Renormalization group flow of Hořava-Lifshitz gravity at low energies

arXiv:hep-th/1309.7273

Adriano Contillo
Radboud Universiteit Nijmegen

Asymptotic Safety Seminar - 4$^{\mathrm{th}}$ November 2013

Outline

Hořava-Lifshitz gravity

Foliated FRGE

Renormalisation Group flow

- Non-gaussian fixed point
- Gaussian fixed point(s)

Trajectory realised by Nature

Conclusions

Hořava-Lifshitz gravity

General Relativity (GR) is a great theory:

- Treats spacetime itself as a dynamical object
- Predicts phenomena previously unexplicable
- Describes phisics over scales spanning 25 o.o.m.

BUT

It turns out to be perturbatively non-renormalisable!

Why is GR non-renormalisable?

$$\mathcal{S} = \frac{1}{16\pi G_N} \int d\tau\, d^d x\, \sqrt{g}\, R$$

gives propagator $\frac{1}{p^2}$ that has to be integrated in $d^{d+1}p$

superficial degree of divergence $D = d + 1 - 2$

the problem can be traced back to $[G_N] = -2$ (in 4D)

Idea of Hořava-Lifshitz (example with scalar field):

$$\mathcal{S} = \frac{1}{\kappa} \int d\tau \, d^d x \left((\partial_\tau \Phi)^2 - \sum_{i=1}^{z} (\Delta \Phi)^i \right)$$

$$[\partial_\tau] = [\nabla]^z \qquad \Rightarrow \qquad [d\tau] = [d\mathbf{x}]^z$$

UV propagator $\frac{1}{p_0^2 - (\mathbf{p}^2)^z} \propto \frac{1}{\mathbf{p}^{2z}}$

$$D = z + d - 2z = d - z$$

only positive for $z < d$

[Hořava '09]

How to extend this to gravity?

must relax the symmetry requirements of the theory, going from Diff($\mathcal{M}$) to Diff($\mathcal{M},\Sigma$)

$\Rightarrow$ use ADM decomposition

$$ds^2 = N^2 d\tau^2 + \sigma_{ij}\left(dx^i + N^i d\tau\right)\left(dx^j + N^j d\tau\right)$$

$$\mathcal{S}_{\mathrm{HL}} = \frac{1}{16\pi G_N}\int d\tau\, d^d x\, N\sqrt{\sigma}\left(K^{ij}K_{ij} - \lambda K^2 + \mathcal{V}_{HD}\left[\sigma_{ij}\right]\right)$$

We are interested in low energy behaviour of HL

$\Rightarrow$ Higher-derivative terms are suppressed "classically"

$$\Gamma_k^{\text{grav}} = \frac{1}{16\pi G_k} \int d\tau \, d^d x \, N \sqrt{\sigma} \left(K^{ij} K_{ij} - \lambda_k K^2 + 2\Lambda_k - {}^{(3)}R \right)$$

In particular, the question is whether GR is recovered

(i.e., $G_k \to G_N$, $\Lambda_k \to \Lambda_{\text{obs}}$ and $\lambda_k \to 1$)

Foliated FRGE

$$k\,\partial_k \Gamma_k = \frac{1}{2}\,\mathrm{Tr}\left[\frac{k\,\partial_k \mathcal{R}_k}{\Gamma_k^{(2)}+\mathcal{R}_k}\right]$$

- Use Landau gauge fixing to force $N = 1$ and $N^i = 0$
- Regulator acting *only* spatially $\mathcal{R}_k(\Delta \equiv \sigma^{ij}\nabla_i\nabla_j)$
- As usual, work with dimensionless couplings

$$\tilde{G} = k^2 G \qquad \tilde{\Lambda} = k^{-2}\Lambda \qquad \lambda$$

need a way to regulate fluctuations in τ-direction

$\Rightarrow$ closed time circle of lenght T

Fourier-sum over modes *inside* beta functions
that now depend parametrically on $m = \frac{2\pi}{kT}$

$$\beta_{\tilde{G}}(\tilde{G}, \tilde{\Lambda}, \lambda; m) \quad , \quad \beta_{\tilde{\Lambda}}(\tilde{G}, \tilde{\Lambda}, \lambda; m) \quad , \quad \beta_{\lambda}(\tilde{G}, \tilde{\Lambda}, \lambda; m)$$

(appearance of two orthogonal correlation lenghts)

QUESTION TIME

†

Speak Now or Forever Hold Your Peace

Renormalisation Group flow

Two correlation lenghts are one correlation length too much...

Need relationship between T and k!
(or equivalently a β_m)

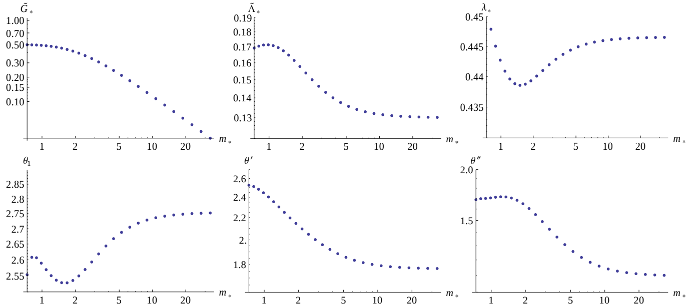
How about $T \propto k^{-1}$? "floating fixed point" scenario

[Rechenberger & Saueressig '12]

$$\Rightarrow \quad \text{fixed point condition } m_k = m_*$$

Non-gaussian fixed point

m_*-dependent



UV-attractive in all directions ⇒ UV completion!

Unfortunately, two main issues:

- Diff($\mathcal{M}$) is not recovered

  Maybe because $\Delta_k \mathcal{S}$ introduces an explicit Lorentz violation?

  We are presently investigating this possibility

- This is *not* the UV completion advocated by Hořava

  Higher-order terms are expected to lead in the UV

  The truncation used here is not reliable

As the NGFP is not "interesting", the following analysis is focused on the other one...

Gaussian fixed point(s)

Line of FPs in $\tilde{G} = 0$, $\tilde{\Lambda} = 0$

beta functions can be linearised (also around $\lambda = 1$)

$$\beta_{\tilde{G}} \simeq \tilde{G}$$

$$\beta_{\tilde{\Lambda}} \simeq \frac{2}{3\pi}\left(m_* + \frac{2}{\tanh(\pi/m_*)}\right)\tilde{G} - 2\tilde{\Lambda}$$

$$\beta_{\lambda} \simeq -\frac{1}{27}\left(\frac{154 m_*}{\pi^2} + \frac{68\pi^2}{45 m_*^3} + \frac{32\pi^4}{945 m_*^5} + \frac{11}{\pi \tanh(\pi/m_*)} - \frac{49}{m_* \sinh(\pi/m_*)^2} - \frac{50\pi}{m_*^2 \tanh(\pi/m_*) \sinh(\pi/m_*)^2}\right)\tilde{G}$$

Integrating this flow we get $\Lambda_k = \bar{\Lambda} + \mathcal{O}(k^3)$ and $\lambda_k = \bar{\lambda} + \mathcal{O}(k)$ but $G_k = \bar{G} k^{-1} + \mathcal{O}(k)$

$$\Longrightarrow \quad G_k \text{ DIVERGES IN THE IR}$$

Pity! A linearised flow with frozen *dimensionful* couplings is a natural candidate for the low energy regime of a theory

Maybe floating fixed point scenario is to blame?

$T \propto k^{-1}$ does not freeze in the IR!

Switch to "interpolating" scenario, *i.e.* link T's flow to G's

$$\partial_k \left(G/T^2 \right) \quad \Rightarrow \quad m_k = 2\pi/\alpha\sqrt{\tilde{G}}$$

being $\alpha = \bar{T}/\sqrt{\bar{G}}$

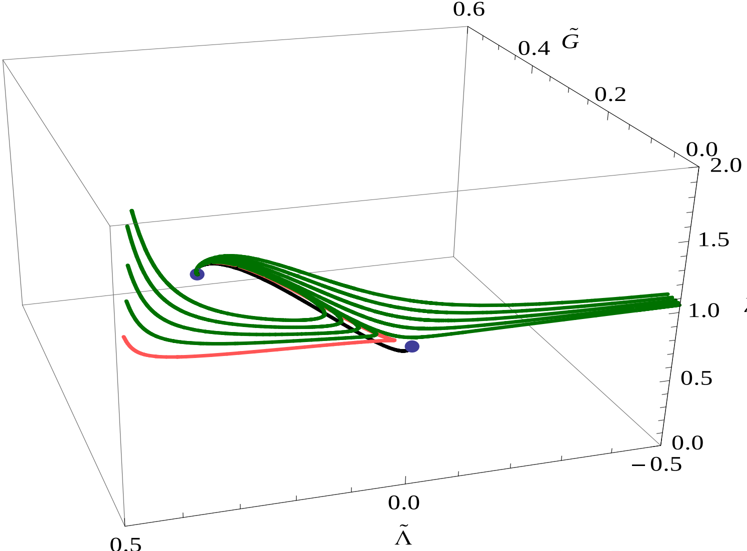
In the interpolating scenario beta functions become

$$\beta_{\tilde{G}} \simeq 2\tilde{G}$$
$$\beta_{\tilde{\Lambda}} \simeq -2\tilde{\Lambda} + \frac{4}{\pi k \bar{T}}\,\tilde{G}$$
$$\beta_{\lambda} \simeq -\frac{332}{27\pi k \bar{T}}\,\tilde{G}$$

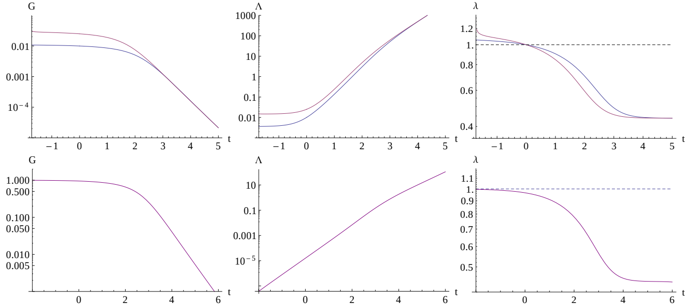
that can be integrated to give

$$G_k \simeq \bar{G}$$
$$\Lambda_k \simeq \bar{\Lambda} + \frac{4}{3\pi}\frac{k^3 \bar{G}}{\bar{T}}$$
$$\lambda_k \simeq \bar{\lambda} - \frac{332}{27\pi}\frac{k\bar{G}}{\bar{T}}$$

Three-dimensional RG flow



Flow of dimensionful couplings



Crossover between NGFP(in the UV) and GFP (in the IR)

Trajectory realised by Nature

Linearised flow:

$$G_k \simeq \bar{G}$$

$$\Lambda_k \simeq \bar{\Lambda} + \frac{4}{3\pi}\frac{k^3 \bar{G}}{\bar{T}}$$

$$\lambda_k \simeq \bar{\lambda} - \frac{332}{27\pi}\frac{k \bar{G}}{\bar{T}}$$

To select a trajectory, one must match

$$G_{\rm obs} = m_{\rm Pl}^{-2}\ , \quad \Lambda_{\rm obs} \approx 10^{-122} m_{\rm Pl}^2\ , \quad \lambda_{\rm obs} \approx 1$$

Take a scale $k_{\rm tp} = \sqrt[3]{\frac{3\pi\,\bar{T}\Lambda_{\rm obs}}{2\,G_{\rm obs}}}$ defined as $k\partial_k\tilde{\Lambda} = 0$

(for $\alpha \sim 1$, one has $k_{\rm tp} \approx 10^{-41} m_{\rm Pl}$)

$$\lambda_{k_{\rm tp}} - 1 \approx -10^{-41}$$

While observational constraints give

$$|\lambda_{\rm obs} - 1| \leq 10^{-7}$$

Conclusions

Using FRGE technique, we derived the RG flow of Hořava-Lifshitz theory (in the low energy regime)

The flow seems to be consistent with observational data, at the price of some fine-tuning

Is Hořava-Lifshitz theory viable? We still do not know, we still have to check the UV (even less trivial)

$\mathcal{THANKS}$