}}^a \not{D}^{ab} \psi_{\mathrm{L}}^b}^{\text{left-handed fermions}} + \overbrace{Z_{\mathrm{R}} \bar{\psi}_{\mathrm{R}} \not{\partial} \psi_{\mathrm{R}}}^{\text{right-handed fermion}})$$
$$+ \underbrace{\bar{h} \bar{\psi}_{\mathrm{R}} \phi^{a\dagger} \psi_{\mathrm{L}}^a - \bar{h} \bar{\psi}_{\mathrm{L}}^a \phi^a \psi_{\mathrm{R}}}_{\text{Higgs-Yukawa coupling}} + \overbrace{\frac{Z_W}{4} F_{\mu\nu}^i F^{i\mu\nu}}^{\text{gauge fields}} + \overbrace{\frac{Z_\phi}{2\alpha} G^i G^i}^{\text{gauge-fixing term}} - \overbrace{\bar{c}^i \mathcal{M}^{ij} c^j}^{\text{ghosts}} \Big]\,.$$

R_α - gauge with gauge fixing parameter α, flow of $\bar{g}$ with background field formalism.

[L. F. Abbott, 1981]
[W. Dittrich & M. Reuter, 1986]

Truncation

Summary: theory space is parametrized by

- wave function renormalizations of matter fields $Z_\phi, Z_\mathrm{L}, Z_\mathrm{R}$
- wave function renormalization of gauge fields Z_W or gauge coupling $\bar{g}$
- Higgs-Yukawa interaction $\bar{h}$, the vev $\bar{v}$ and the Higgs effective potential U

Questions so far ?

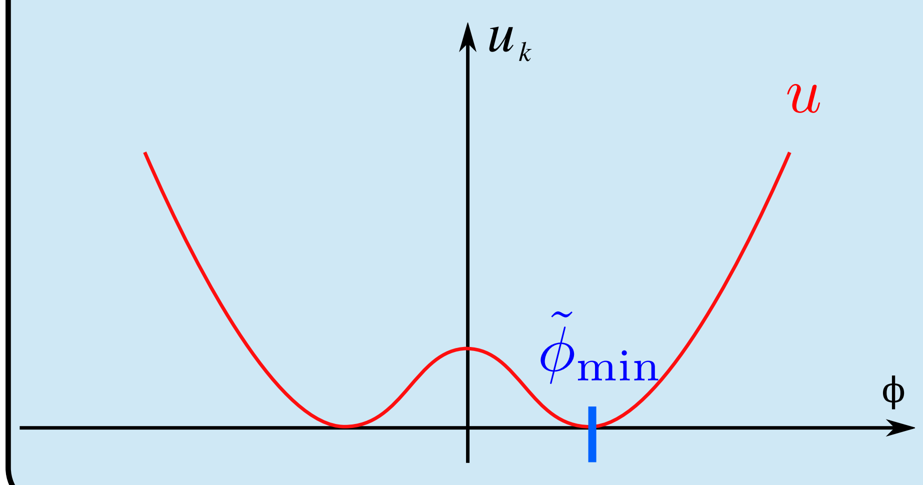
Dimensionless parameters

Dimensionless renormalized parameters:

- Yukawa/gauge couplings: $h^2 = \frac{k^{d-4}\bar{h}^2}{Z_\phi Z_\mathrm{L} Z_\mathrm{R}}, \quad g^2 = \frac{\bar{g}^2}{Z_W k^{4-d}},$
- Effective potential: $u(\tilde{\rho}) = k^{-d} U(Z_\phi^{-1} k^{d-2} \tilde{\rho}), \quad \tilde{\rho} = \frac{Z_\phi \rho}{k^{d-2}}.$
- Minimum of the potential: $\kappa = \frac{Z_\phi \bar{v}^2}{2k^{d-2}} = \tilde{\rho}_\mathrm{min}$
- Expansion of effective potential: $u = \frac{\lambda_2}{2!}(\tilde{\rho} - \kappa)^2 + \frac{\lambda_3}{3!}(\tilde{\rho} - \kappa)^3 + \cdots .$
- Anomalous dimensions:

$$\eta_\phi = -\partial_t \log Z_\phi \,, \qquad \eta_W = -\partial_t \log Z_W$$
$$\eta_\mathrm{L} = -\partial_t \log Z_\mathrm{L} \,, \qquad \eta_\mathrm{R} = -\partial_t \log Z_\mathrm{R} \,.$$

Effective potential:



Flow equations and fixed points

Fixed point and critical exponents:

- Flow of couplings and β-functions: $\partial_t g_i = \beta_{g_i}(g_1, g_2, \dots)$
- Fixed point: $\beta_i(g_1^*, g_2^*, ...) = 0\,,\ \forall\ i,$
- Linearized flow and stability matrix: $\partial_t g_i = B_i{}^j (g_j - g_j^*) + \dots, \quad B_i{}^j = \left.\frac{\partial \beta_{g_i}}{\partial g_j}\right|_{g=g^*},$

This model gives flow equations for:

- Effective potential u → self-interactions λ_i and minimum κ
- Yukawa coupling h
- anomalous dimensions (can be solved as function of u, h, λ_i, κ)
- gauge coupling g is kept fixed for the moment!

ϕ^4

Set of flow equations:

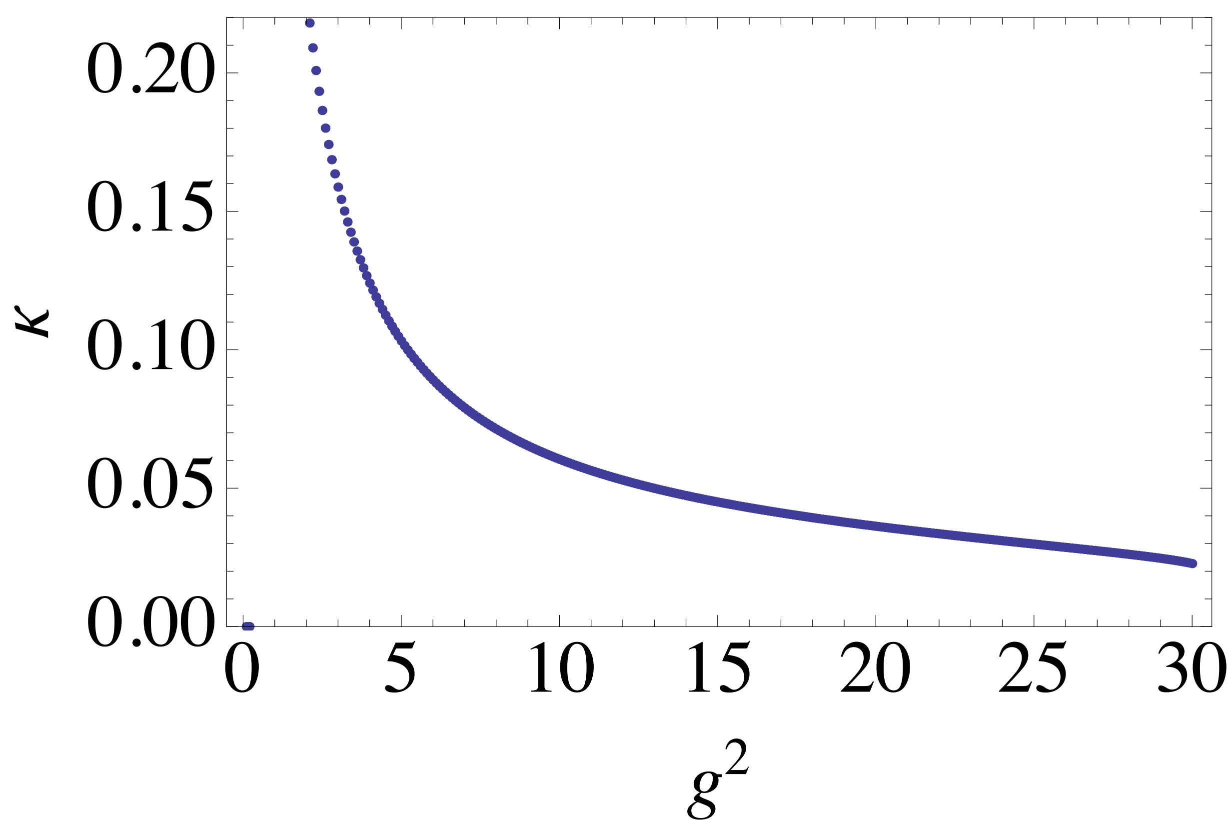
$$\partial_t \kappa = \beta_\kappa(\kappa, \lambda_2, h^2, g^2),$$
$$\partial_t \lambda_2 = \beta_\lambda(\kappa, \lambda_2, h^2, g^2),$$
$$\partial_t h^2 = \beta_h(\kappa, \lambda_2, h^2, g^2),$$

NGFP at finite fixed gauge coupling **$SU(N_L=2)$**

- Choose finite gauge coupling g (no FP for flow of g)
- *For a given g matter system shows NGFP in SSB regime*
- Strong dependence of FP on value of g
- Position of the *minimum κ diverges as $g \to 0$*

$$\begin{aligned}\partial_t \kappa &= \beta_\kappa(\kappa, \lambda_2, h^2, g^2) = 0,\\ \partial_t \lambda_2 &= \beta_\lambda(\kappa, \lambda_2, h^2, g^2) = 0,\\ \partial_t h^2 &= \beta_h(\kappa, \lambda_2, h^2, g^2) = 0,\\ \partial_t g^2 &\neq 0\end{aligned}$$

FP value of minimum κ

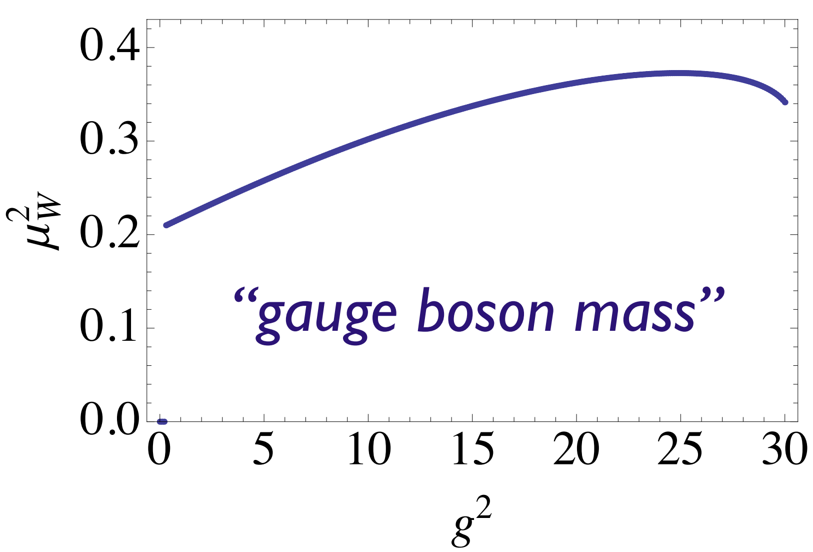


NGFP at finite fixed gauge coupling $SU(N_L=2)$

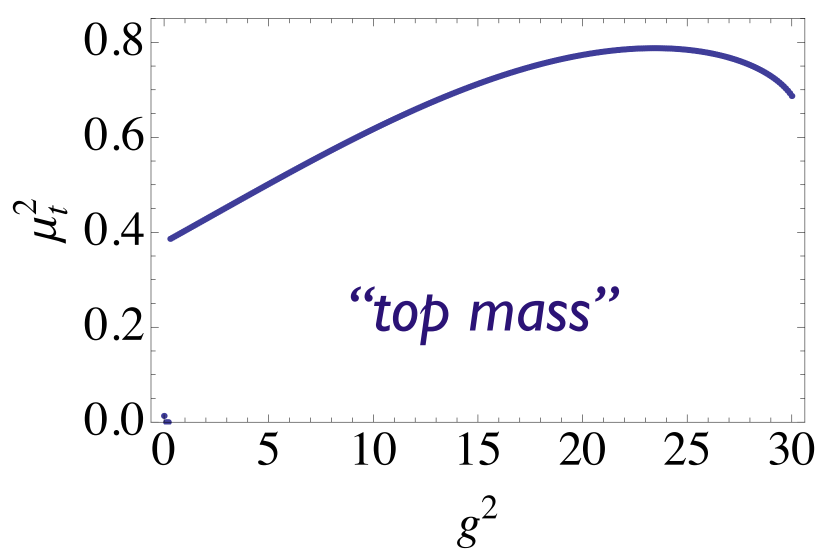
- Position of the *minimum κ diverges as $g \to 0$*
- Asymptotically free couplings as $g \to 0$
- *Combinations of couplings approach finite FP as $g \to 0$*
  - ➡ *dimensionless mass parameters approach finite FP*

$$\partial_t \kappa = \beta_\kappa(\kappa, \lambda_2, h^2, g^2) = 0,$$
$$\partial_t \lambda_2 = \beta_\lambda(\kappa, \lambda_2, h^2, g^2) = 0,$$
$$\partial_t h^2 = \beta_h(\kappa, \lambda_2, h^2, g^2) = 0,$$
$$\partial_t g^2 \neq 0$$

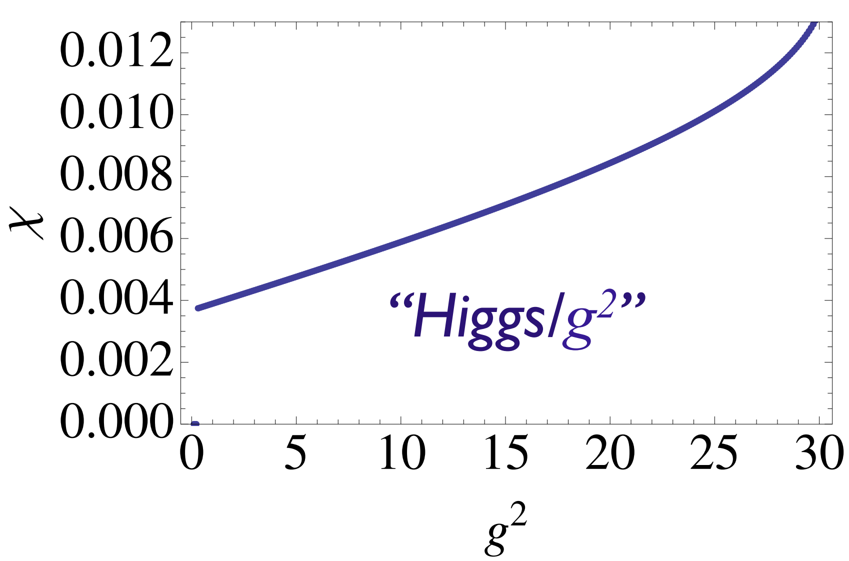
FP value of $(g^2\kappa/2)$



FP value of $(h^2\kappa)$



FP value of $(2\lambda\kappa/g^2)$



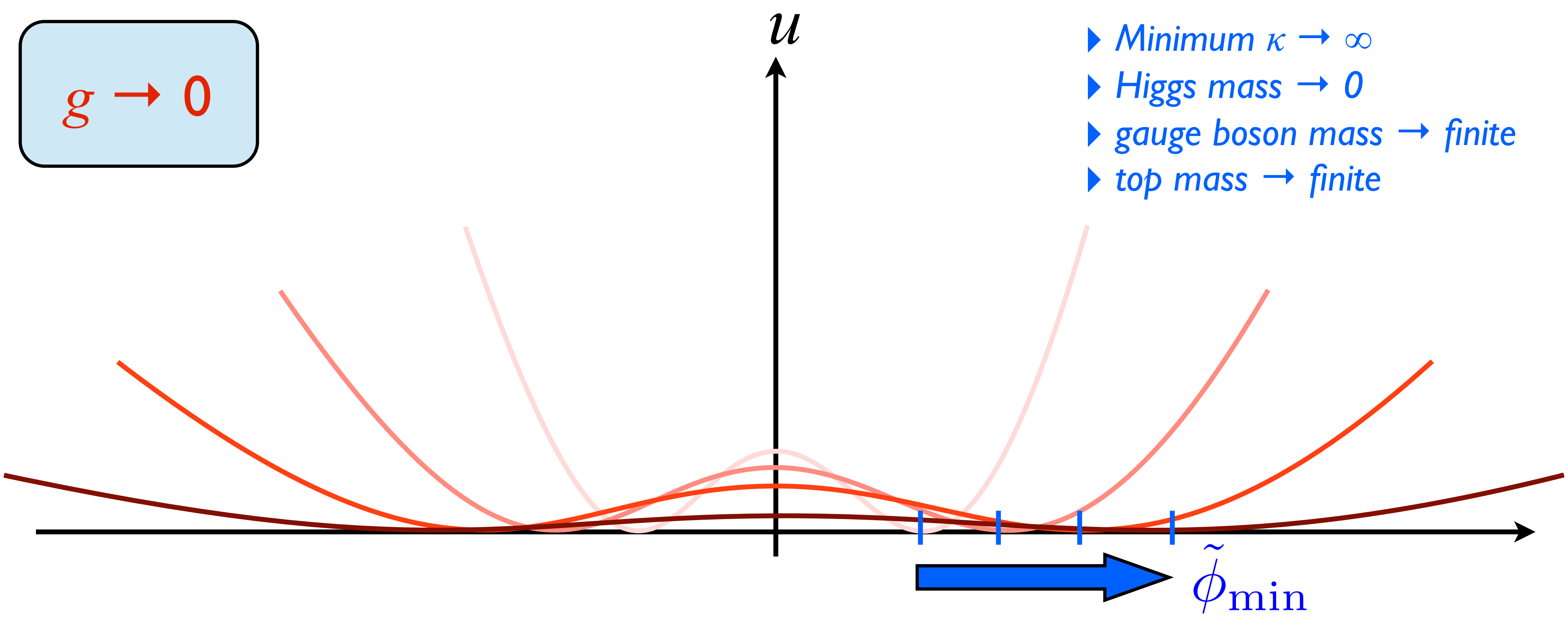
NGFP at finite fixed gauge coupling ***SU(N_L=2)***

Is this the Gaussian FP ($g \to 0$)? No, it's not!

- True GFP of present model has massless gauge bosons and massless chiral fermions
- Mass parameters arise from interplay of interaction terms in the flow equation in the weak-coupling limit → *genuine interaction effect*
- *Critical exponents* do not agree with canonical dimensions at GFP (see below!)

$g \to 0$

- *Minimum $\kappa \to \infty$*
- *Higgs mass → 0*
- *gauge boson mass → finite*
- *top mass → finite*



- Scalar potential approaches flatness in the UV with nonvanishing minimum!

Mass parametrization **$SU(N_L=2)$**

For further investigation → mass parametrization:

	gauge boson	Higgs	top
‣ dimensionless renormalized mass parameters:	$\mu_W^2 = \frac{1}{2} g^2 \kappa,$	$\mu_{\mathrm{H}}^2 = 2\lambda_2 \kappa,$	$\mu_t^2 = \kappa h^2.$
‣ dimensionful renormalized masses:	$m_W^2 = \mu_W^2 k^2,$	$m_{\mathrm{H}}^2 = \mu_{\mathrm{H}}^2 k^2,$	$m_{\mathrm{t}}^2 = \mu_t^2 k^2.$

Flow equations in mass parametrization:

$$\partial_t \mu_{\mathrm{H}}^2 = 2(\partial_t \kappa)\lambda_2 + 2\kappa(\partial_t \lambda_2),$$

$$\partial_t \mu_{\mathrm{t}}^2 = (\partial_t \kappa) h^2 + \kappa(\partial_t h^2), \qquad \partial_t g^2 = g^2 \eta_W,$$

$$\partial_t \mu_{\mathrm{W}}^2 = \frac{1}{2}(\partial_t \kappa) g^2 + \frac{1}{2}\kappa(\partial_t g^2),$$

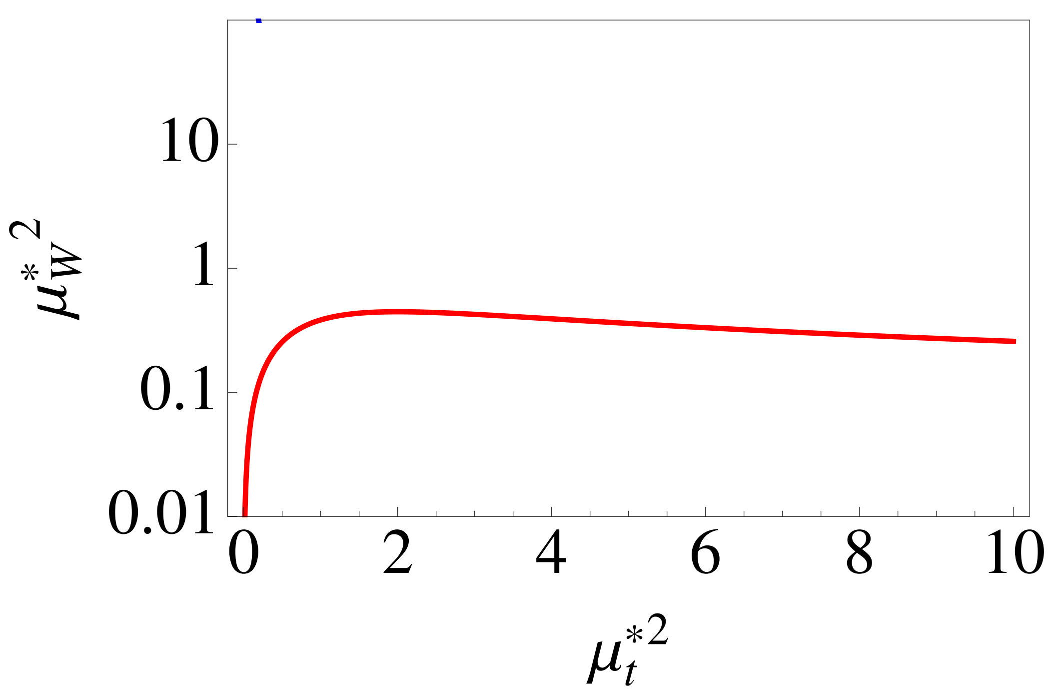
Fixed point in mass parametrization SU(N_L=2)

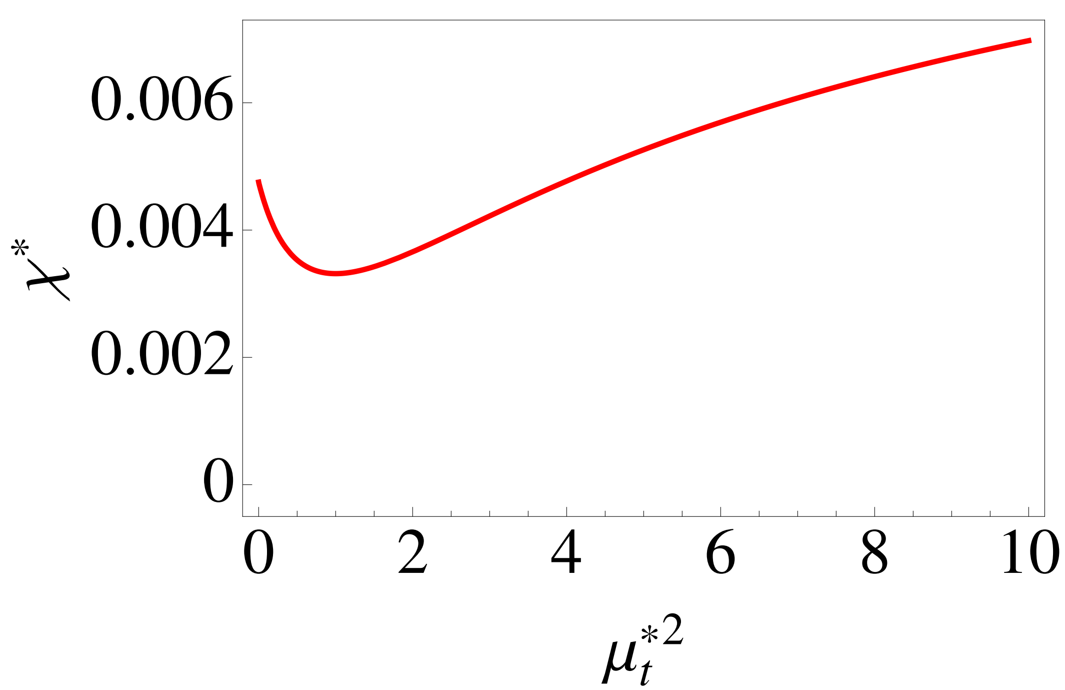
- NGFP in mass parametrization in the limit $g \to 0$ (now also $\partial_t g^2 = 0$):

$$\partial_t g^2 = 0, \quad \partial_t \mu_{\mathrm{H}} = 0,$$

$$\chi = \frac{\mu_{\mathrm{H}}^2}{g^2}, \qquad \partial_t \chi^2 = \frac{1}{g^2}\partial_t \mu_{\mathrm{H}}^2 - \frac{\mu_{\mathrm{H}}^2}{g^4}\partial_t g^2, \qquad \chi^* = -\frac{1}{16\pi^2}\left(\frac{\mu_{\mathrm{t}}^2(1+3\mu_{\mathrm{t}}^2)}{(1+\mu_{\mathrm{t}}^2)^3\mu_W^2} - \frac{9(1+3\mu_W^2)}{4(1+\mu_W^2)^3}\right).$$

Mass parametrization uncovers line of NGFPs:

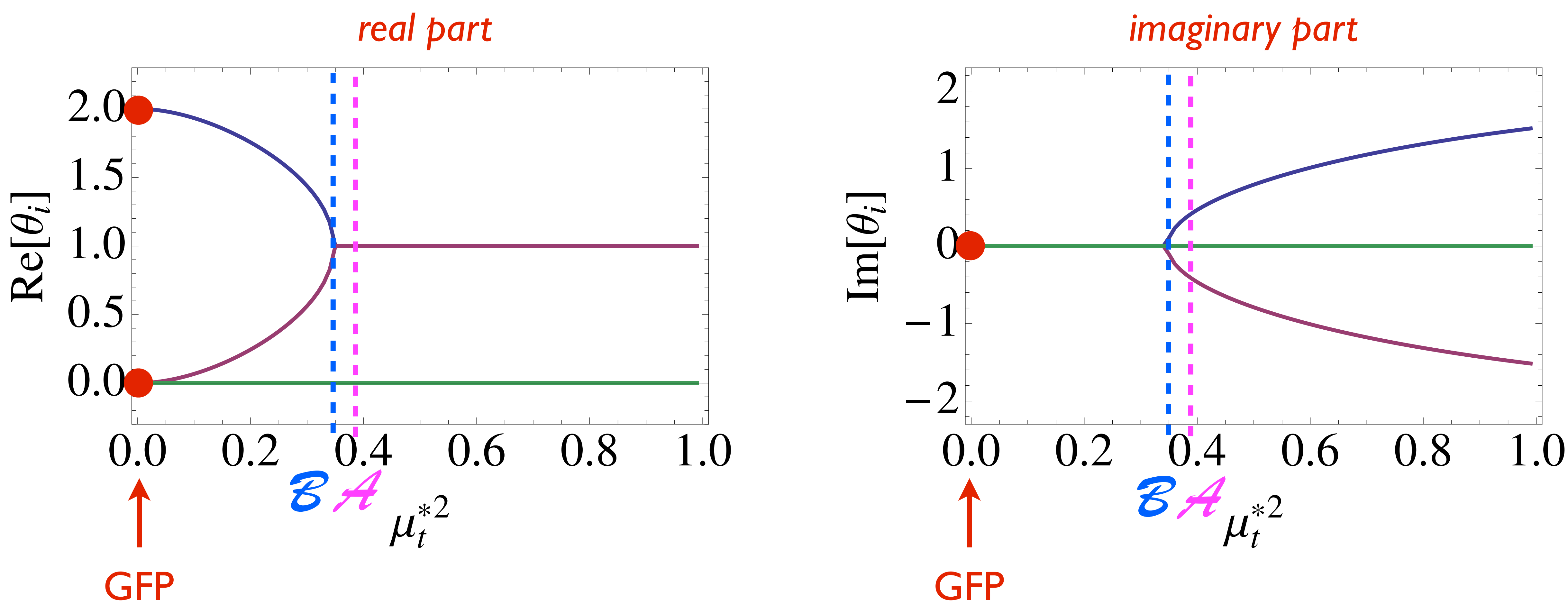




▸ Includes NGFP from standard parametrization: $\mathcal{A}: \quad (\mu_{\mathrm{t}}^{*2}, \mu_W^{*2}, \chi^*) \simeq (0.38, 0.21, 0.0037)$

Critical exponents $SU(N_L=2)$

Critical exponents on the line of fixed points as a function of the top mass parameter:



- Recover GFP for vanishing top mass parameter with canonical dimensions
- Complex pair of critical exponents for NGFP

$$\mathcal{A}: \quad (\mu_{\rm t}^{*2}, \mu_W^{*2}, \chi^*) \simeq (0.38, 0.21, 0.0037)$$
$$\mathcal{A}: \quad \theta_{1/2} = 1 \pm 0.36i\,, \theta_3 = \theta_4 = 0$$

- Special NGFP

$$\mathcal{B}: \quad (\mu_{\rm t}^{*2}, \mu_W^{*2}, \chi^*) \simeq (0.35, 0.19, 0.0037)$$
$$\mathcal{B}: \quad \theta_{1/2} = 1, \theta_{3,4} = 0$$

Fixed points & critical exponents ***SU(N_L=2)***

Summary: We found a *line of UV stable fixed points*

- *3 physical parameters + 1 FP choice* on the line of FPs
- FPs can serve to define UV complete QFT of a gauged Higgs-Yukawa model
- Specifying an RG trajectory yields fully predictive long-range theory

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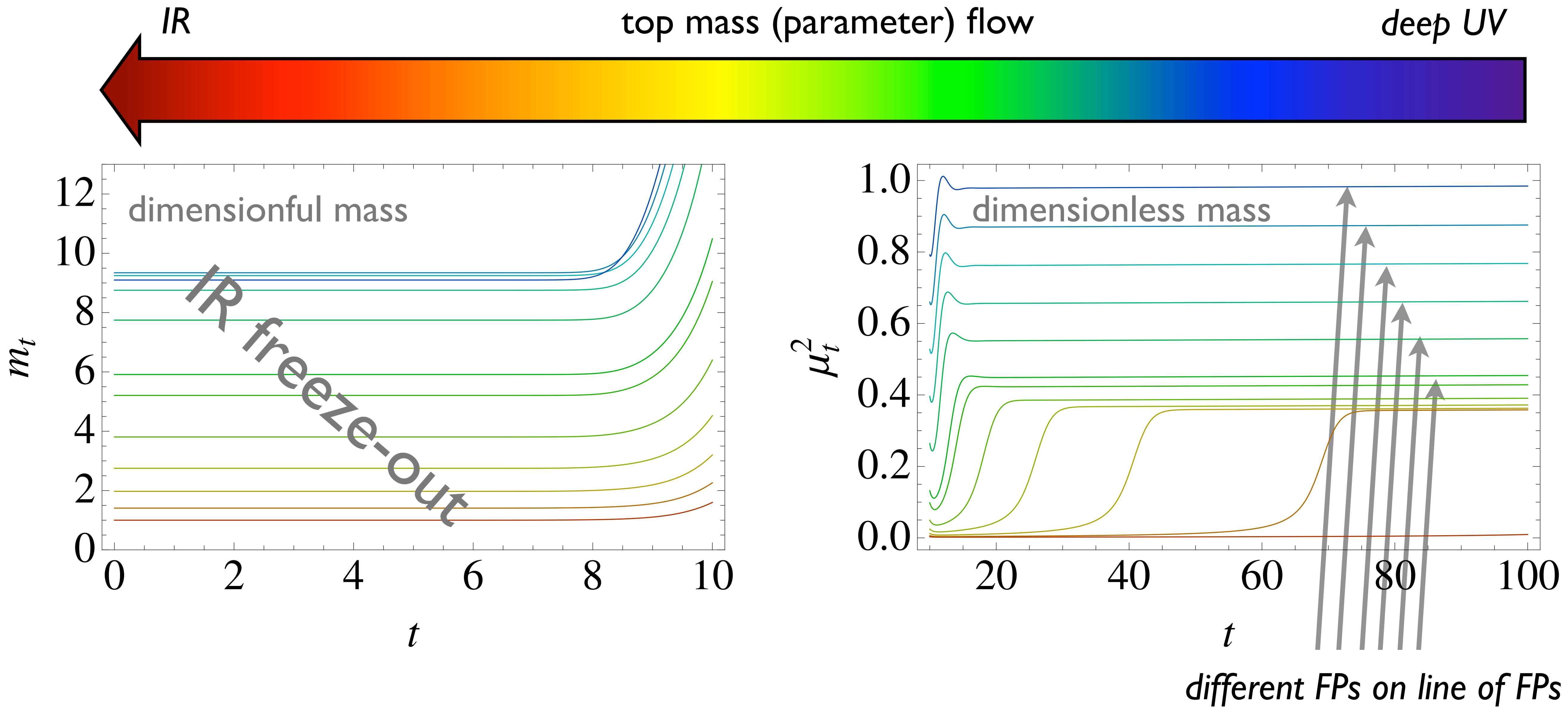
Questions so far ?

Flow from the UV to the electroweak scale ***SU(N_L=2)***

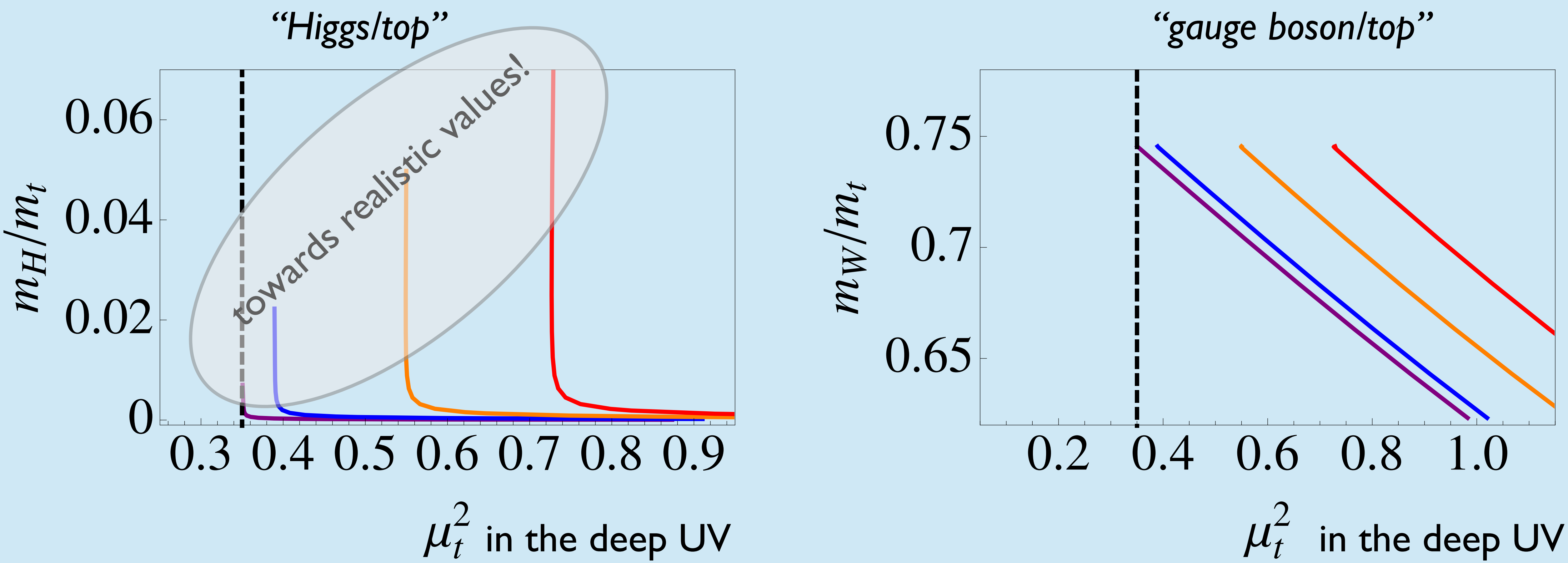


- Similar flows for gauge boson mass and Higgs mass
- Gauge coupling exhibits perturbative asymptotically free running in the UV
- *Asymptotically safe trajectories:*
  - IR exhibits standard Higgs phase as in perturbative scenario
  - UV controlled by FP at which continuum limit can be taken

Flow from the UV to the electroweak scale ***SU(N_L=2)***

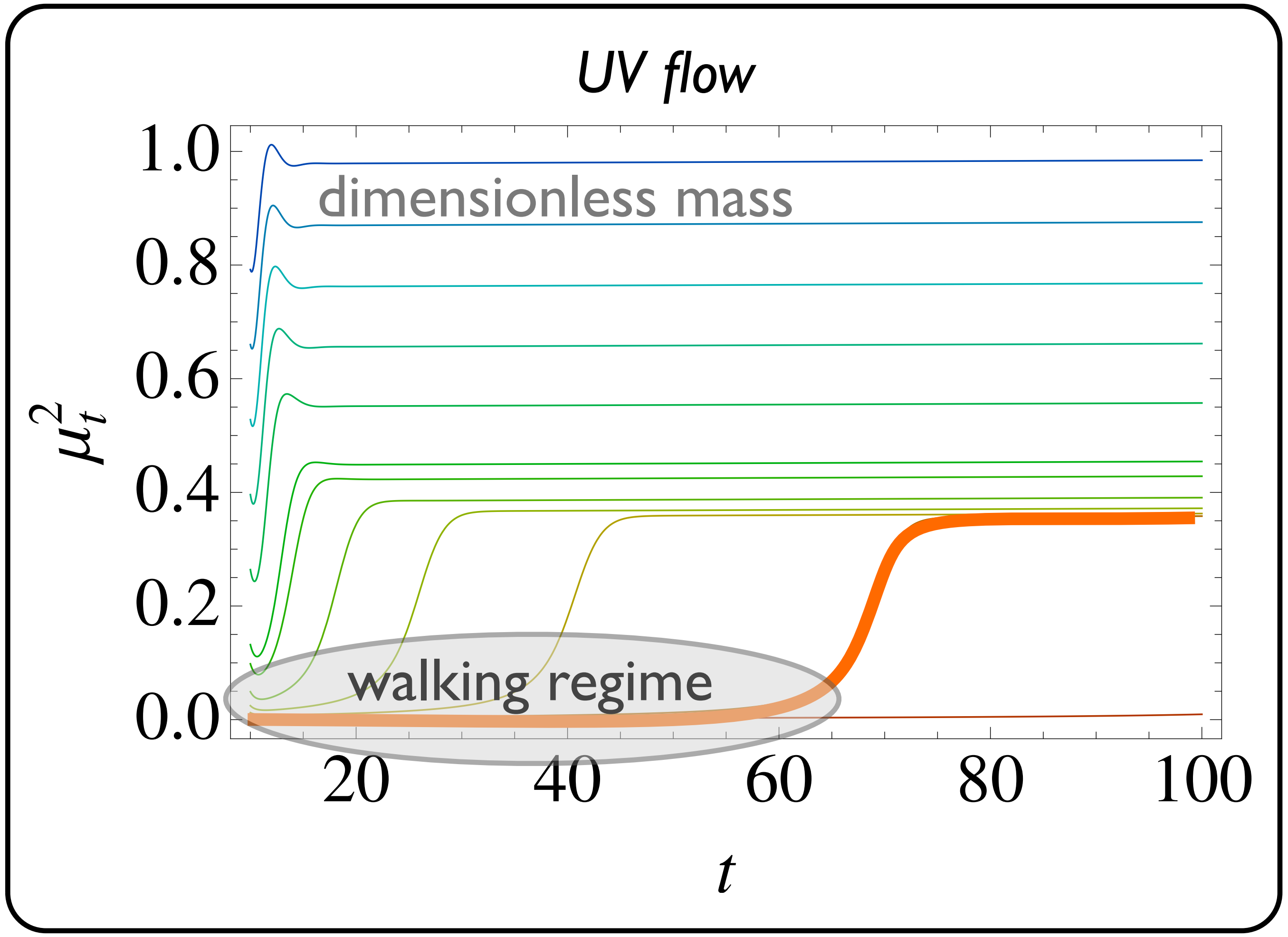
- Observation: Typical flows feature Higgs mass of about two orders of magnitude smaller than top and gauge boson mass
- Only particulary tuned trajectories *Higgs mass to top mass ratio* approach realistic value ~125/175

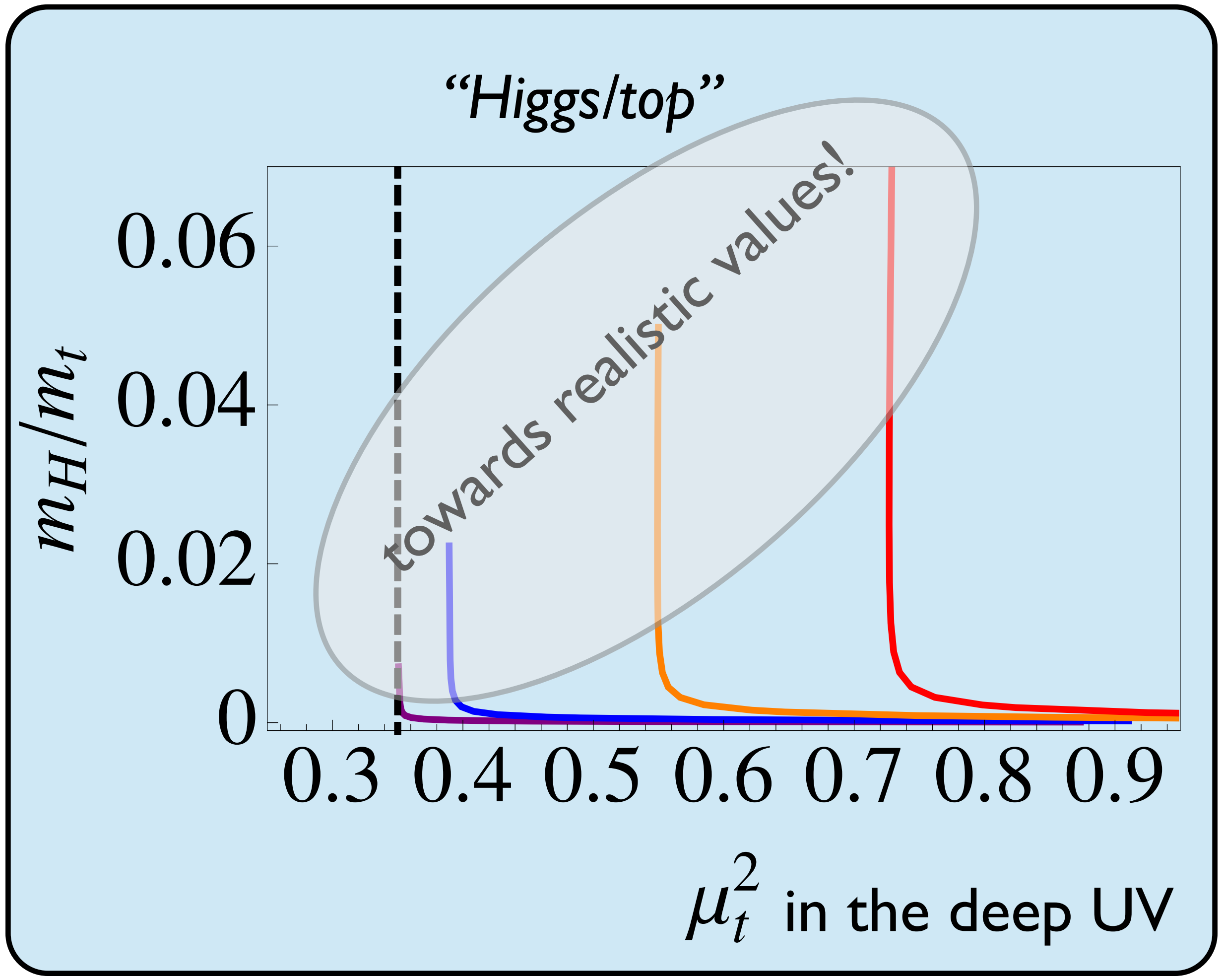
Mass ratios for different initial conditions in the UV
for 4 different UV choices of gauge coupling (colors)



Flow from the UV to the electroweak scale ***SU(N$_L$=2)***

- Realistic values for "Higgs/top" can be achieved via *walking regime on intermediate scales*





- *Walking regime* on intermediate scales between *deep UV FP regime* and *IR freeze-out regime*
- Walking regime = "quasi-fixed-point regime" which extends over wide range of scales
- β-functions small but non-vanishing
  - Remnant of line of FPs at finite gauge coupling

Conclusions

- *Line of weak-coupling FPs from threshold effects* in RG flow of gauged chiral Higgs-Yukawa models
- Weak-coupling FPs *well-controlled* → *No strong couplings, anomalous dimensions vanish*
- *NGFPs* define UV complete asymptotically safe QFTs including an *elementary scalar*
- Critical exponents: *2 relevant + 1 marginally relevant* direction + *1 exactly marginal* direction
  - linear instead of quadratically running renormalization constants
- Typically small Higgs masses → Realistic masses by particularly tuned flow with *walking regime*
- Methods to deal with threshold phenomena are also available within perturbation theory
  - Reproduce our results e.g. with a mass dependent RG scheme

Thank you for your attention!

Link to paper:

http://arxiv.org/abs/1306.6508

More questions ?

Backup slide 1: Truncation

- FRG flow equation: $\partial_t \Gamma_k[\Phi] = \frac{1}{2}\mathrm{STr}\{[\Gamma_k^{(2)}[\Phi] + R_k]^{-1}(\partial_t R_k)\}\,.$

- Truncation:

$$\Gamma_k = \int d^dx \Big[\underbrace{U(\rho)}_{\text{Higgs effective potential}} + \underbrace{Z_\phi (D^\mu \phi)^\dagger (D_\mu \phi)}_{\text{Higgs}} + i(\overbrace{Z_\mathrm{L} \bar{\psi}_\mathrm{L}^a \not{D}^{ab} \psi_\mathrm{L}^b}^{\text{left-handed fermions}} + \overbrace{Z_\mathrm{R} \bar{\psi}_\mathrm{R} \not{\partial} \psi_\mathrm{R}}^{\text{right-handed fermion}})$$

$$+ \underbrace{\bar{h}\bar{\psi}_\mathrm{R}\phi^{a\dagger}\psi_\mathrm{L}^a - \bar{h}\bar{\psi}_\mathrm{L}^a \phi^a \psi_\mathrm{R}}_{\text{Higgs-Yukawa coupling}} + \overbrace{\frac{Z_W}{4} F_{\mu\nu}^i F^{i\mu\nu}}^{\text{gauge fields}} + \overbrace{\frac{Z_\phi}{2\alpha} G^i G^i}^{\text{gauge-fixing term}} - \overbrace{\bar{c}^i \mathcal{M}^{ij} c^j}^{\text{ghosts}} \Big]\,.$$

R_α - gauge with gauge fixing parameter α, flow of $\bar{g}$ with background field method:

vev

fluctuations about vev

- Gauge fixing condition: $G^i(W) = \partial_\mu W_\mu^i + i\alpha \bar{v}\bar{g}(T^i_{\hat{n}\check{a}}\Delta\phi_1^{\check{a}} + iT^i_{\hat{n}a}\Delta\phi_2^a) = 0$

- Fadeev-Popov operator: $\mathcal{M}^{ij} = -\partial^2\delta^{ij} - \bar{g} f^{ilj}\partial_\mu W^{l\mu} + \sqrt{2}\alpha\bar{v}\bar{g}^2 T^i_{\hat{n}\check{a}} T^j_{\check{a}b}\Delta\phi^b,$

Backup Slide 2: Masses

Masses in SB regime:

- Gauge boson mass matrix: $\bar{m}_W^2{}^{ij} = \frac{1}{2} Z_\phi \bar{g}^2 \bar{v}^2 \{T^i, T^j\}_{\hat{n}\hat{n}}.$
- Basis in adjoint color space: $\bar{m}_W^2{}^{ij} = \bar{m}_{W,i}^2 \delta^{ij} \quad (\text{no sum over } i)\,.$
- Scalar mass matrix: $\bar{m}_\phi^{2\,ab} = \bar{v}^2 U''\left(\frac{\bar{v}^2}{2}\right) \hat{n}^a \hat{n}^{\dagger b}\,.$
- “top mass:” $\bar{m}_\mathrm{t} = \frac{\bar{h}\bar{v}}{\sqrt{2}}\,.$