Asymptotically safe matrix models

International Asymptotic Safety Seminar

Tim A. Koslowski

University of New Brunswick, Fredericton, NB, Canada

work with A. Sfondrini: IJMP A 26 (2011) 4009
and with A. Eichhorn: PRD 88 (2013) 084016

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Are we there yet?

Why are we not there yet?

Possible answers

- Quantum Field Theory of General Relativity is unattainable
  $\Rightarrow$ redefine the goal
- Quantum Field Theory of GR is more subtle than we thought
  $\Rightarrow$ renormizability, unitarity, causality, classical limit, ...

Combine approaches

each approach has built-in features and hard problems
$\Rightarrow$ combine approaches that solve each others hard problems

Example: LQG and renormalization group

hard LQG problem: semiclassical continuum limit
hard FRGE problem: unitary quantum theory

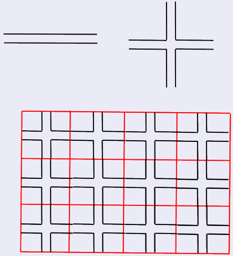
BUT: there is no simple way to combine with real LQG!

A combination that is starting to work

Topic of talk: combine FRGE with tensor-inspired models

hard problem: phase diagram in tensor models
⇒ FRGE gives access to approximate phase diagram

2D-Matrix models motivate tensor program



matrix Feynman graphs ⇒ 't Hooft expansion
⇒ large N-limit corresponds to Riemann surfaces
⇒ use matrix partition function to describe Riemann surfaces
⇒ generalization: use tensor partition function to describe Euclidean geometries

⇒ Investigate asymptotic safety in matrix models using the FRGE

Plan of the talk

Asymptotic Safety and the Grosse-Wulkenhaar model

• GW-model = noncommutative ϕ_4^{*4} model
• Constructive program: GW model is asymptotically safe (bounded RG flow near GFP, no Landau pole)
$\Rightarrow$ Confirm this using the FRGE methods of AS program
(with A. Sfondrini: IJMPA 26 (2011) 4009)

Double Scaling Limit

• matrix model for 2D Euclidean gravity
• Constructive program: There is a nontrivial (critical) large N-limit
$\Rightarrow$ there is a non-Gaussian FP (if N is cut-off) $\Rightarrow$ find with FRGE
aim: extract recipe for tensor FRGE
(with A. Eichhorn: PRD 88 (2013) 084016)

General goals

(1) test FRGE as a tool for discrete geometry models
(2) test use of FRGE in AS by testing known results

Grosse Wulkenhaar model

Grosse-Wulkenhaar model

scalar field ϕ on $\mathbb{R}^{*4}$ = "2 copies of QM phase space"

Moyal product $f * g := f \exp(\overleftarrow{\partial}_\mu \theta^{\mu\nu} \overrightarrow{\partial}_\nu) g$ with $\theta^{\mu\nu} = \theta(\epsilon^{\mu\nu 34} + \epsilon^{12\mu\nu})$

ϕ^{*4} interaction generates $(\tilde{x}_\mu \phi) * (\tilde{x}^\mu \phi)$ term (where: $\tilde{x}_\mu = 2(\theta^{-1})_{\mu\nu} x^\nu$)

$$S = \int d^4x \left(\frac{1}{2}(\partial_\mu \phi) * (\partial^\mu \phi) + \frac{\Omega^2}{2}(\tilde{x}_\mu \phi) * (\tilde{x}^\mu \phi) + \frac{m^2}{2}\phi^{*2} + \frac{\lambda}{4!}\phi^{*4} \right)$$

Langmann-Szabo trf. "$x \leftrightarrow p$": $S(\Omega, m, \lambda) \to \Omega^2 S(\frac{1}{\Omega}, \frac{m}{\Omega}, \frac{\lambda}{\Omega^2})$

bare action is self-dual for $\Omega = 1 \Rightarrow$ smaller self-dual theory space

harm. osc. base $\phi(x) = \sum \phi_{n_1 m_1, n_2 m_2} h^{n_1 m_1, n_2 m_2}(x) \Rightarrow$ tensor model

- $*$-product is mapped into matrix product
- $\int d^4x(...)$ is mapped into $\nu \mathrm{Tr}(...)$ (where: $\nu = (2\pi\theta)^2$)
- self-dual kinetic term is mapped into $\frac{1}{2}\nu \mathrm{Tr}(\phi.XX.\phi)$ (where: $XX = \tilde{x}_\mu \tilde{x}^\nu$)

$$S_{sd} = \nu \mathrm{Tr}\left(\frac{1}{2}\phi.XX.\phi + \frac{m^2}{2}\phi.\phi + \frac{\lambda}{4!}\phi.\phi.\phi.\phi \right)$$

Known Results

perturbative UV attractivity of $\Omega = 1$

i.e. the self-dual GW model is UV attractive

Constructive Asymptotic Safety

the integrated RG flow of λ is bounded in a neighborhood of the GFP (in the self-dual model)

$\Rightarrow$ derive these results with FRGE

- UV attractivity in tree-level truncation
- bound on RG flow by estimating β-functions near GFP

FRGE setup for GW model

FRGE from $e^{-W[j]} = \int [d\phi]_\Lambda e^{-S[\phi]-\Delta_k S[\chi]+\nu \mathrm{Tr}(j^T.\phi)}$ (Λ is UV cut-off in XX)

$$k\partial_k \Gamma_k[\phi] = \frac{1}{2}(\Gamma_k^{(2)}[\phi] + R_k)^{-1}_{m_1 \dots n_2, a_1 \dots b_2} (k\partial_k R_k)^{m_1 \dots n_2, a_1 \dots b_2}$$

IR suppression term

cut-off on $\Delta_{sd} = XX$ is effectively a cut-off on matrix size (finite dim. integrals!)
$\Rightarrow$ use standard cut-off $\Delta_k S[\phi] = \frac{k^2 \nu}{2} \mathrm{Tr}(\phi . r(\frac{XX}{k^2}).\phi)$

Theory space

$\Gamma_k[\phi] = \sum g^1_{a_1 b_1 \dots} \nu \mathrm{Tr}(\phi^{a_1} X X^{b_1} \dots) + \sum g^2_{a_1 b_1} \frac{\nu}{\theta^2} \mathrm{Tr}(\dots)\mathrm{Tr}(\dots) +$ "more traces"

Canonical dimensions (from position representation)

roughly: $[\phi] = 1$, $[\nu \mathrm{Tr}] = -4$, $[XX] = 2$

Duality covariant β-functions (in GW truncation)

Truncation: $S = Z\nu \mathrm{Tr}\left(\frac{(1+\omega/2)}{2}\phi.XX.\phi + \frac{\omega}{2}X.\phi.X.\phi + \frac{m^2}{2}\phi.\phi + Z\frac{\lambda}{4!}\phi.\phi.\phi.\phi\right)$

$$\begin{aligned}
\eta &= \frac{\lambda}{192\pi^2} + O(\lambda^2, \bar\theta^{-2}) \\
\beta(\omega) &= -\frac{\lambda\omega}{192\pi^2} + O(\lambda^2, \bar\theta^{-2}) \\
\beta(m^2) &= -2m^2 + \frac{\lambda m^2}{192\pi^2} + O(\lambda^2, \bar\theta^{-2}) \\
\beta(\lambda) &= O(\lambda^2, \bar\theta^{-2})
\end{aligned}$$

(using dimensionless $\bar\theta$, which vanishes in UV)

(important: $\beta(\lambda)$ vanishes to all orders in λ in self-dual GW truncation)

this matches the perturbative result:

ω is asymptotically free $\Rightarrow$ UV behavior due to self-dual theory

Remark:

FRGE calculation is significantly simpler than perturbative calculation (and easily generalizes to include nonperturbative effects)

Self-dual β-functions (near GFP)

main (unproven) assumption:

theory space can be given a Banach space topology s.t. FRGE flow is differentiable

Rough sketch of strategy:

1. split couplings into GW truncation and "rest"
2. $\beta(\lambda) = 0$ in GW truncation
3. estimate effect of GW operators on "rest" β-fkt. (assume near GFP!)
4. estimate the effect of "rest" operators on GW flow

$\Rightarrow$ put together $|\beta(\lambda)| < e^{-(2+\epsilon)\tau}(c_o + c_1\lambda + c_2\lambda^2)$

inspection of bound yields:

for λ small enough the integrated flow of λ is bounded

Summary of Grosse-Wulkenhaar FRGE

1. FRGE setup for tensor representation can be read-off from position rep.
2. UV attractivity of self-dual model can be found with significantly less work than in perturbation theory
3. λ is exactly marginal at GFP
4. flow of λ can be bounded in small region around GFP (asymptotically safe region)

Double scaling limit

Original Hermitian matrix model for Euclidean 2D quantum gravity

$\phi^2 + g\,\phi^3$ matrix model for 2D-QG

- Hermitian $N \times N$ matrices ϕ as fund. deg. of freed.
  model: $N^2 Z_{grav.} = \log\left(\int [d\phi]_N e^{-\frac{1}{2}\mathrm{Tr}(\phi^2) + N^{-1/2} g \mathrm{Tr}(\phi^3)}\right)$ take $N \to \infty$
- action generates random triangulations: $Z = \sum_h N^{2(1-h)} Z_h$
  $\Rightarrow$ planar scaling limit (i.e. only sphere, genus $h = 0$, remains)
- 2D QG = critical theory ("double scaling limit"):
  $N \to \infty$, $g \to g_c$ s.t. N^{-2h} is compensated
  $\Rightarrow$ keep $N(g - g_c)^{1-\gamma_{str.}/2} = \text{const.}$
  $\Rightarrow$ continuum limit $N \to \infty$ corresponds to FP g_* (in large N scaling)
- analytic result: $\theta = N\partial_N \beta(g)\big|_{g=g_*} = 4/5$ (to reproduce with FRGE)
- **problem ϕ^3-interaction:**
  (1) is odd (large theory space)
  (2) unbounded (only formal path integral)

Revised matrix models

Hermitian $\phi^2 + g\,\phi^4$

- corresponds to random tasselation by squares (pic. before)
- action has Z_2-symmetry $\Rightarrow$ smaller theory space
- analytic result $\theta = 4/5$ (what we actually want to reproduce)
- perturbative calculations available as benchmarks
(Brezin, Zinn-Justin: PLB 288 (1992) 54; Ayala: PLB 311 (1993) 55)

Equivalent formulation:

- $N \to \infty$ turns out equivalent to real matrix model with $\phi \to O_1^T \phi O_2$ gauge symmetry
- correspondence: Hermitian matrices $\phi^2 \leftrightarrow$ real matrices $\rho := \phi^T \phi$
- gauge symmetry: real setup very close to colored tensor models!
(cf. Gurau: various publications in 2010,20111)

$\Rightarrow$ base FRGE setup on bare action $\phi^2 + g\,\phi^4$ and cut-off in matrix size

FRGE setup for matrix model

Theory space

general invariant functional: $\Gamma_k[\phi] = F_k\left(\mathrm{Tr}(\phi^2), \mathrm{Tr}(\phi^4), ...\right)$
suitable expansion: number of traces (connected components) and powers of ϕ

Problem: no "Laplacian" in theory space

- "invent Laplacian" (s.t. cut-off on matrix size) $\Rightarrow$ larger theory space
- unfeasible Ward-id: $G_\epsilon \Gamma_k = \frac{1}{2} tr_{op}\left((\Gamma_k^{(2)} + R_k)^{-1} G_\epsilon R_k\right)$

  all regulators break $U(N)$ resp. $O(N)^2$ symmetry G_ϵ
- practically: good results on orig. theory sp. $\Rightarrow$ potential optimization

  (choice of Laplacian and regulator profile)

Canonical dimensions (from perturbative large N-limit):

- operator basis: $\Gamma_k[\phi] = \sum g^i_{n_1 \ldots n_i} \mathrm{Tr}(\phi^{n_1}) \ldots \mathrm{Tr}(\phi^{n_i})$

  $\Rightarrow$ dimensions $[g^i_{n_1 \ldots n_i}] = N^{i-2+\frac{1}{2}\sum_{j=1}^{i} n_j}$

Single trace truncation

extract β-functions using vertex expansion

- assume $[P, \phi] = 0$ and take large N-limit (reasonable propagator commutes w/h low-energy ϕ)

$\Rightarrow$ scheme dependence in $O(1)$ numbers $[\dot{R}P^n]$

Single trace truncation $\Gamma_k[\phi] = \frac{Z}{2}\mathrm{Tr}(\phi^2) + \sum_{n\geq 2} \frac{\bar{g}_{2n}}{2n}\mathrm{Tr}(\phi^{2n})$

- dimensionless couplings $\bar{g}_i = Z^{i/2} N^{i/2-1} g_i$
- β functions: $\eta = g_4[\dot{R}P^2]$

$$\beta(g_{2n}) = ((1+\eta)n - 1)g_{2n}$$
$$+2n \sum_{i;\{\vec{m}\}=\sum_i i m_i} (-1)^{\sum_i m_i} [\dot{R}P^{1+\sum_i m_i}] \begin{pmatrix} \sum m_i \\ m_1\, m_2 \ldots \end{pmatrix} \Pi_i g_{2(i+1)}^{m_i}$$

Fixed point analysis:

- fixed point with **single** relevant direction appears in all truncations
- positive $\theta_1 \to\sim 1_+$ (reasonably aligned with g_4)
- all other exponents $\theta_i \to\sim -i+1$ (reasonably aligned with g_{2i+2})

ϕ^6 double trace truncation

Operators that directly renormalize bare action $O(\phi^4)$

- only tadpole over $\mathrm{Tr}(\phi^2)\mathrm{Tr}...$ receives N^2 (necessary to overcome dimensional suppression)

$\Rightarrow$ include $\frac{1}{4}\bar{g}_{2,2}(\mathrm{Tr}(\phi^2))^2$ and $\frac{1}{2}\bar{g}_{2,4}\mathrm{Tr}(\phi^2)\mathrm{Tr}(\phi^4)$

β-functions

- η receives $+2g_{2,2}[\dot{R}P^2]$ contribution
- $\beta(g_4)$ receives $-2g_{2,4}[\dot{R}P^2]$ contribution
- double trace β-fkt. become messy

Fixed point:

- FP with single relevant direction

$$\theta_1 = 1.21, \quad \theta_2 = -0.69, \quad \theta_3 = -1.01, \quad \theta_4 = -1.88$$

- slight improvement over pert. calculation (but possibility for optimization not yet explored)

Summary of double scaling limit:

FRGE is a tool to investigate phases of matrix/tensor QG models

bottom-line: FRGE finds double scaling limit with reasonable accuracy.
(calculations appear simpler than pert. calc. !)

General recipe for matrix/tensor models

- Theory space from field content and symmetries of bare action
- Standard regulator (invent Laplacian; min. gauge sym. breaking)
- Canonical dim. partly determined by requiring $1/N$ expansion of β-fkt.
- β-fkt. found from vertex expansion, $[P, \phi] \approx 0$ for large N

Summary

- Starting point: desire to explore continuum limit in LQG related models
  $\Rightarrow$ combine FRGE with discrete geometry models
- First step: investigate matrix models
- Matrix model results:
  1. AS of GW model can be confirmed with FRGE, but method does not straightforwardly generalize to NGFP
  2. FRGE finds double scaling limit
     $\Rightarrow$ establish FRGE as tool in matrix models
     $\Rightarrow$ do optimization and precision calculations to benchmark FRGE
  3. practical experience leads to general FRGE "recipe" in tensor model (and GFT) framework
- countless applications of FRGE in discrete approaches to QG
  in particular: FRGE allows systematic investigation of theory space
  $\Rightarrow$ phase diagram

Thank you!