Scalar field theory and background fields

Implications for asymptotic safety?

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Asymptotic Safety Online Seminar – 10/02/2014

Tim Morris, I. Hamzaan Bridle, JD: arXiv:1312.2846

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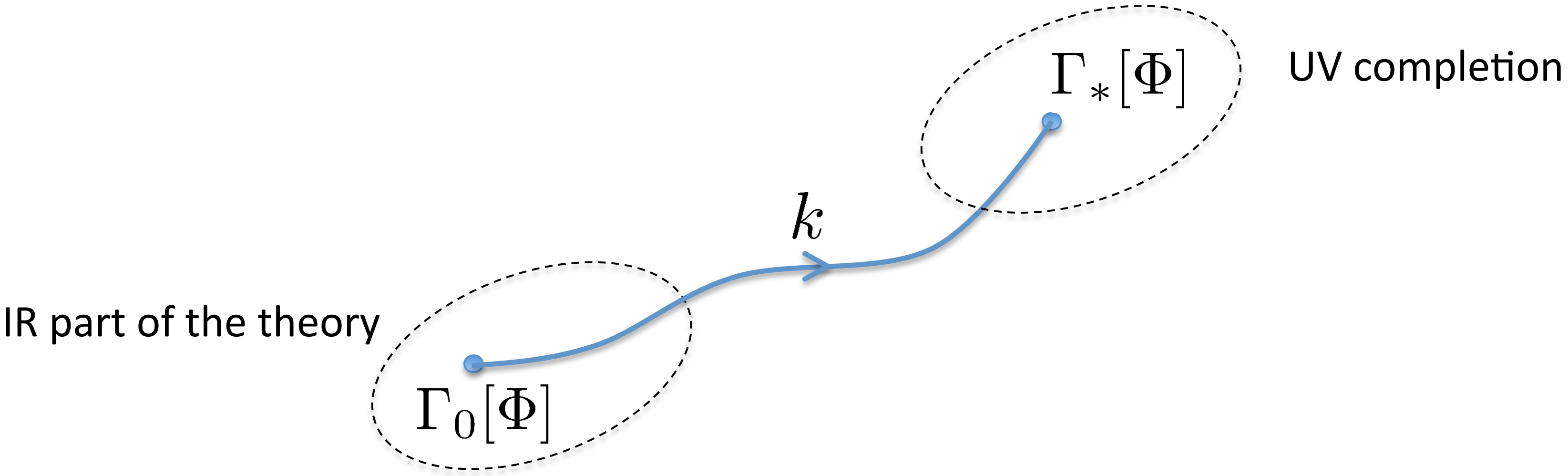
Outline

- Motivation for our work
- Problems of the single field approximation
- The shift Ward identity and its use in the LPA of scalar field theory
- Comments on the shift Ward identity in gravity

Asymptotic safety

- A non-perturbative RG trajectory defining quantum gravity:



- If this works, gravity is non-perturbatively renormalisable
- It is safe to remove the cutoff at the non-perturbative fixed point, whence asymptotic safety Weinberg, 1979

Motivation

Problem: too many fixed points

Lines of fixed points were previously found in the f(R) truncation

JD, T. Morris

Reason: redundancy in the equations

Equations of motion don’t have vacuum solutions,
leading to all eigenoperators being redundant

Fix redundancy in a scalar field theory setting

→ this talk

The problem

- Eigenoperators become redundant, if they describe an infinitesimal change of field variable for the effective action

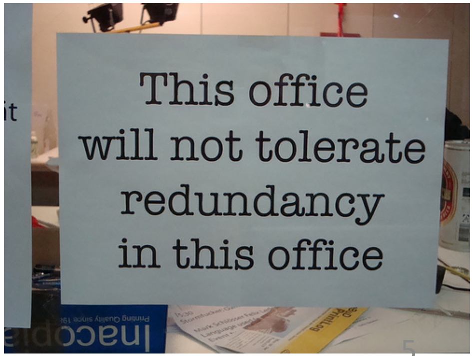
- In the f(R) truncation this happens because the equations of motion never vanish on fixed point solutions:

$$E(R) = 2f_*(R) - R\,f_*'(R) \neq 0$$

- This leads to a collapse of eigenspaces for the f(R) truncation

Where could this redundancy come from and how can it be fixed?

Possible answer: Treatment of background field!



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Background field formalism

- The effective action is a functional of two metrics:

$$\Gamma_k = \Gamma_k[g_{\mu\nu}, \bar{g}_{\rho\sigma}]$$

- Here, $g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$ is the total metric split into the background metric $\bar{g}_{\mu\nu}$ and the fluctuation field $h_{\mu\nu}$.

This is necessary for various reasons, e.g.

- the background Laplacian $-\bar{\nabla}^2$ defines the momenta which are compared to k^2,
- the background field is needed for gauge fixing

M. Reuter

single metric
single field

versus

bi-metric
bi-field

Bi-metric results in gravity

M. Reuter et al. recognised the need to keep both metrics:

- Bi-metric conformal gravity (Reuter, Manrique ’09)
- Matter induced bi-metric gravity (Reuter, Manrique, Saueressig ’10)
- Bi-metric Einstein-Hilbert truncation (Reuter, Manrique, Saueressig ’10)

- In each case the results point towards asymptotic safety
- But: calculation for an $f(R, \bar{R})$ - type truncation would be hard

→ Investigate the role of the background field in the simpler setting of scalar field theory.

Back to scalar field theory

- Scalar field theory is much simpler
- Established results are available (e.g. Wilson-Fisher fixed point)

1. Make the single field approximation and show things go wrong (additional fixed points, redundant eigenoperators)

$$\Gamma[\phi]$$

2. Perform the corresponding bi-field calculations and show that this reproduces the correct results

$$\Gamma[\varphi, \bar{\varphi}]$$

$$\phi = \bar{\varphi} + \varphi$$

In doing so, it is important to mimic the approach adopted for gravity!

Single field approximation

The effective action is decomposed as

$$\Gamma_k[\varphi,\bar\varphi] = \Gamma_k[\phi] + \hat\Gamma_k[\varphi,\bar\varphi],$$

where $\phi = \bar\varphi + \varphi$ is the total field and

$$\Gamma_k[\phi] = \Gamma_k[0,\phi].$$

The effect of this is:

$$\tfrac{1}{2}m^2(\varphi+\bar\varphi)^2 + \tfrac{1}{2}\bar m^2\bar\varphi^2 \quad \rightarrow \quad \underbrace{\tfrac{1}{2}m^2\phi^2}_{\Gamma_k[\phi]} + \underbrace{\tfrac{1}{2}\bar m^2\bar\varphi^2}_{\hat\Gamma_k[\varphi,\bar\varphi]}$$

$\hat\Gamma_k[\varphi,\bar\varphi]$ captures the deviation of $\Gamma_k[\varphi,\bar\varphi]$ from being a function of the total field $\varphi+\bar\varphi$ only.

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$\hat\Gamma_k[\varphi,\bar\varphi]$ captures the deviation of $\Gamma_k[\varphi,\bar\varphi]$ from being a function of the total field $\varphi+\bar\varphi$ only.

$\Gamma_k[\phi]$

$\hat\Gamma_k[\varphi,\bar\varphi]$

Local potential approximation (LPA)

- The LPA in scalar field theory is given by

$$\Gamma_k[\phi] = \int dx \left\{ \frac{1}{2} \left(\partial_\mu \phi\right)^2 + V(\phi) \right\}$$

- In Gravity the cutoff depends on the background metric. In particular, we use the replacement

$$-\bar{\nabla}^2 \mapsto -\bar{\nabla}^2 + c\bar{R}.$$

- Implement the same idea in scalar field theory:

$$R_k\left(-\partial^2, \bar{\varphi}\right) = \left(k^2 + \partial^2 - \alpha k^{4-d} \bar{\varphi}^2\right) \theta\left(k^2 + \partial^2 - \alpha k^{4-d} \bar{\varphi}^2\right)$$

- In the single field approximation the background field $\bar{\varphi}$ in this cutoff turns into a ϕ

Background field dependent flows

Fixed-point equations with background field

$$3V_* - \frac{1}{2}\phi V_*' = \frac{\left(1-\alpha\phi^2\right)^{3/2}\left(1-\frac{1}{2}\alpha\phi^2\right)}{1-\alpha\phi^2+V_*''}\theta\left(1-\alpha\phi^2\right)$$

and without background field:

$$3V_* - \frac{1}{2}\phi V_*' = \frac{1}{1+V_*''}$$

The Wilson-Fisher fixed point

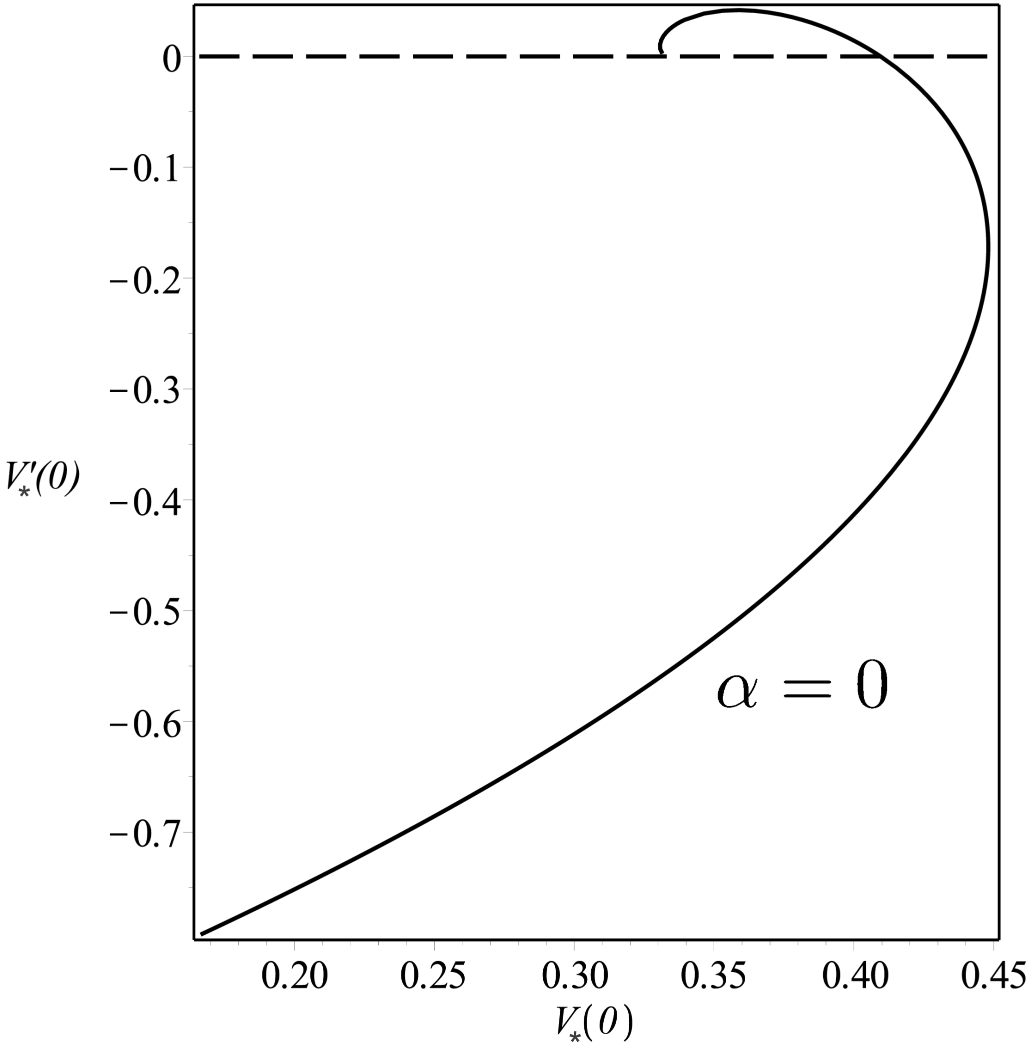
- For large field,

$$V_*(\phi) \approx A\phi^6$$

- We vary A to get this curve

- The Wilson-Fisher fixed point is described by an even potential:

$$V'_{WF}(0) = 0$$



Things do go wrong (1)

- For large field,

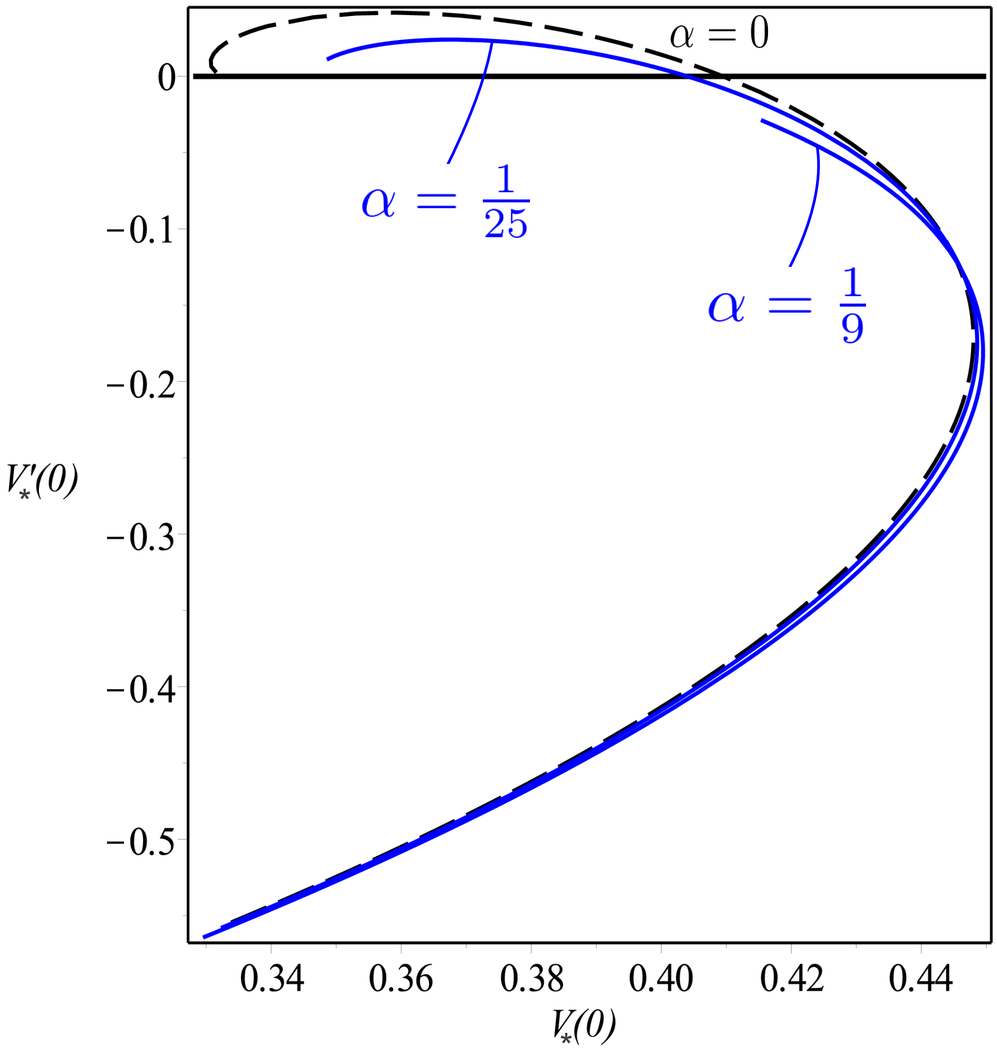
$$V_*(\phi) \approx A\phi^6$$

- We vary A to get these curves

- The Wilson-Fisher fixed point has an even potential:

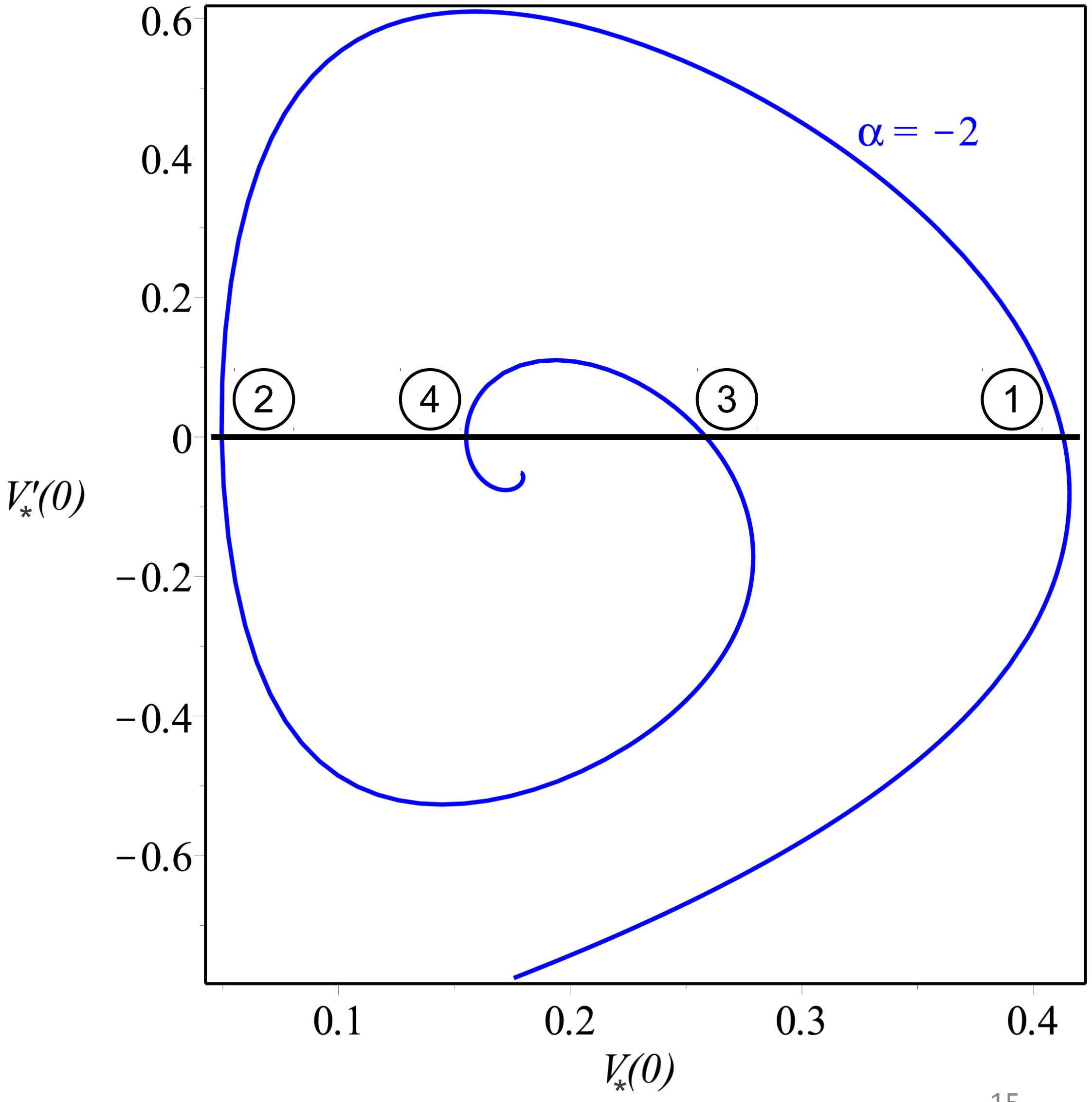
$$V'_{WF}(0) = 0$$

First the Gaussian, then the Wilson-Fisher fixed point disappears!



Things do go wrong (2)

- For negative α **additional fixed points** appear
- Decreasing α further leads to more and more fixed points



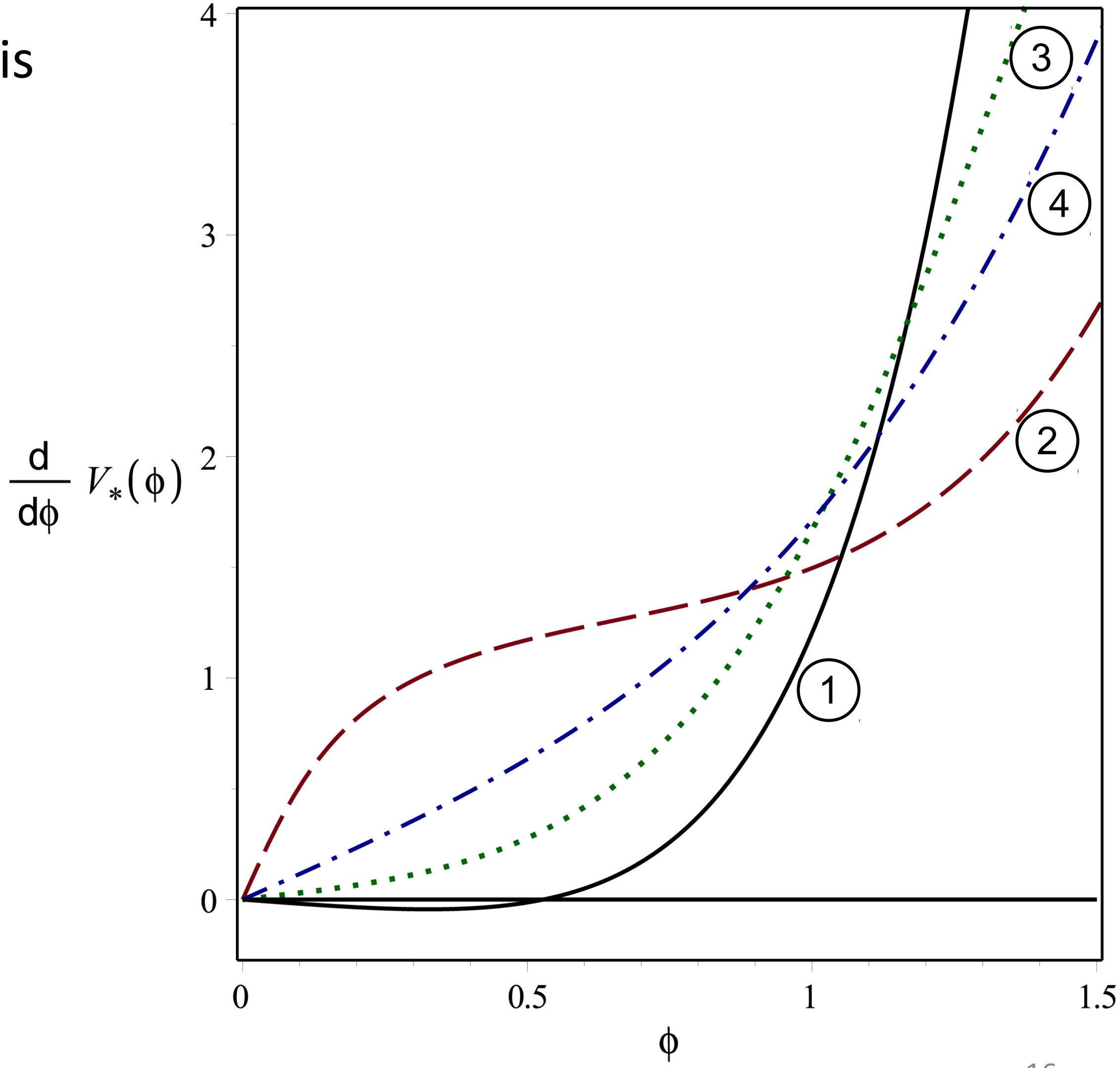
Things do go wrong (3)

- Here, an eigenoperator is redundant if

$$v(\phi) = V_*'(\phi)\zeta(\phi)$$

- For the fixed points ② - ④ all odd eigenoperators are redundant

This is similar to what happened for gravity!



A second choice of regulator

- We also considered a second choice of regulator:

$$R(-\partial^2,\bar{\varphi}) = \left(k^2+\partial^2-\alpha V''(\bar{\varphi})\right)\theta\left(k^2+\partial^2-\alpha V''(\bar{\varphi})\right)$$

Again we find:

  - Additional fixed points appear for $\alpha > 0$
  - There appear to be no non-trivial solutions to the eigenoperator equation for non-Gaussian fixed points

The need to keep $\hat{\Gamma}_k[\varphi, \bar{\varphi}]$

- Neglecting $\hat{\Gamma}_k[\varphi, \bar{\varphi}]$ and thereby adopting the single field approximation leads to inaccurate results

- If we keep $\hat{\Gamma}_k[\varphi, \bar{\varphi}]$ we have to deal with an effective action depending on two fields: $\Gamma_k[\varphi, \bar{\varphi}]$

What determines the background field dependence of the effective action?

Related work

- Scalar field theory
  - Litim, Pawlowski, 2002:
    - Polynomial potentials in the LPA
    - Qualitative agreement but large quantitative deviations for critical exponent
    - No conclusive result for the $V''(\bar{\varphi})$ case
  - Litim, 2002:
    - Introduces an additional t-dependent effective mass term in the optimised cutoff: $m_t = V''(\phi_0)$
    - This is shown not to affect the results

- Yang Mills
  - Gies 2002, Litim 2002
    - Background field affects the one-loop beta function

…

Here: Possible problems in the single field approximation can be much more severe.

Any questions?

Back to the path integral

$$Z[J,\bar{\varphi}] = \int \mathcal{D}\varphi \exp\left(-S[\varphi+\bar{\varphi}] - S_k[\varphi,\bar{\varphi}] + J\cdot\varphi\right)$$

- The bare action depends only on the total field $\phi = \bar{\varphi} + \varphi$
- The cutoff action S_k introduces a separate $\bar{\varphi}$-dependence
- The cutoff action breaks the shift symmetry

$$\varphi \mapsto \varphi + \varepsilon, \qquad \bar{\varphi} \mapsto \bar{\varphi} - \varepsilon$$

of the bare action.

The modified shift Ward identity

- This broken symmetry leads to the modified shift Ward identity (sWI)

$$\frac{\delta\Gamma_k}{\delta\bar{\varphi}(x)} - \frac{\delta\Gamma_k}{\delta\varphi(x)} = \frac{1}{2}\mathrm{Tr}\left[\left(\frac{\delta^2\Gamma_k}{\delta\varphi\delta\varphi} + R_k\right)^{-1} \frac{\delta R_k}{\delta\bar{\varphi}(x)}\right].$$

- It keeps track of the separate background field dependence introduced by the cutoff
- It "knows" about the the fact that $S[\varphi + \bar{\varphi}]$ depends only on the total field
- It is conserved along the flow

The sWI must hold in addition to the usual flow equation; it is an extra constraint on Γ_k

sWI as a constraint

- Suppose we have a solution $\Gamma_k[\varphi, \bar{\varphi}]$ of the flow equation
- Then $\tilde{\Gamma}_k = \Gamma_k[\varphi, \bar{\varphi}] + F[\bar{\varphi}]$ is another solution
- But the sWI no longer holds as $\tilde{\Gamma}_k$ corresponds to a bare action

$$S[\varphi + \bar{\varphi}] - F[\bar{\varphi}]$$

- This violates the shift symmetry

In full bi-field computations the sWI ensures uniqueness of the effective action.

Bi-field LPA

With background-field dependence the LPA becomes

$$\Gamma_k[\varphi,\bar{\varphi}] = \int dx \left\{ \frac{1}{2}\left(\partial_\mu \varphi\right)^2 \; + \frac{1}{2}\left(\partial_\mu \bar{\varphi}\right)^2 + \gamma \partial_\mu \varphi \partial^\mu \bar{\varphi} + V(\varphi,\bar{\varphi}) \right\}$$

and we choose the cutoff operator

$$R_k(-\partial^2,\bar{\varphi}) = \left(k^2 + \partial^2 - h_k(\bar{\varphi})\right)\theta\left(k^2 + \partial^2 - h_k(\bar{\varphi})\right)$$

with a general t-dependent function $h_k(\bar{\varphi})$.

The previous two choices where:

$$h_k(\bar{\varphi}) \to \alpha k^{4-d}\bar{\varphi} \qquad\qquad h_k(\bar{\varphi}) \to \alpha V''(\bar{\varphi})$$

RG flow and sWI

The flow equation becomes:

$$\partial_t V - \frac{1}{2}(d-2)\left(\varphi\partial_\varphi V + \bar{\varphi}\partial_{\bar{\varphi}} V\right) + dV$$

$$= \frac{(1-h)^{d/2}}{1-h+\partial_\varphi^2 V}\left(1 - h - \frac{1}{2}\partial_t h + \frac{1}{4}(d-2)\bar{\varphi}h'\right)\theta(1-h)$$

As opposed to just ($h = 0$):

$$\partial_t V - \tfrac{1}{2}(d-2)\phi V' + dV = \tfrac{1}{1+V''}$$

And the sWI is:

$$\partial_\varphi V - \partial_{\bar{\varphi}} V = \frac{h'}{2}\,\frac{(1-h)^{d/2}}{1-h+\partial_\varphi^2 V}\,\theta(1-h)$$

Flow equations: single versus bi-field

The single-field flow is:

$$\partial_t V + 3V - \frac{1}{2}\phi V' = \frac{\left(1-\alpha\phi^2\right)^{3/2}\left(1-\frac{1}{2}\alpha\phi^2\right)}{1-\alpha\phi^2+V''}\theta\left(1-\alpha\phi^2\right)$$

The bi-field flow is (remember $\phi = \varphi + \bar{\varphi}$):

$$\partial_t V + 3V - \frac{1}{2}\left(\varphi\partial_\varphi V + \bar{\varphi}\partial_{\bar{\varphi}} V\right) = \frac{\left(1-\alpha\bar{\varphi}^2\right)^{3/2}\left(1-\frac{1}{2}\alpha\bar{\varphi}^2\right)}{1-\alpha\bar{\varphi}^2+\partial_\varphi^2 V}\theta\left(1-\alpha\bar{\varphi}^2\right)$$

Flow equations: single versus bi-field

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The bi-field flow complemented by the sWI (remember $\phi = \varphi + \bar{\varphi}$):

$$\partial_t V + 3V - \frac{1}{2}\left(\varphi\partial_\varphi V + \bar{\varphi}\partial_{\bar{\varphi}} V\right) = \frac{\left(1-\alpha\bar{\varphi}^2\right)^{3/2}\left(1-\frac{1}{2}\alpha\bar{\varphi}^2\right)}{1-\alpha\bar{\varphi}^2+\partial_\varphi^2 V}\theta\left(1-\alpha\bar{\varphi}^2\right)$$

$$\partial_\varphi V - \partial_{\bar{\varphi}} V = \bar{\varphi}\frac{(1-\alpha\bar{\varphi})^{3/2}}{1-\alpha\bar{\varphi}^2+\partial_\varphi^2 V}\,\theta(1-\alpha\bar{\varphi}^2)$$

The sWI is crucial

$$\partial_t V - \frac{1}{2}(d-2)\left(\varphi\partial_\varphi V + \bar{\varphi}\partial_{\bar{\varphi}} V\right) + dV$$

$$= \frac{(1-h)^{d/2}}{1-h+\partial_\varphi^2 V}\left(1-h-\frac{1}{2}\partial_t h + \frac{1}{4}(d-2)\bar{\varphi}h'\right)\theta(1-h)$$

$$\partial_\varphi V - \partial_{\bar{\varphi}} V = \frac{h'}{2}\frac{(1-h)^{d/2}}{1-h+\partial_\varphi^2 V}\theta(1-h)$$

Change of variables:

$$V = (1-h)^{d/2}\hat{V}, \qquad \varphi = (1-h)^{\frac{d-2}{4}}\hat{\varphi} - \bar{\varphi}, \qquad t = \hat{t} - \ln\sqrt{1-h}$$

The sWI is crucial

Flow equation

+

shift Ward identity

change of variables

$$\partial_{\hat{t}}\hat{V} + d\hat{V} - \tfrac{1}{2}(d-2)\hat{\varphi}\partial_{\hat{\varphi}}\hat{V} = \frac{1}{1+\partial^2_{\hat{\varphi}}\hat{V}}$$

This is back to the standard d-dim. flow!

The sWI is crucial

- There is a one to one correspondence between the fixed points of both flows:

$$V_*(\varphi,\bar\varphi) = (1-h_*(\bar\varphi))^{d/2}\,\hat V_*\Big((1-h_*(\bar\varphi))^{\frac{2-d}{4}}(\varphi+\bar\varphi)\Big)$$

- Looking at eigenoperators
  - Before change of variables

$$V_t(\varphi,\bar\varphi) = V_*(\varphi,\bar\varphi) + \varepsilon\, v(\varphi,\bar\varphi)\exp(-\lambda t)$$

  - After change of variables

$$\hat V_{\hat t}(\hat\varphi) = \hat V_*(\hat\varphi) + \varepsilon\,\hat v(\hat\varphi)\exp(-\lambda\hat t)$$

  - The change of variables then implies

$$\begin{aligned} h_t(\bar\varphi) &= h_*(\bar\varphi) + \varepsilon\,\delta h(t,\bar\varphi) \\ &= h_*(\bar\varphi) + \varepsilon\,\kappa(\bar\varphi)\exp(-\lambda t) \end{aligned}$$

The sWI is crucial

- The linearisation of the complicated system reduces to the linearisation of the standard flow equation
- The eigenspectra are **identical** and the eigenoperators are related via

$$v = (1 - h_*)^{\frac{d-\lambda}{2}} \hat{v} - \frac{\kappa}{2} \frac{(1-h_*)^{\frac{d}{2}-1}}{1+\partial_{\hat{\varphi}}^2 \hat{V}_*}$$

Statement of universality

In particular: in d=3 these relations completely resolve all previously described issues of the single field approximation.

The sWI in the literature

- Scalar field theory
  - Litim, Pawlowski, 2002
- Yang-Mills theory
  - Reuter, Wetterich, 1994, 1997
  - Litim, Pawlowski, 1998, 2002
- Scalar QED
  - Reuter, Wetterich, 1994
- Conformal gravity
  - Manrique, Reuter, 2010

Here: In scalar field theory the sWI is enough to recover exact universality!

The sWI and gravity

- In gravity, the dependence on the background field is much more involved: gauge fixing, ghosts, auxiliary fields

- The sWI is far more complicated

- In scalar field theory the sWI effectively removes the background-field dependence as introduced by the cutoff

- In gravity, the background field is an intrinsic component of the construction of the effective action and not just put in by hand via the cutoff

Conclusions

- Single field approximation can lead to inaccurate results if there is a background field dependence in the regulator
- This can include additional fixed points, previously existing fixed point can disappear, eigenspectra can be modified and redundant eigenoperators can appear
- In bi-field calculations the sWI determines the background field dependence of the effective action and ensures its uniqueness
- In the LPA of scalar field theory the sWI as a complement to the flow equation is enough to recover exact universality

Thank you!