The effective field theory of general relativity and running couplings

1) GR as an EFT

2) Gravitational corrections to gauge coupling running?

3) Can we define a good running G in the perturbative region?

4) AS to one loop - matching to EFT

Overall goal: Understanding how gravity works in the perturbative regime
Does gravity lead to well-defined corrections to running couplings?

“Running” work with M. Anber

AMHERST CENTER FOR FUNDAMENTAL INTERACTIONS
Physics at the interface: Energy, Intensity, and Cosmic frontiers
University of Massachusetts Amherst

John Donoghue
AS skype seminar
Oct 7, 2013

General Relativity as a quantum Effective Field Theory

We do have a quantum theory of GR at ordinary energies

It has the form of an Effective Field Theory

Rigorous well-established procedure

EFT falls apart beyond Planck scale

Well-defined calculations at ordinary energy

A lot of portentous drivel has been written about the quantum theory of gravity, so I'd like to begin by making a fundamental observation about it that tends to be obfuscated. There is a perfectly well-defined quantum theory of gravity that agrees accurately with all available experimental data.

Frank Wilczek
Physics Today
August 2002

Effective Field Theory

Effective Field Theory

- general and practical technique
- separates known low energy physics from high energy physics
- I will present only EFT with dimensionful coupling (gravity)

Goal

- field theory with only the light D.O.F. and their interactions
- heavy particle effects contained in local operators of effective Lagrangian
- most often refers to non-renormalizable subset of a full theory
- complete field theory – loops and all

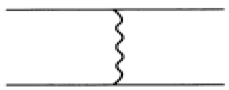
What to watch for:

- presence of new operators in Lagrangian of higher order in energy expansion
- loops generate higher powers of the energy
- what gets renormalized (hint: the higher order operators)

Key Steps

1) High energy effects are local (when viewed at low E)

Example = W exchange



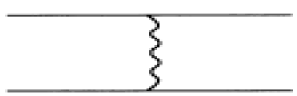
=> local 4 Fermi interaction

Even loops

=> local mass counterterm

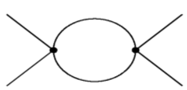
Low energy particles propagate long distances:

Photon:



Not local

$$V \sim \frac{1}{q^2} \sim \frac{1}{r}$$



Even in loops – cuts, imag. parts….

Result: High energy effects in **local** Lagrangian

$$L = g_1 L_1 + g_2 L_2 + g_3 L_3 + \ldots.$$

Even if you don't know the true effect, you know that it is local

-use most general local Lagrangian

2) Energy Expansion

Order lagrangians by powers of (low scale/high scale)^N

Only a finite number needed to a given accuracy

Then:

Quantization: use lowest order Lagrangian

Renormalization: **

-U.V. divergences are **local**

- can be absorbed into couplings of local Lagrangian

Remaining effects are predictions

General Procedure

1) Identify Lagrangian

-- most general (given symmetries)

-- order by energy expansion

2) **Calculate and renormalize**

-- start with lowest order

-- renormalize parameters

3) **Phenomenology**

-- measure parameters

-- residual relations are predictions

Note: Two differences from textbook renormalizable field theory:

1) no restriction to renormalizable terms only

2) energy expansion

QCD and the physics of pions – Chiral Perturbation Theory

The chiral symmetry of QCD, with pions as pseudo-Golstone bosons requires a non-linear lagrangian with **all** powers of the pion field

$$U = \exp(i\boldsymbol{\tau}\cdot\boldsymbol{\pi}/F_\pi) \qquad \text{with} \qquad U \to LUR^\dagger$$

Construct most general lagrangian consistent with symmetry – order by energy expansion:

1) Only a constant at zero derivatives: $U^\dagger U = 1$

2) Unique term at two derivatives

$$\mathcal{L}_2 = \frac{F_\pi^2}{4}\,\mathrm{Tr}\,\left(D_\mu U D^\mu U^\dagger\right) \qquad D_\mu U \equiv \partial_\mu U + i\ell_\mu U - iUr_\mu$$

3) Many terms with more derivatives:

$$\mathcal{L} = F_\pi^2 Tr(D_\mu U D^\mu U^\dagger) + L_1[Tr(D_\mu U D^\mu U^\dagger)]^2 + L_2 Tr(D_\mu U D_\nu U^\dagger)\, Tr(D^\mu U D^\nu U^\dagger) + \ldots\ldots..$$

At low energy only the lowest lagrangian is relevant:

$$\mathcal{L}_2 = \frac{F_\pi^2}{4} \mathrm{Tr} \left(D_\mu U D^\mu U^\dagger \right)$$

All depends on one parameter – must be measured

$$\left\langle 0 \left| A_\mu^j(0) \right| \pi^k(\mathbf{p}) \right\rangle = i F_\pi p_\mu \delta^{jk}$$

$$F_\pi \simeq 92 \text{ MeV}$$

$$\Gamma_{\pi^+ \to \mu^+ \nu_\mu} = \frac{G_F^2}{4\pi} F_\pi^2 m_\mu^2 m_\pi |V_{\mathrm{ud}}|^2 \left(1 - \frac{m_\mu^2}{m_\pi^2} \right)^2$$

Then very large number of predictions:

$$\mathrm{Re}\, T_\ell^I = \left(\frac{q^2}{m_\pi^2} \right)^\ell \left(a_\ell^I + b_\ell^I \frac{q^2}{m_\pi^2} + \dots \right)$$

$$a_0^0 = \frac{7 m_\pi^2}{32 \pi F_\pi^2}, \quad a_0^2 = -\frac{m_\pi^2}{16 \pi F_\pi^2}, \quad a_1^1 = \frac{m_\pi^2}{24 \pi F_\pi^2},$$

$$b_0^0 = \frac{m_\pi^2}{4 \pi F_\pi^2}, \quad b_0^2 = -\frac{m_\pi^2}{8 \pi F_\pi^2},$$

$$T_0^0 = \frac{1}{32 \pi F_\pi^2} (2s - m_\pi^2)$$

Table VII–2. The radiative complex of pion and kaon transitions.

Pions	Kaons
$\gamma \to \pi^+ \pi^-$	$\gamma \to K^- K^+$
$\gamma \pi^+ \to \gamma \pi^+$	$\gamma K^+ \to \gamma K^+$
$\pi^+ \to e^+ \nu_e \gamma$	$K^+ \to e^+ \nu_e \gamma$
$\pi^+ \to \pi^0 e^+ \nu_e$	$K \to \pi e^+ \nu_e$
$\pi^+ \to e^+ \nu_e e^+ e^-$	$K^+ \to e^+ \nu_e e^+ e^-$
	$K^+ \to \pi^0 e^+ \nu_e \gamma$

Weinberg's power counting theorem

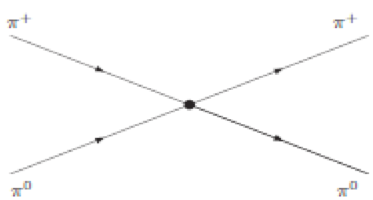
$\boldsymbol{M \sim E^D}$ with $$D = 2 + \sum_n N_n(n-2) + 2N_L$$

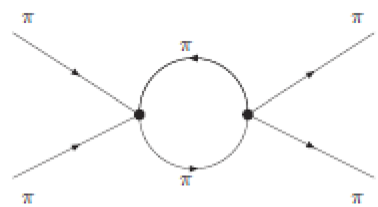
Translation: Each loop add a power of $\mathbf{E^2}$

Lowest order in energy: Tree level L_2
Order E^4 - one loop L_2 (and tree level L_4)
Order E^6 – two loops L_2 (and one loop $L_{4,}$ tree L_6)

Consequence: Loop diagram does not renormalize the lowest order Lagrangian

Example:





$$\mathcal{M} = \frac{s}{F_\pi^2}$$

$$\mathcal{M} = \frac{1}{F_\pi^4} I(s) = \frac{s}{F_\pi^2}\frac{s}{16\pi^2 F_\pi^2}(a + b\ln s)$$

Result: Loop expansion is expansion in $$\frac{E^2}{16\pi^2 F_\pi^2}$$

Background field renormalization

Expansion about background field preserves symmtries of the theory

$$U = \bar{U} e^{i\Delta/F}$$

Integrate out quantum fluctuations

$$S = \int d^4x \left[\mathcal{L}(\bar{U}) + \frac{1}{2}\Delta(D^2 + \sigma)\Delta \right]$$

Result is local E^4 lagrangian

$$\Delta\mathcal{L} = \frac{1}{192\pi^2(d-4)} \left[[Tr(D_\mu U D^\mu U^\dagger)]^2 + 2Tr(D_\mu U D_\nu U^\dagger)\ Tr(D^\mu U D^\nu U^\dagger) \right]$$

$$L_1^{ren} = L_1 + \frac{1}{192\pi^2(d-4)}$$

$$L_2^{ren} = L_2 + \frac{2}{192\pi^2(d-4)}$$

Actually, richer set of operators than I have been showing:

$$\mathcal{L}_2 = \frac{F_\pi^2}{4}\,\mathrm{Tr}\,\left(D_\mu U D^\mu U^\dagger\right) + \frac{F_\pi^2}{4}\,\mathrm{Tr}\,\left(\chi U^\dagger + U\chi^\dagger\right)$$

$$\begin{aligned}
\mathcal{L}_4 &= \sum_{i=1}^{10} L_i O_i \\
&= l_1 \left[\mathrm{Tr}\,\left(D_\mu U D^\mu U^\dagger\right)\right]^2 + L_2\,\mathrm{Tr}\,\left(D_\mu U D_\nu U^\dagger\right)\cdot\mathrm{Tr}\,\left(D^\mu U D^\nu U^\dagger\right) \\
&+ L_3\,\mathrm{Tr}\,\left(D_\mu U D^\mu U^\dagger D_\nu U D^\nu U^\dagger\right) \\
&+ L_4\,\mathrm{Tr}\,\left(D_\mu U D^\mu U^\dagger\right)\,\mathrm{Tr}\,\left(\chi U^\dagger + U\chi^\dagger\right) \\
&+ L_5\,\mathrm{Tr}\,\left(D_\mu U D^\mu U^\dagger\left(\chi U^\dagger + U\chi^\dagger\right)\right) + L_6\left[\mathrm{Tr}\,\left(\chi U^\dagger + U\chi^\dagger\right)\right]^2 \\
&+ L_7\left[\mathrm{Tr}\,\left(\chi^\dagger U - U\chi^\dagger\right)\right]^2 + L_8\,\mathrm{Tr}\,\left(\chi U^\dagger \chi U^\dagger + U\chi^\dagger U\chi^\dagger\right) \\
&+ iL_9\,\mathrm{Tr}\,\left(L_{\mu\nu} D^\mu U D^\nu U^\dagger + R_{\mu\nu} D^\mu U^\dagger D^\nu U\right) + L_{10}\,\mathrm{Tr}\,\left(L_{\mu\nu} U R^{\mu\nu} U^\dagger\right),
\end{aligned}$$

$$\begin{aligned}
S_4^{\rm div} = -\lambda \int d^4x \Bigg[&\frac{3}{32}\left[\mathrm{Tr}\,\left(D_\mu U D^\mu U^\dagger\right)\right]^2 \\
&+ \frac{3}{16}\,\mathrm{Tr}\,\left(D_\mu U D_\nu U^\dagger\right)\,\mathrm{Tr}\,\left(D^\mu U D^\nu U^\dagger\right) \\
&+ \frac{1}{8}\,\mathrm{Tr}\,\left(D_\mu U D^\mu U^\dagger\right)\,\mathrm{Tr}\,\left(\chi^\dagger U + U^\dagger\chi\right) \\
&+ \frac{3}{8}\,\mathrm{Tr}\,\left[D_\mu U D^\mu U^\dagger\left(\chi U^\dagger + U\chi^\dagger\right)\right] \qquad (2.5) \\
&+ \frac{11}{144}\left[\mathrm{Tr}\,\left(\chi U^\dagger + U\chi^\dagger\right)\right]^2 + \frac{5}{48}\,\mathrm{Tr}\,\left(\chi U^\dagger\chi U^\dagger + U\chi^\dagger U\chi^\dagger\right) \\
&+ \frac{i}{4}\,\mathrm{Tr}\,\left(L_{\mu\nu} D^\mu U D^\nu U^\dagger + R_{\mu\nu} D^\mu U^\dagger D^\nu U\right) - \frac{1}{4}\,\mathrm{Tr}\,\left(L_{\mu\nu} U R^{\mu\nu} U^\dagger\right)\Bigg]
\end{aligned}$$

with

$$\lambda \equiv \frac{1}{32\pi^2}\left\{\frac{2}{d-4} - \ln 4\pi - 1 + \gamma\right\} \quad . \qquad (2.6)$$

Nature of predictions:

Not the coefficients - measured
Relationships between different reactions
Residual kinematics from loops – logs in particular

Example – pion form factor

$$G_\pi(q^2) = 1 + q^2 \left[\frac{2L_9^{(2)r}}{F_\pi^2} - \frac{1}{96\pi^2 F_\pi^2} \left(\ln \frac{q^2}{\mu^2} + 1 \right) \right] + \ldots$$

$$L_9^r(\mu = m_\eta) = (7.1 \pm 0.3) \times 10^{-3}$$

Note: Lowest order coefficient F_π does not have any scale dependence
but next order coefficients have log scale dependence

$$L_9^r(\mu') = \begin{cases} L_9^r(\mu) - \dfrac{1}{192\pi^2} \ln \left(\dfrac{\mu'^2}{\mu^2} \right) & (SU(2)) \\ L_9^r(\mu) - \dfrac{1}{128\pi^2} \ln \left(\dfrac{\mu'^2}{\mu^2} \right) & (SU(3)) \end{cases} .$$

What if we used dimensional cutoff?

Exactly the same formulas result, when expressed in physical parameters

Intermediate renormalization different:

$$F_\pi^2 = F_0^2[1 + a\frac{\Lambda^2}{16\pi^2 F_0^2}] \qquad\qquad L_i = L_i^{(0)} + \frac{\gamma_i}{32\pi^2 F_0^2}\ln\frac{\Lambda^2}{\mu^2}$$

But disappears from final result

Gravity as an effective theory

Weinberg
JFD

Both General Relativity and Quantum Mechanics known and tested over common range of scales

Is there an incompatibility **at those scales** ?

Or are problems only at uncharted high energies?

Need to study GR with a careful consideration of scales

The general Lagrangian

The Einstein action: $$S_{grav} = \int d^4x \sqrt{-g} \left[\frac{2}{\kappa^2} R \right]$$

$\kappa^2 = 32\pi G, g = det g_{\mu\nu}, g_{\mu\nu}$ is the metric tensor and $R = g^{\mu\nu} R_{\mu\nu}$

$$\begin{aligned} R_{\mu\nu} &= \partial_\nu \Gamma^\lambda_{\mu\lambda} - \partial_\lambda \Gamma^\lambda_{\mu\nu} + \Gamma^\sigma_{\mu\lambda} \Gamma^\lambda_{\nu\sigma} - \Gamma^\sigma_{\mu\nu} \Gamma^\lambda_{\lambda\sigma} \\ \Gamma^\lambda_{\alpha\beta} &= \frac{g^{\lambda\sigma}}{2} \left(\partial_\alpha g_{\beta\sigma} + \partial_\beta g_{\alpha\sigma} - \partial_\sigma g_{\alpha\beta} \right) \end{aligned}$$

But this is not the most general lagrangian consistent with general covariance.

Key: R depends on <u>two derivatives</u> of the metric
- Energy expansion – expansion in number of derivatives

$$S_{grav} = \int d^4x \sqrt{-g} \left\{ \Lambda + \frac{2}{\kappa^2} R + c_1 R^2 + c_2 R_{\mu\nu} R^{\mu\nu} + \ldots \right\}$$

Parameters

$$S_{grav} = \int d^4x \sqrt{-g} \left\{ \Lambda + \frac{2}{\kappa^2} R + c_1 R^2 + c_2 R_{\mu\nu} R^{\mu\nu} + \ldots \right\}$$

1) Λ = cosmological constant

$$\Lambda = (1.2 \pm 0.4) \times 10^{-123} M_P^4$$

$$M_P = 1.22 \times 10^{19} \text{ GeV}$$

-this is observable only on cosmological scales
-neglect for rest of talk
-interesting aspects

2) Newton's constant

$$\kappa^2 = 32\pi G$$

3) Curvature –squared terms c_1, c_2

- studied by Stelle
- modify gravity at very small scales
-essentially unconstrained by experiment

$$c_1, c_2 \leq 10^{74}$$

Quantization

"Easy" to quantize gravity:

-Covariant quantization Feynman deWitt

- -gauge fixing
- -ghosts fields

-Background field method 't Hooft Veltman

- -retains symmetries of GR
- -path integral

Background field:

$$
\begin{aligned}
g_{\mu\nu} &= \bar{g}_{\mu\nu} + \kappa h_{\mu\nu} \\
g^{\mu\nu} &= \bar{g}^{\mu\nu} - \kappa h^{\mu\nu} + \kappa^2 h^{\mu}_{\lambda} h^{\lambda\nu} + \dots
\end{aligned}
$$

Expand around this background:

$$
S_{grav} = \int d^4x \sqrt{-\bar{g}} \left[\frac{2\bar{R}}{\kappa^2} + \mathcal{L}_g^{(1)} + \mathcal{L}_g^{(2)} + \dots \right]
$$

$$
\begin{aligned}
\mathcal{L}_g^{(1)} &= \frac{h_{\mu\nu}}{\kappa} \left[\bar{g}^{\mu\nu} \bar{R} - 2\bar{R}^{\mu\nu} \right] \\
\mathcal{L}_g^{(2)} &= \frac{1}{2} h_{\mu\nu;\alpha} h^{\mu\nu;\alpha} - \frac{1}{2} h_{;\alpha} h^{;\alpha} + h_{;\alpha} h^{\alpha\beta}{}_{;\beta} - h_{\mu\beta;\alpha} h^{\mu\alpha;\beta} \\
&\quad + \bar{R} \left(\frac{1}{4} h^2 - \frac{1}{2} h_{\mu\nu} h^{\mu\nu} \right) + \left(2h^{\lambda}_{\mu} h_{\nu\lambda} - h h_{\mu\nu} \right) \bar{R}^{\mu\nu}
\end{aligned}
$$

Linear term vanishes by Einstein Eq. $\bar{R}^{\mu\nu} - \frac{1}{2} \bar{g}^{\mu\nu} \bar{R} = -\frac{\kappa^2}{4} T^{\mu\nu}$

Gauge fixing:
-harmonic gauge

$$\mathcal{L}_{gf} = \sqrt{-\bar{g}} \left\{ \left(h_{\mu\nu}{}^{;\nu} - \frac{1}{2} h_{;\mu} \right) \left(h^{\mu\lambda}{}_{;\lambda} - h^{;\mu} \right) \right\}$$

$$h \equiv h^{\lambda}_{\lambda}$$

Ghost fields:

$$\mathcal{L}_{ghost} = \sqrt{-\bar{g}} \eta^{*\mu} \left\{ \eta_{\mu;\lambda}{}^{;\lambda} - \bar{R}_{\mu\nu} \eta^{\nu} \right\}$$

vector fields
anticommuting,
in loops only

Performing quantum calculations

Quantization was straightforward, but what do you do next?
- calculations are not as simple

<u>Next step: Renormalization</u>
-divergences arise at high energies
-not of the form of the basic lagraingian

Solution: Effective field theory and renormalization
- renormalize divergences into parameters of the most general lagrangian (c_1,c_2…)

Power counting theorem: (pure gravity, Λ=0)
-each graviton loop → 2 more powers in energy expansion
-1 loop → Order $(\partial g)^4$
-2 loop → Order $(\partial g)^6$

Renormalization

One loop calculation: 'tHooft and Veltman

$$Z[\phi, J] = Tr ln D$$

Divergences are local:

dim. reg. preserves symmetry

$$\Delta\mathcal{L}_0^{(1)} = \frac{1}{8\pi^2}\frac{1}{\epsilon}\left\{\frac{1}{120}R^2 + \frac{7}{20}R_{\mu\nu}R^{\mu\nu}\right\} \qquad \epsilon = 4 - d$$

Renormalize parameters in general action:

$$c_1^{(r)} = c_1 + \frac{1}{960\pi^2\epsilon}$$
$$c_2^{(r)} = c_2 + \frac{7}{160\pi^2\epsilon}$$

Pure gravity "one loop finite" since $R_{\mu\nu}$=0

Note: Two loop calculation known in pure gravity Goroff and Sagnotti

$$\Delta\mathcal{L}^{(2)} = \frac{209\,\kappa}{2880(16\pi^2)^2}\frac{1}{\epsilon}\sqrt{-g}R^{\alpha\beta}{}_{\gamma\delta}R^{\gamma\delta}{}_{\rho\sigma}R^{\rho\sigma}{}_{\alpha\beta}$$

Order of six derivatves

What are the quantum predictions?

Not the divergences

- they come from the Planck scale
- unreliable part of theory

Not the parameters

- local terms in L
- we would have to measure them

$$Amp \sim q^2 \ln(-q^2) \quad , \quad \sqrt{-q^2}$$

Low energy propagation

- not the same as terms in the Lagrangian
- most always non-analytic dependence in momentum space
- can't be Taylor expanded – can't be part of a local Lagrangian
- long distance in coordinate space

Corrections to Newtonian Potential

JFD 1994
JFD, Holstein, Bjerrum-Bohr 2002
Khriplovich and Kirilin
Other references later

Here discuss scattering potential of two heavy masses.

$$\begin{aligned} \langle f|T|i\rangle &\equiv (2\pi)^4\delta^{(4)}(p-p')(\mathcal{M}(q)) \\ &= -(2\pi)\delta(E-E')\langle f|\tilde{V}(\mathbf{q})|i\rangle \end{aligned}$$

Potential found using from

$$V(\mathbf{x}) = \frac{1}{2m_1}\frac{1}{2m_2}\int \frac{d^3q}{(2\pi)^3} e^{i\mathbf{q}\cdot\mathbf{x}}\mathcal{M}(\vec{q})$$

Classical potential has been well studied

Iwasaki
Gupta-Radford
Hiida-Okamura

What to expect:

General expansion:

$$V(r) = -\frac{GMm}{r}\left[1 + a\frac{G(M+m)}{rc^2} + b\frac{G\hbar}{r^2c^3}\right] + cG^2Mm\delta^3(r)$$

Classical expansion parameter

Quantum expansion parameter

Short range

Relation to momentum space:

$$\int \frac{d^3q}{(2\pi)^3}\; e^{i\mathbf{q}\cdot\mathbf{r}}\frac{1}{|\mathbf{q}|^2} = \frac{1}{4\pi r}$$

$$\int \frac{d^3q}{(2\pi)^3}\; e^{i\mathbf{q}\cdot\mathbf{r}}\frac{1}{|\mathbf{q}|} = \frac{1}{2\pi^2 r^2}$$

$$\int \frac{d^3q}{(2\pi)^3}\; e^{i\mathbf{q}\cdot\mathbf{r}}\ln(\mathbf{q}^2) = \frac{-1}{2\pi r^3}$$

Momentum space amplitudes:

$$V(q^2) = \frac{GMm}{q^2}\left[1 + a'G(M+m)\sqrt{-q^2} + b'G\hbar q^2\ln(-q^2) + c'Gq^2\right]$$

Classical

quantum

short range

Non-analytic

analytic

Parameter free and divergence free

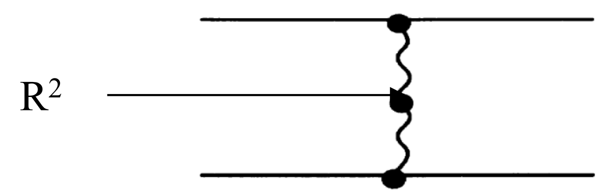
Recall: divergences like local Lagrangian $\sim R^2$

Also unknown parameters in local Lagrangian $\sim c_1, c_2$

But this generates only "short distance term"

Note: R^2 has 4 derivatives $R^2 \sim q^4$

<u>Then:</u>
Treating R^2 as perturbation

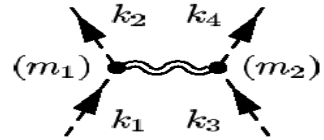


$$V_{R^2} \sim G^2 Mm \;\; \frac{1}{q^2}\, q^4 \, \frac{1}{q^2} \sim \text{const.} \to G^2 Mm\delta^3(x)$$

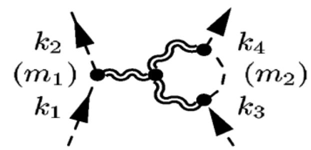
Local lagrangian gives only short range terms

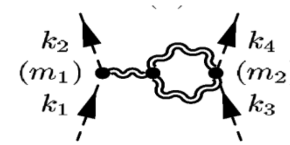
The calculation:

Lowest order:

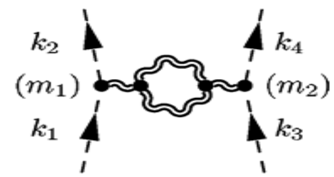


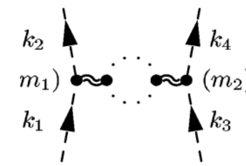
Vertex corrections:



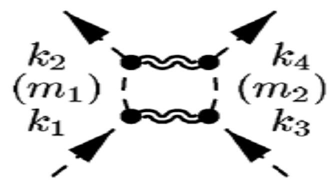


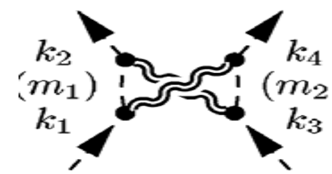
Vacuum polarization:
(Duff 1974)



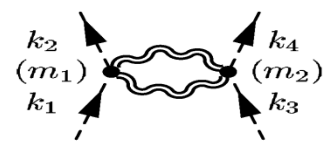


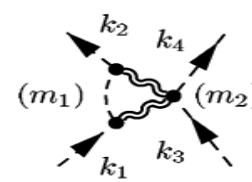
Box and crossed box





Others:





Results:

Pull out non-analytic terms:
-for example the vertex corrections:

$$M_{5(a)+5(b)}(\vec{q}) = 2G^2 m_1 m_2 \left(\frac{\pi^2(m_1+m_2)}{|\vec{q}|} + \frac{5}{3}\log \vec{q}^2\right)$$

$$M_{5(c)+5(d)}(\vec{q}) = -\frac{52}{3} G^2 m_1 m_2 \log \vec{q}^2$$

Sum diagrams:

$$V(r) = -\frac{G m_1 m_2}{r}\left[1 + 3\frac{G(m_1+m_2)}{r} + \frac{41}{10\pi}\frac{G\hbar}{r^2}\right]$$

Gives precession
of Mercury, etc
(Iwasaki ;
Gupta + Radford)

Quantum
correction

Comments

1) Both classical and quantum emerge from a one loop calculation!
   - classical first done by Gupta and Radford (1980)

1) Unmeasurably small correction:
   - best perturbation theory known(!)

3) Quantum loop well behaved - no conflict of GR and QM

4) Other calculations
   (Radikowski, Duff, JFD; Muzinich and Vokos; Hamber and Liu; Akhundov, Bellucci, and Sheikh ; Khriplovich and Kirilin)
   -other potentials or mistakes

5) Why not understood 30 years ago?
   - power of effective field theory reasoning

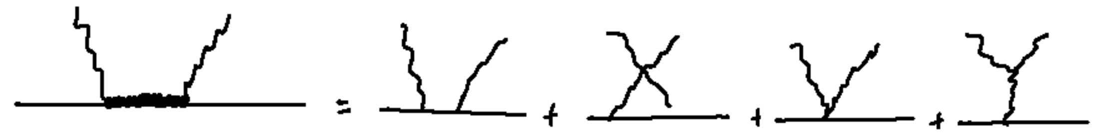
Loops without loops - unitarity techniques:

arXiv:1309.0804

(with E. Bjerrum-Bohr and P. Vanhove)

1) **Dispersion relations** -on-shell cut involves gravitational Compton amplitude

with



$$\rho_g(s,t) = \frac{-1}{32\pi}\int \frac{d\Omega_k}{4\pi}\,\mathcal{M}_A^{\mu\nu,\lambda\sigma}(p_1,-k,p_1-q,q-k)\mathcal{M}_B^{\alpha\beta,\gamma\delta}(p_3,-k,p_3+q,k-q)P_{\mu\nu,\alpha\beta}P_{\lambda\sigma,\gamma\delta}$$

On-shell gravity amplitudes are squares of gauge theory amplitudes – consistency check

2) Full modern helicity formalism calculation

$$M_0^{\rm tree}(p_1,p_2,k_1^+,k_2^+) = \frac{\kappa_{(4)}^2}{16}\,\frac{1}{(k_1\cdot k_2)}\,\frac{m^4\,[k_1\,k_2]^4}{(k_1\cdot p_1)(k_1\cdot p_2)}\,,$$

$$M_0^{\rm tree}(p_1,p_2,k_1^-,k_2^+) = \frac{\kappa_{(4)}^2}{16}\,\frac{1}{(k_1\cdot k_2)}\,\frac{\langle k_1|p_1|k_2]^2\,\langle k_1|p_2|k_2]^2}{(k_1\cdot p_1)(k_1\cdot p_2)}$$

$$M^{1-{\rm loop}}\Big|_{disc} = \int \frac{d^D\ell}{(2\pi)^D}\,\frac{\sum_{\lambda_1,\lambda_2} M_{\lambda_1\lambda_2}^{\rm tree}(p_1,p_2,-\ell_2^{\lambda_2},\ell_1^{\lambda_1})(M_{\lambda_1\lambda_2}^{\rm tree}(p_3,p_4,\ell_2^{\lambda_2},-\ell_1^{\lambda_1}))^*}{\ell_1^2\ell_2^2}\Bigg|_{cut},$$

Reproduce usual result without calculating loops – low energy only

$$V(r) = -\frac{GMm}{r}\left[1+3\frac{G(M+m)}{rc^2}+\frac{41}{10\pi}\frac{G\hbar}{r^2c^3}\right]$$

Graviton –graviton scattering

Fundamental quantum gravity process

Lowest order amplitude:

$$\mathcal{A}^{tree}(++;++) = \frac{i}{4}\frac{\kappa^2 s^3}{tu}$$

Cooke;
Behrends Gastmans
Grisaru et al

One loop:

Incredibly difficult using field theory

Dunbar and Norridge –string based methods! (just tool, not full string theory)

$$\begin{aligned}
\mathcal{A}^{1-loop}(++;--) &= -i\frac{\kappa^4}{30720\pi^2}\left(s^2+t^2+u^2\right)\\
\mathcal{A}^{1-loop}(++;+-) &= -\frac{1}{3}\mathcal{A}^{1-loop}(++;--)\\
\mathcal{A}^{1-loop}(++;++) &= \frac{\kappa^2}{4(4\pi)^{2-\epsilon}}\frac{\Gamma^2(1-\epsilon)\Gamma(1+\epsilon)}{\Gamma(1-2\epsilon)}\mathcal{A}^{tree}(++;++)\times(s\,t\,u) \qquad (3)\\
&\quad\times\left[\frac{2}{\epsilon}\left(\frac{\ln(-u)}{st}+\frac{\ln(-t)}{su}+\frac{\ln(-s)}{tu}\right)+\frac{1}{s^2}f\left(\frac{-t}{s},\frac{-u}{s}\right)\right.\\
&\quad\left.+2\left(\frac{\ln(-u)\ln(-s)}{su}+\frac{\ln(-t)\ln(-s)}{tu}+\frac{\ln(-t)\ln(-s)}{ts}\right)\right]
\end{aligned}$$

where

$$\begin{aligned}
f\left(\frac{-t}{s},\frac{-u}{s}\right) &= \frac{(t+2u)(2t+u)\left(2t^4+2t^3u-t^2u^2+2tu^3+2u^4\right)}{s^6}\left(\ln^2\frac{t}{u}+\pi^2\right)\\
&\quad+\frac{(t-u)\left(341t^4+1609t^3u+2566t^2u^2+1609tu^3+341u^4\right)}{30s^5}\ln\frac{t}{u}\\
&\quad+\frac{1922t^4+9143t^3u+14622t^2u^2+9143tu^3+1922u^4}{180s^4}, \qquad (4)
\end{aligned}$$

Infrared safe:

JFD +
Torma

The $1/\epsilon$ is from infrared
-soft graviton radiation
-made finite in usual way
$1/\epsilon$ -> ln(1/resolution) (gives scale to loops)
-cross section finite

$$\left(\frac{d\sigma}{d\Omega}\right)_{tree} + \left(\frac{d\sigma}{d\Omega}\right)_{rad.} + \left(\frac{d\sigma}{d\Omega}\right)_{nonrad.} = \tag{29}$$

$$= \frac{\kappa^4 s^5}{2048\pi^2 t^2 u^2}\left\{1 + \frac{\kappa^2 s}{16\pi^2}\left[\ln\frac{-t}{s}\ln\frac{-u}{s} + \frac{tu}{2s^2} f\left(\frac{-t}{s}, \frac{-u}{s}\right)\right.\right.$$
$$\left.\left. - \left(\frac{t}{s}\ln\frac{-t}{s} + \frac{u}{s}\ln\frac{-u}{s}\right)\left(3\ln(2\pi^2) + \gamma + \ln\frac{s}{\Lambda^2} + \frac{\sum_{ij}\eta_i\eta_j\mathcal{F}^{(1)}(\gamma_{ij})}{\sum_{ij}\eta_i\eta_j\mathcal{F}^{(0)}(\gamma_{ij})}\right)\right]\right\}.$$

*

finite

Beautiful result:
-low energy theorem of quantum gravity

Reformulate problem of quantum gravity

Old view: GR and Quantum Mechanics incompatible

Unacceptable

New view: We need to find the right "high energy" limit for gravity – new physics or new treatment

Less shocking:

-not a conflict of GR and QM

-just incomplete knowledge

THIS IS PROGRESS!

Summary for purpose of this talk:

$$S_{grav} = \int d^4x \sqrt{-g} \left\{ \Lambda + \frac{2}{\kappa^2} R + c_1 R^2 + c_2 R_{\mu\nu} R^{\mu\nu} + \ldots \right\}$$

$$\updownarrow$$

$$V(r) = -\frac{GMm}{r} \left[1 + a \frac{G(M+m)}{rc^2} + b \frac{G\hbar}{r^2 c^3} \right] + cG^2 Mm \delta^3(r)$$

1) **Loops do not modify the original coupling**

$$\Delta\mathcal{L}_0^{(1)} = \frac{1}{8\pi^2} \frac{1}{\epsilon} \left\{ \frac{1}{120} R^2 + \frac{7}{20} R_{\mu\nu} R^{\mu\nu} \right\}$$

2) **Loops involved in renormalization of higher order coupling**

$$c_1^{(r)} = c_1 + \frac{1}{960\pi^2\epsilon}$$
$$c_2^{(r)} = c_2 + \frac{7}{160\pi^2\epsilon}$$

3) **Matrix elements expanded in powers of the momentum**

$$V(q^2) = \frac{GMm}{q^2} \left[1 + a' G(M+m)\sqrt{-q^2} + b' G\hbar q^2 \ln(-q^2) + c' G q^2 \right]$$

4) **Corrections to lowest order have two features**

- higher order operators and power dependence
- loops also generate logarithms **at higher order**

A motivation for the gravity and running couplings:

PRL **96,** 231601 (2006) PHYSICAL REVIEW LETTERS week ending 16 JUNE 2006

Gravitational Correction to Running of Gauge Couplings

Sean P. Robinson* and Frank Wilczek†
Center for Theoretical Physics, Laboratory for Nuclear Science and Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA
(Received 30 March 2006; published 15 June 2006)

We calculate the contribution of graviton exchange to the running of gauge couplings at lowest nontrivial order in perturbation theory. Including this contribution in a theory that features coupling constant unification does not upset this unification, but rather shifts the unification scale. When extrapolated formally, the gravitational correction renders all gauge couplings asymptotically free.

$$\beta(g, E) \equiv \frac{dg}{d\ln E} = -\frac{b_0}{(4\pi)^2} g^3 + a_0 \frac{E^2}{M_{\rm P}^2} g,$$

$$G \sim \frac{1}{M_P^2} \sim \kappa^2$$

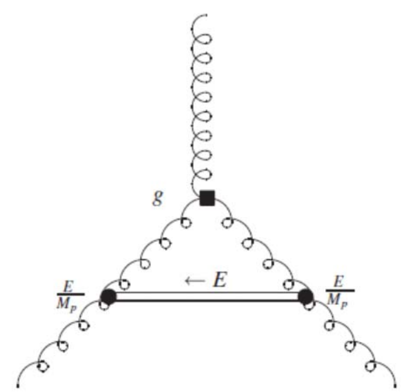


FIG. 1. A typical Feynman diagram for a gravitational process contributing to the renormalization of a gauge coupling at one loop. Curly lines represent gluons. Double lines represent gravitons. The three-gluon vertex ■ is proportional to g, while the gluon-graviton vertex ● is proportional to $E/M_{\rm P}$.

A hint of asymptotic freedom for all couplings

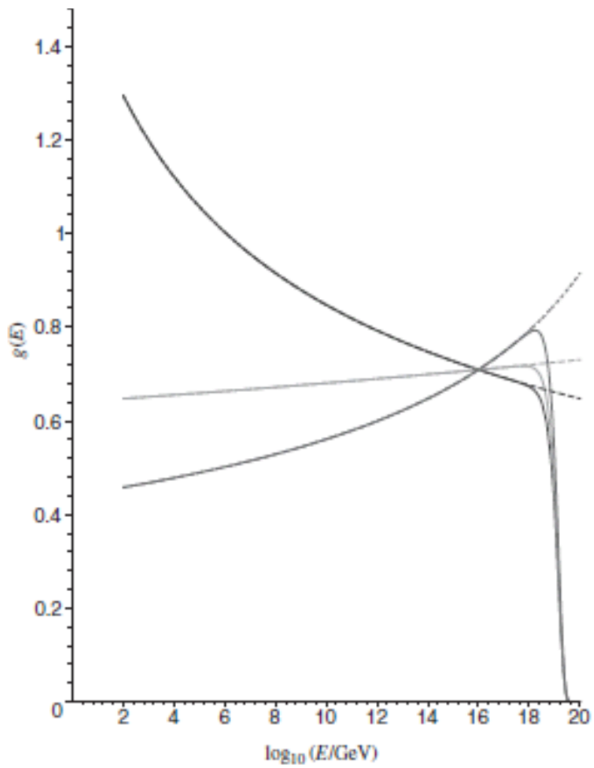


FIG. 2. When gravity is ignored, the three gauge couplings of a model theory evolve as the inverse logarithm of E at one-loop order (dashed curves). Initial values at 100 GeV were set so that the curves exactly intersect at approximately 10^{16} GeV. When gravity is included at one loop (solid curves), the couplings remain unified near 10^{16} GeV, but evolve rapidly towards weaker coupling at high E.

when the energy approaches the Planck scale, and soon after that one loses the right to neglect higher-order graviton exchanges. Though neglect of additional corrections is not justified beyond $E \ll M_P$, it is entertaining to consider some consequences of extrapolating Eq. (2) as it stands to these energies, taking into account $a_0 < 0$. The integral on the right-hand side converges as $E \rightarrow \infty$, and so Eq. (20) arises as an asymptotic relation. Thus, the effective coupling vanishes rapidly beyond the Planck scale, rendering the gauge sector approximately free at these energies. In **

A Rough History:

Prehistory: Fradkin, Vilkovisky, Tseytlin, Diennes, Kiritsis, Kounnas…

Start of "modern era":

S. P. Robinson and F. Wilczek, "Gravitational correction to running of gauge couplings," Phys. Rev. Lett. **96**, 231601 (2006) [arXiv:hep-th/0509050].

Claims that couplings do not run

-analysis in dimensional regularization

A. R. Pietrykowski, "Gauge dependence of gravitational correction to running of gauge couplings," Phys. Rev. Lett. **98**, 061801 (2007) [arXiv:hep-th/0606208].

D. J. Toms, "Quantum gravity and charge renormalization," Phys. Rev. D **76**, 045015 (2007) [arXiv:0708.2990 [hep-th]].

D. Ebert, J. Plefka and A. Rodigast, "Absence of gravitational contributions to the running Yang-Mills coupling," Phys. Lett. B **660**, 579 (2008) [arXiv:0710.1002 [hep-th]].

Y. Tang and Y. L. Wu, "Gravitational Contributions to the Running of Gauge Couplings," arXiv:0807.0331 [hep-ph].

Claims that couplings do run:

- analysis using cutoff regularization

D. J. Toms, "Quantum gravitational contributions to quantum electrodynamics," Nature **468**, 56-59 (2010). [arXiv:1010.0793 [hep-th]].

H. -J. He, X. -F. Wang, Z. -Z. Xianyu, "Gauge-Invariant Quantum Gravity Corrections to Gauge Couplings via Vilkovisky-DeWitt Method and Gravity Assisted Gauge Unification," [arXiv:1008.1839 [hep-th]].

Y. Tang, Y. -L. Wu, "Quantum Gravitational Contributions to Gauge Field Theories,"

[arXiv:1012.0626 [hep-ph]].

S. Folkerts, D. F. Litim, J. M. Pawlowski, "Asymptotic freedom of Yang-Mills theory with gravity," [arXiv:1101.5552 [hep-th]].

Claims that running couplings do not work

M. M. Anber, J. F. Donoghue, M. El-Houssieny, "Running couplings and operator mixing in the gravitational corrections to coupling constants," Phys. Rev. D83, 124003 (2011) [arXiv:1011.3229 [hep-th]].

J. Ellis, N. E. Mavromatos, "On the Interpretation of Gravitational Corrections to Gauge Couplings," [arXiv:1012.4353 [hep-th]].

Agreement

D. J. Toms, "Quadratic divergences and quantum gravitational contributions to gauge coupling constants," Phys. Rev. D **84**, 084016 (2011).

What is going on?

1) Dim-reg vs cutoff regularization – why the difference?

2) Running with $(\text{Energy})^2$
   - dimensional coupling constant

3) Why don't other similar effective field theories use running couplings?

4) **Application in physical processes**
   - does the running coupling work?

Asymptotic Safety motivation #1:

If Λ =0 at low energy, is it non-zero and running at higher energy?

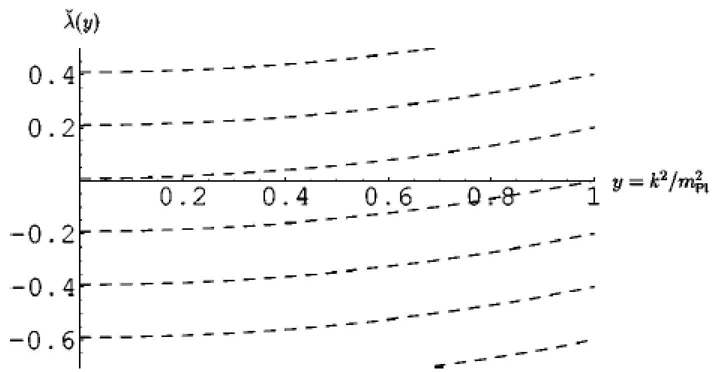


FIG. 3. Solution (4.5) to the naive flow equation for different initial values $\check{\lambda}(\hat{y})$ and $\check{G}(0)=1$.

Applications – cosmology
i.e. Entropy generation
from variable CC. (Reuter)

Does G run in the perturbative regime?

Can we see indications of AS in Lorentzian perturbation theory?

Asymptotic safety motivation #2:

Can we see signs of AS in real cross-sections?

$$\sigma(\Lambda, G, c_i, p_a) = \frac{1}{\mu^2}\sigma(\frac{\Lambda}{\mu^4}, G\mu^2, \tilde{c}_i, \frac{p_a}{\mu})$$

Dimensionless running couplings – running to UV fixed point

Weinberg's original phrasing

$$\tilde{\lambda} \to \tilde{\lambda}^*, \qquad \tilde{G} = G\mu^2 \to \tilde{G}_*, \qquad \tilde{c}_i \to \tilde{c}_i^*$$

Apparent conflict between EFT and gravity contribution to running couplings:

Gravitational corrections modify **different** operator

- at higher order in energy expansion
- R^2 rather than R

But certainly **logically possible**

– renormalize at higher energy scale E

$$\begin{aligned}
\mathrm{Amp}_i &= a_i g^2 + b_i g^2 \kappa^2 q^2 \\
&= a_i g^2\left(1 + \frac{b_i}{a_i}\kappa^2 E^2\right) + b_i g^2 \kappa^2 (q^2 - E^2) \\
&= a_i g^2(E) + b_i g^2(E) \kappa^2 (q^2 - E^2),
\end{aligned}$$

R+W

Result.—At this point the Gaussian integrals over the quantum fields in Eq. (10) are formally defined, but the resulting functional determinants contain ultraviolet divergences. We subtract them at a reference energy E_0. We find the one-loop effective action at energy scale E is

$$S_{\text{eff}}[g, a] \approx -\frac{1}{4}\int d^4x\left[\frac{1}{g^2} + \frac{\kappa^2}{g^2}\frac{3}{(4\pi)^2}(E^2 - E_0^2) + \frac{b_0}{(4\pi)^2}\ln\frac{E^2}{E_0^2}\right]F^{\mathbf{a}}_{ab}F^{\mathbf{a}ab}, \tag{18}$$

where b_0 depends on the gauge and matter content independently of whether gravitation is included in the calculation. Taking E differentially close to E_0, we read off the one-loop β function

$$\beta(g, E) = -\frac{b_0}{(4\pi)^2}g^3 - 3\frac{\kappa^2}{(4\pi)^2}gE^2. \tag{19}$$

Conditions for success:

1) Definition is **useful**
   - like $\alpha_s\,(q^2)$ sums up a set of radiative corrections
2) Definition is **universal**
   - like $\alpha_s\,(q^2)$ comes from the universal renormalization of α_s

We will find two big obstructions

1) The crossing problem – kinematic

q^2 does not have a definite sign - renormalize at $q^2 = +E^2$ or $q^2 = -E^2$.

- occurs differently in different processes – with different signs
- since really higher order operator

$$\begin{aligned} \text{Amp}_i &= a_i g^2 + b_i g^2 \kappa^2 q^2 \\ &= a_i g^2\left(1 + \frac{b_i}{a_i}\kappa^2 E^2\right) + b_i g^2 \kappa^2 (q^2 - E^2) \\ &= a_i g^2(E) + b_i g^2(E)\kappa^2(q^2 - E^2), \end{aligned}$$

2) **The universality problem**

- b_i/a_i highly process dependent
- no definition works for other process
- since really not renormalization of original operator

My claim: No definition is useful and universal (in perturbative regime)

Consider gravity corrections to gauge interactions:

(Anber, El Houssienny, JFD)

-we have done this in great detail for Yukawa
- I will be schematic for gauge interactions in order to highlight key points

Lowest order operator:

$$\mathcal{L}_{l.o} = g\bar{\psi}\gamma_\mu\psi A^\mu \qquad\qquad \mathcal{L}_{l.o.} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu}$$

Higher order operator

$$\mathcal{L}_{h.o} = c_2\bar{\psi}\gamma_\mu\psi\partial^2 A^\mu \qquad\qquad \mathcal{L}_{h.o.} = -k\partial_\mu F^{\mu\nu}\partial^\lambda F_{\lambda\nu}$$

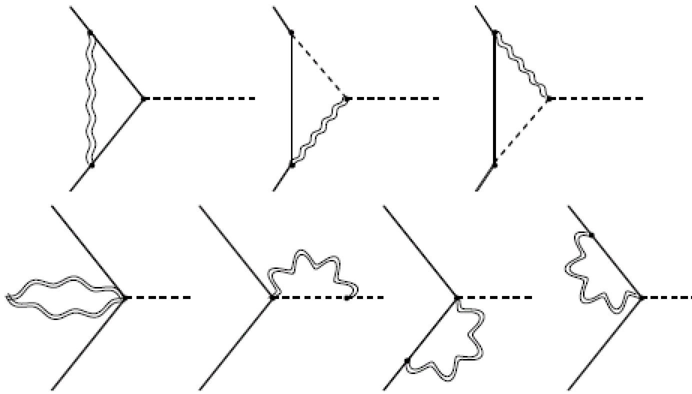
Equations of motion

$$\partial^2 A^\mu = J^\mu$$

Equivalent contact operator:

$$\mathcal{L}_{h.o} = c_2\bar{\psi}\gamma_\mu\psi\bar{\psi}\gamma^\mu\psi = c_2 J_\mu J^\mu$$

Direct calculation



Vertex (fermions on shell) found to be:

$$\mathcal{V} = \bar{u}\left[e\gamma^\mu + a(q^2)e\kappa^2 q^2\gamma^\mu\right]u$$

with

$$a(q^2) = a_0\left[\frac{1}{\epsilon} + \frac{1}{2}\ln 4\pi - \frac{\gamma}{2} - \frac{1}{2}\ln(-q^2/\mu^2)\right]$$

Physical process:

FIG. 4: Tree diagram for the on-shell scattering processes involving fermion.The filled circle denotes the set of vertex renormalization diagrams.

Overall matrix element:

$$\begin{aligned}\mathcal{M} &= \bar{u}\left[e^2\gamma^\mu + e^2 a(q^2)\kappa^2 q^2\gamma^\mu\right] u \, \frac{1}{q^2} \, \bar{u}\gamma_\mu u + h.c. + c_2\bar{u}\gamma^\mu u\bar{u}\gamma_\mu u \\ &= \bar{u}\gamma^\mu u \; \bar{u}\gamma_\mu u \left[\frac{e^2}{q^2} + \left(c_2^r(\mu) - e^2 a_0\kappa^2 \ln(-q^2/\mu^2)\right)\right]\end{aligned}$$

Describes the two reactions:

$$\begin{aligned} q^2 &> 0 \;\; \text{for}\, f + \bar{f} \to f + \bar{f} \\ &< 0 \;\; \text{for}\, f + f \to f + f \end{aligned}$$

Renormalization of higher order operator:

$$c_2^r = c_2 - a_0\left[\frac{1}{\epsilon} + \frac{1}{2}\ln 4\pi - \frac{\gamma}{2}\right]$$

Similar in the modification of photon properties

Photon propagator correction:

$$\Pi = c\kappa^2 q^4$$

Like

$$\mathcal{L}_{h.o.} = -k\partial_\mu F^{\mu\nu}\partial^\lambda F_{\lambda\nu}$$

Again looks like contact interaction:

$$\mathcal{M} = \bar{u}\gamma^\mu u \, \frac{1}{q^2} \, \left[e^2 + e^2 c\kappa^2 q^2\right] \, \frac{1}{q^2} \, \bar{u}\gamma_\mu u + c_2 \bar{u}\gamma^\mu u \bar{u}\gamma_\mu u$$

Can this be packaged as a running coupling?

Propose:

$$e^2(M^2) = e^2\left[1 + b_0\kappa^2 M^2\right]$$

Is the amplitude equal to?

$$\begin{aligned}\mathcal{M} \quad ?=? \quad & \bar{u}\gamma^\mu u\ \bar{u}\gamma_\mu u\left[\frac{e^2(M^2)}{q^2} + (c_2'))\right] \\ ?=? \quad & \bar{u}\gamma^\mu u\ \bar{u}\gamma_\mu u\left[\frac{e^2}{q^2} + \frac{e^2 2b_0\kappa^2 M^2}{q^2} + (c_2'))\right]\end{aligned}$$

$$\begin{aligned} q^2 &> 0 \ \text{for} f + \bar{f} \to f + \bar{f} \\ &< 0 \ \text{for} f + f \to f + f\end{aligned}$$

Recall

$$\begin{aligned}\mathcal{M} &= \bar{u}\left[e^2\gamma^\mu + e^2 a(q^2)\kappa^2 q^2\gamma^\mu\right]u\ \frac{1}{q^2}\ \bar{u}\gamma_\mu u + h.c. + c_2\bar{u}\gamma^\mu u\bar{u}\gamma_\mu u \\ &= \bar{u}\gamma^\mu u\ \bar{u}\gamma_\mu u\left[\frac{e^2}{q^2} + \left(c_2^r(\mu) - e^2 a_0\kappa^2\ln(-q^2/\mu^2)\right)\right]\end{aligned}$$

You can make the definition work for either process but not for both

- No universal definition

Other forms of non-universality:

Other processes have other divergences and other operators:

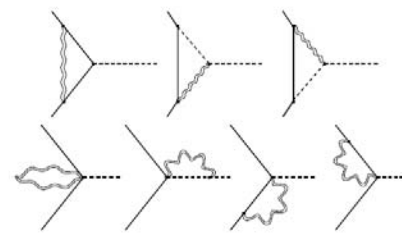
Lowest order:

$$\mathcal{L}_{l.o} = g\bar{\psi}\gamma_\mu\psi A^\mu$$

Different higher order operator is relevant

$$\mathcal{L}_{h.o.} = c_3 A^\mu \bar{\psi}\gamma_\mu \partial^2 \psi$$

Calculation of the vertex corrections:



$$\mathcal{V} = \bar{u}\left[e\gamma^\mu + b(p^2)e\kappa^2 p^2 \gamma^\mu\right] u$$

Different value for the correction (verified in Yukawa case)

$$b(q^2) \neq a(q^2)$$

Different correction to matrix element

$$\mathcal{M} = e^2\epsilon_\mu\epsilon_\nu \left(\bar{u}\gamma^\mu \left[1 + b((q+p_1)^2)\kappa^2(q+p_1)^2\right] \frac{1}{\not{q} + \not{p}_1} \gamma^\nu u + h.c. + c_3\bar{u}\gamma^\mu(\not{q} + \not{p}_1)\gamma^\nu\gamma_\mu u\right)$$

What about calculations with dimensionful cutoff?

- above agrees with EFT logic and dim-reg conclusions
- new papers with cutoff make very different claim

Work with:

$$A_\mu \to \frac{1}{e_0} A_\mu \quad , \quad D_\mu \to \partial_\mu + i A_\mu \quad , \quad \mathcal{L} \to \frac{1}{4e_0^2} F_{\mu\nu} F^{\mu\nu} + \bar{\psi} i \not{D} \psi$$

Quadradic dependence on the cutoff:

- different methods but find effective action

$$\mathcal{L} = \frac{1 + c\kappa^2 \Lambda^2}{4e_0^2} F_{\mu\nu} F^{\mu\nu} + b \ln(\Lambda^2) F_{\mu\nu} \partial^2 F^{\mu\nu}$$

Toms and others interpret this as a running coupling constant

$$e^2(\Lambda) = e^2(1 - c\kappa^2 \Lambda^2)$$

$$\beta(e^2) = \Lambda \frac{\partial e^2}{\partial \Lambda} = -c\kappa^2 e^2 \Lambda^2$$

But this cutoff dependence is unphysical artifact

- wavefuntion/charge renormalization
- disappears from physical processes

$$\frac{e_0}{4\pi(1+c\kappa^2\Lambda^2)} = \frac{e^2}{4\pi} = \frac{1}{137}$$

The quadratic cutoff dependence disappears in physical processes

$$\mathcal{M} = \frac{e_0^2(1-c\kappa^2\Lambda^2)}{q^2} + a\kappa^2 e^2 \frac{1}{q^2} q^4 \frac{1}{q^2}\left[\ln\frac{-q^2}{\Lambda^2} + ...\right] + c_2$$

After renormalization, obtain exactly the dim-reg result:

$$\mathcal{M} = \frac{e^2}{q^2} + \left(c_2^r(\mu) + a\kappa^2 e^2 \ln\frac{-q^2}{\mu}\right)$$

1) **Quadratic cutoff dependence is NOT running of charge**
2) **Agreement of different schemes**

Summary of gauge coupling section:

-We have addressed renormalization of effective field theories

-Organized as a series of operators

- **Running coupling is NOT an accurate description of quantum loops in the EFT regime**

- Confusion in the literature is understood as misunderstanding of results calculated with a dimensionful cutoff

-There is no scheme dependence to physical processes

Could gravity influenced running couplings **eventually** play a role?
- **Maybe after EFT regime**

Test cases for running of G:

Anber, JFD

Phys. Rev. D 85, 104016 (2012)

1) Graviton propagator – vacuum polarization

2) Graviton-graviton scattering

3) Massless scalars scattering gravitationally
   a) identical scalars – permutation symmetry
   b) non-identical scalars – channel dependence

4) Massive gravitational potential

1) Vacuum polarization in the graviton propagator:

Including vacuum polarization and renormalizing

$$\Delta\mathcal{L} = \frac{\sqrt{g}}{16\pi^2\epsilon}\left[\frac{1}{120}R^2 + \frac{7}{20}R_{\alpha\beta}R^{\alpha\beta}\right]$$

we find the one-loop corrected propagator to be

$$\begin{aligned} i\mathcal{D}'^{\alpha\beta,\mu\nu} &= i\mathcal{D}^{\alpha\beta,\mu\nu} + i\mathcal{D}^{\alpha\beta,\gamma\delta} i\Pi_{\gamma\delta,\rho\tau} i\mathcal{D}^{\rho\tau,\mu\nu} \\ &= \frac{i}{2q^2}(1 + 2B(q^2))[L^{\alpha\mu}L^{\beta\nu} + L^{\alpha\nu}L^{\beta\mu} \\ &\quad - L^{\alpha\beta}L^{\mu\nu}] - i\frac{A(q^2)}{4}L^{\alpha\beta}L^{\mu\nu}. \end{aligned}$$

$$L^{\mu\nu}(q) = \eta^{\mu\nu} - q^\mu q^\nu / q^2.$$

$$A(q^2) = -\frac{1}{30\pi}G\ln\left(\frac{-q^2}{\mu_1^2}\right) - \frac{7}{10\pi}G\ln\left(\frac{-q^2}{\mu_2^2}\right),$$

$$B(q^2) = \frac{7}{40\pi}Gq^2\ln\left(\frac{-q^2}{\mu_2^2}\right).$$

Not a unique definition:

a) 00,00 component (non-rel. masses)

$$G(q^2) = G\left[1 + \frac{1}{60\pi}Gq^2\ln\left(\frac{-q^2}{\mu_2^2}\right) + \frac{7}{10\pi}Gq^2\ln\left(\frac{-q^2}{\mu_1^2}\right)\right].$$

b) Proportional to original propagator

$$G(q^2) = G(1 + 2B(q^2)) = G\left(1 + \frac{7}{20\pi}Gq^2\ln\left(\frac{-q^2}{\mu_2^2}\right)\right).$$

Has **crossing problem**: which sign of q^2 should be used?

If Euclidean-ized, $q^2 \rightarrow -q_E^2$ the effective strength **increases**

Also know that μ_i drop out in pure gravity

2) Lets look at graviton –graviton scattering

Lowest order amplitude:

$$\mathcal{A}^{tree}(++;++) = \frac{i}{4}\frac{\kappa^2 s^3}{tu}$$

One loop: Dunbar and Norridge

$$\begin{aligned}
\mathcal{A}^{1-loop}(++;--) &= -i\,\frac{\kappa^4}{30720\pi^2}\left(s^2+t^2+u^2\right) \\
\mathcal{A}^{1-loop}(++;+-) &= -\frac{1}{3}\mathcal{A}^{1-loop}(++;--) \\
\mathcal{A}^{1-loop}(++;++) &= \frac{\kappa^2}{4(4\pi)^{2-\epsilon}}\,\frac{\Gamma^2(1-\epsilon)\Gamma(1+\epsilon)}{\Gamma(1-2\epsilon)}\,\mathcal{A}^{tree}(++;++)\times(s\,t\,u) \qquad (3) \\
&\times\left[\frac{2}{\epsilon}\left(\frac{\ln(-u)}{st}+\frac{\ln(-t)}{su}+\frac{\ln(-s)}{tu}\right)+\frac{1}{s^2}f\left(\frac{-t}{s},\frac{-u}{s}\right)\right. \\
&\left.+2\left(\frac{\ln(-u)\ln(-s)}{su}+\frac{\ln(-t)\ln(-s)}{tu}+\frac{\ln(-t)\ln(-s)}{ts}\right)\right]
\end{aligned}$$

where

$$\begin{aligned}
f\left(\frac{-t}{s},\frac{-u}{s}\right) &= \frac{(t+2u)(2t+u)\left(2t^4+2t^3u-t^2u^2+2tu^3+2u^4\right)}{s^6}\left(\ln^2\frac{t}{u}+\pi^2\right) \\
&+\frac{(t-u)\left(341t^4+1609t^3u+2566t^2u^2+1609tu^3+341u^4\right)}{30s^5}\ln\frac{t}{u} \\
&+\frac{1922t^4+9143t^3u+14622t^2u^2+9143tu^3+1922u^4}{180s^4}, \qquad (4)
\end{aligned}$$

Infrared divergences are not issue:

JFD +
Torma

-soft graviton radiation
-made finite in usual way
1/ϵ -> ln(1/resolution) (gives scale to loops)
-cross section finite

$$\left(\frac{d\sigma}{d\Omega}\right)_{tree} + \left(\frac{d\sigma}{d\Omega}\right)_{rad.} + \left(\frac{d\sigma}{d\Omega}\right)_{nonrad.} = \tag{29}$$

$$= \frac{\kappa^4 s^5}{2048\pi^2 t^2 u^2}\left\{1 + \frac{\kappa^2 s}{16\pi^2}\left[\ln\frac{-t}{s}\ln\frac{-u}{s} + \frac{tu}{2s^2} f\left(\frac{-t}{s},\frac{-u}{s}\right)\right.\right.$$

$$\left.\left. - \left(\frac{t}{s}\ln\frac{-t}{s} + \frac{u}{s}\ln\frac{-u}{s}\right)\left(3\ln(2\pi^2) + \gamma + \ln\frac{s}{\Lambda^2} + \frac{\sum_{ij}\eta_i\eta_j\mathcal{F}^{(1)}(\gamma_{ij})}{\sum_{ij}\eta_i\eta_j\mathcal{F}^{(0)}(\gamma_{ij})}\right)\right]\right\}.$$

<u>Correction is positive in physical region:</u>

- **increases** strength of interaction

$$s = 2E^2 \quad t = u = -E^2$$

$$\text{“}G(E)\text{”} = G\left[1 + \frac{4GE^2}{\pi}\left(\ln^2 2 + \frac{2297}{1440}\right)\right]$$

3) Gravitational scattering of a massless scalar

- permutation symmetry
- here there is a non-vanishing higher order operator which gets renormalized

$$\mathcal{L} = c \ (D_\mu \phi D^\mu \phi)^2$$

Lowest order:

$$\mathcal{M}^{tree} = \frac{\kappa^2}{2} \left[\frac{1}{s}(t^2 + u^2) + \frac{1}{t}(s^2 + u^2) + \frac{1}{u}(s^2 + t^2) \right]$$

One loop order:

Dunbar and Norridge

- done by unitarity method
- IR portion identified (and here subtracted off)

$$\mathcal{M}^{1-loop} = -\frac{\kappa^4}{(4\pi)^2} M_{tot}$$

$$M_h = M_{tot} - M_{IR}$$

$$\begin{aligned}
M_h \;=\; & \frac{1}{2}\left(\frac{s^2+t^2}{2u} + \frac{s^2+u^2}{2t} + \frac{u^2+t^2}{2s}\right) \\
& \quad \times \left[-\frac{1}{2}u\log^2(-u) - \frac{1}{2}s\log^2(-s) - \frac{1}{2}t\log^2(-t)\right] \\
+ \; & \frac{1}{2}\left[\frac{s^4+t^4}{st}\log(-s)\log(-t) + \frac{s^4+u^4}{su}\log(-s)\log(-u)\right. \\
& \quad \left. + \frac{u^4+t^4}{tu}\log(-t)\log(-u)\right] \\
+ \; & \frac{1}{4}\left[\left(s^2+2t^2+2u^2\right)\log^2(-s) + \left(2s^2+t^2+2u^2\right)\log^2(-t\right. \\
+ \; & \left(c^{ren}(\mu_r)\left(s^2+t^2+u^2\right)\right. \\
& -\frac{1}{240}\left[\left(43s^2+283t^2+283u^2\right)\log(-s/\mu_r^2)\right. \\
& \quad + \left(283s^2+43t^2+283u^2\right)\log(-t/\mu_r^2) \\
& \quad \left.\left. + \left(283s^2+283t^2+43u^2\right)\log(-u/\mu_r^2)\right]\right)
\end{aligned}$$

Evaluate this at the central physical point:

$$\mathcal{M} = \frac{-9}{4}\kappa^2 E^2 \left[1 - \frac{1}{180}\frac{\kappa^2 E^2}{16\pi^2}\Big(170(-\log^2(2) + \pi^2) + 609\log(E^2/\mu_r^2) + 123\log(2)\Big)\right]$$

Really associated
with higher order operator

Negative effect

4) Two different types of massless particles

M. Anber

$\underline{A + B \ \rightarrow A + B}$

$$\mathcal{M}_{\text{tree}} = \frac{i\kappa^2 su}{4t},$$

$$\mathcal{M}_h = i\frac{\kappa^4}{(4\pi)^2}\Big[\frac{1}{8}\Big(\frac{s^3}{t}\ln(-s)\ln(-t) + \frac{u^3}{t}\ln(-u)\ln(-t)\Big) - \frac{1}{16t}(s^3 + u^3 + tsu)\ln(-t) + \frac{1}{16}(s^2\ln^2(-s) + u^2\ln^2(-u))$$
$$+ \frac{us}{16t}(s\ln^2(-s) + t\ln^2(-t) + u\ln^2(-u)) + \frac{1}{240}(71us - 11t^2)\ln(-t) - \frac{1}{16}(s^2\ln(-s) + u^2\ln(-u))\Big], \quad (4$$

This reaction is not crossing symmetric

Again try to define effective gravitational strength for these reactions

A + B → A + B

$$\mathcal{M}_{\text{total}} = \frac{i\kappa^2 E^2}{2}\left[1 - \frac{\kappa^2 E^2}{10(4\pi)^2}\left((19 + 10\ln 2)\ln\left(\frac{E^2}{\mu^2}\right) + 5(\pi^2 - (\ln 2 - 1)\ln 2)\right)\right].$$

A + A → B + B

$$\mathcal{M}_{\text{total}} = \frac{i\kappa^2 E^2}{8}\left[1 + \frac{\kappa^2 E^2}{10(4\pi)^2}\left(9\ln\left(\frac{E^2}{\mu^2}\right) - 5\pi^2 + (19 + 5\ln 2)\ln 2\right)\right].$$

Both crossing problem and universality problem

5) Gravity matter coupling – non-relativistic masses:

Recall: $$V(r) = -\frac{Gm_1m_2}{r}\left[1 + 3\frac{G(m_1+m_2)}{r} + \frac{41}{10\pi}\frac{G\hbar}{r^2}\right]$$

Including all diagrams:

$$\text{“}G(r)\text{”} = G\left[1 + \frac{41}{10\pi}\frac{G}{r}\right]$$

Excluding box plus crossed box:

$$\text{“}G(r)\text{”} = G\left[1 - \frac{347}{60\pi}\frac{G}{r}\right]$$

Vac. pol. + vertex corrections:

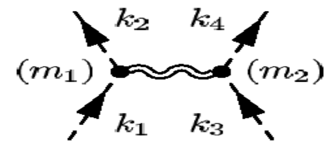
$$\text{“}G(r)\text{”} = G\left[1 - \frac{167}{15\pi}\frac{G}{r}\right]$$

- **Only vacuum polarization”**

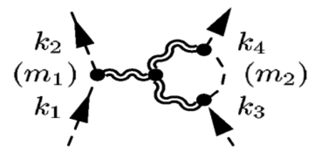
$$\text{“}G(r)\text{”} = G\left[1 + \frac{43}{15\pi}\frac{G}{r}\right]$$

Components of log in matter coupling

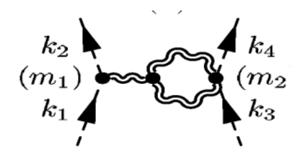
Lowest order:



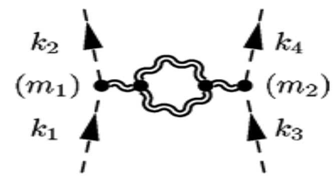
Vertex corrections:



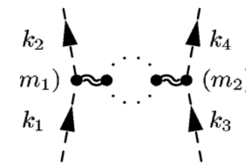
-42/3



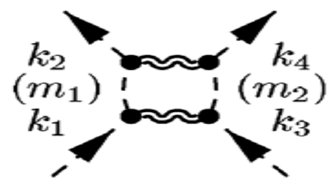
Vacuum polarization:
(Duff 1974)



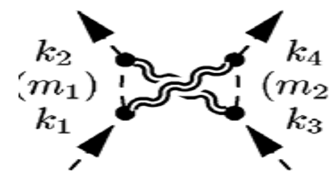
+43/15



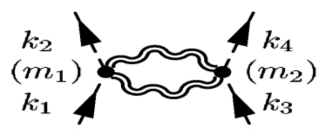
Box and crossed box



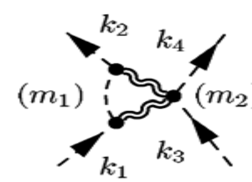
+94/3



Others:



+44-56



Summary – gravitational running in EFT

1) No useful, universal running coupling seen in perturbative calculations

2) Kinematic ambiguity resurfaces

3) Tasks for asymptotic safety program
   - continuing back to Lorentzian spacetimes
   - addition of matter couplings
   - universality of effects
   - matching to perturbative results

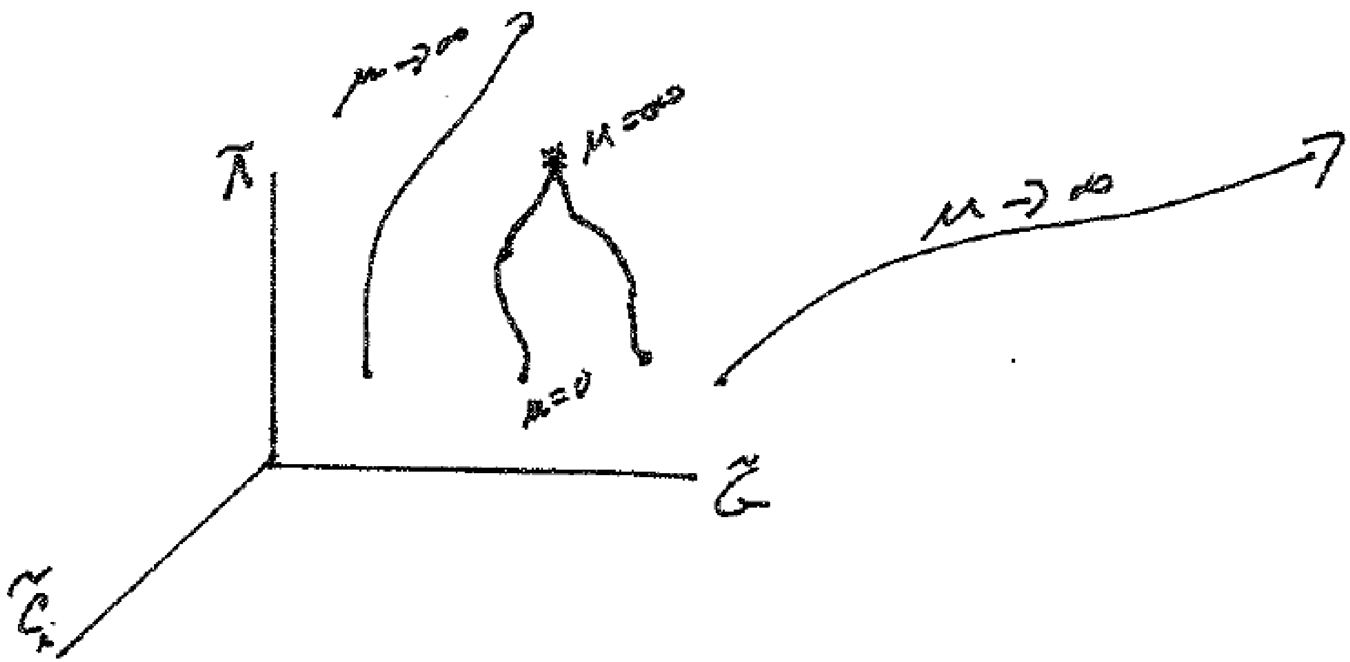
But already some conclusions:

- no evidence for generating a cosmological constant if $\Lambda = 0$ at low energy
- no evidence for scattering behavior that Weinberg was after

Asymptotic Safety to one loop

Thanks for discussions with Percacci, Cordello, Reuter but they are not to be held responsible

Defining a Euclidean theory – scaling to $\mu = 0, \mu = \infty$:
Integrate out modes above μ



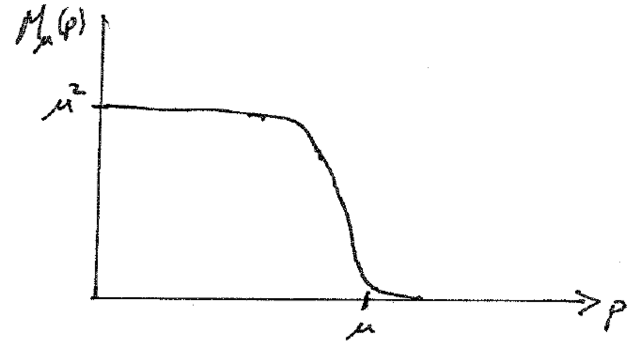
$$\tilde{G} = G\mu^2$$

UV fixed point = **finite** bare theory

In practice:

Expand around background field $g_{\alpha\beta} = \bar{g}_{\alpha\beta} + h_{\alpha\beta}$

Define function that suppresses all low momentum modes, integrate out high modes
- bi-linear – mass-like



$$\Delta\mathcal{L} = h(p) M_{\{\mu\}}(p) h(p)$$

Calculate effective action variation with scale

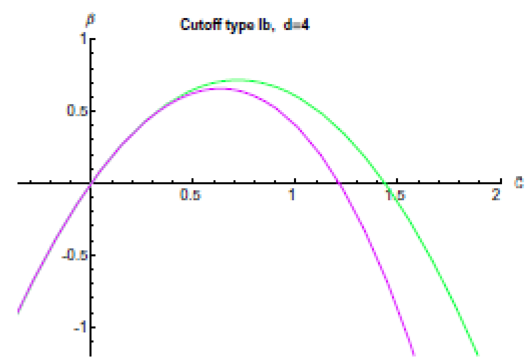
$$\mu \frac{\partial}{\partial \mu} \Gamma_{\{\mu\}} = \frac{1}{2} Tr \left[\frac{1}{D + M_{\{\mu\}}} \right] \mu \frac{\partial}{\partial \mu} M_{\{\mu\}}$$

μ is scale, not Lorentz index

Truncate action to manageable set of terms
- simplest is just the Einstein action
- locate UV fixed pt. – for example:

$$\mu \frac{\partial \tilde{G}}{\partial \mu} = 2\tilde{G} - \frac{167}{15\pi} \tilde{G}^2$$

$$\tilde{G} = G\mu^2$$



Running G $\quad G(\mu) = \dfrac{G_N}{1 + \frac{167}{30\pi} G_N \mu^2}$

Matching to Effective Field Theory

- one-loop running down to $\mu = 0$

Running of effective action:

$$\partial_t \, \Gamma_{\{\mu\}} = -\frac{1}{2} \, \partial_t \, Tr \ln \left[D(\bar{\phi}) + M_{\{\mu\}} \right]$$

Expand w.r.t background field to get individual matrix elements

- evaluated with running coupling

$$\partial_t \, \mathcal{M} = -\frac{1}{2} G^2(\mu^2) \partial_t \, Tr \left[V \frac{1}{D + M_{\{\mu\}}} V \frac{1}{D + M_{\{\mu\}}} \right]$$

Integrate from fixed point down to $\mu = 0$

$$\mathcal{M}_{\mu=0} = \mathcal{M}_\infty + \frac{1}{2} \int_0^\infty \frac{d\mu}{\mu} G^2(\mu) \; \mu \frac{\partial}{\partial \mu} \int \frac{d^4 q}{(2\pi)^4} \frac{1}{q^2 + M_{\{\mu\}}(q^2)} \frac{1}{(q+p)^2 + M_{\{\mu\}}((q+p)^2)} f(q,p)$$

Reproduces structure of effective field theory

$$\mathcal{M}_{\mu=0} = \mathcal{M}_{\infty} + \frac{1}{2}\int_0^\infty \frac{d\mu}{\mu} G^2(\mu)\ \mu\frac{\partial}{\partial\mu}\int \frac{d^4q}{(2\pi)^4}\frac{1}{q^2+M_{\{\mu\}}(q^2)}\frac{1}{(q+p)^2+M_{\{\mu\}}((q+p)^2)}f(q,p)$$

External momentum can be continued to Lorentzian space

Long distance parts of loops reproduced

-at low scales $G(\mu) \sim G_N = \mathrm{const.}$

$$\int_0 \frac{d\mu}{\mu}\ \mu\frac{\partial}{\partial\mu}[\ldots] \sim \frac{1}{2}G_N^2 \int \frac{d^4q}{(2\pi)^4}\frac{1}{q^2}\frac{1}{(q+p)^2}f(q,p)$$

High energy dependence on $G(\mu)$ appears local at low energy

- match to coefficients of EFT

More formally shown by 'non-local heat kernel expansion'

Satz, Cordello, Mazzitelli
Cordello, Zanusso

Regularized version of EFT

Weinberg’s original goal does not seem satisfied

$$\sigma(\Lambda, G, c_i, p_a) = \frac{1}{\mu^2}\sigma(\frac{\Lambda}{\mu^4}, G\mu^2, \tilde{c}_i, \frac{p_a}{\mu})$$

This is **not** the form of the Lorentzian result in the perturbative region

In principle, end up with EFT with infinite numbers of local terms in Lagrangian, but with related coefficients and with regularized loops.

Will this make infinite number of reactions well-behaved in all kinematic regions?

Summary of AS section

AS appears capable of reproducing EFT

In principle predictive (relationships among couplings). In practice, not yet.

But, Euclidean running with μ does not translate into running with energy
in the physical perturbative region

Implication - not appropriate to assume running G, Λ in FLRW applications

Still unsure if Weinberg's asymptotic hypothesis emerges from present AS practice
- well defined Lorentzian QFT at high scales?

Needed tests:

- run to k=0
- then test several Lorentzian processes with $E \rightarrow \infty$

Overall:

EFT treatment useful in perturbative region

Running couplings with gravity fail two criteria:

Not **useful** – do not encapsulate a well defined set of quantum corrections
- e.g. the crossing problem even within related reactions

Not **universal** – not a renormalization of the basic coupling
- quantum effects very different in different reactions

Many cutoff calculations mis-applied

Asymptotic Safety
- compatible with EFT treatment
- difference between Euclidean running to define theory and physical processes
- but no special definition of running couplings in Lorentzian perturbative region.