Coarse graining and renormalization: the bottom up approach

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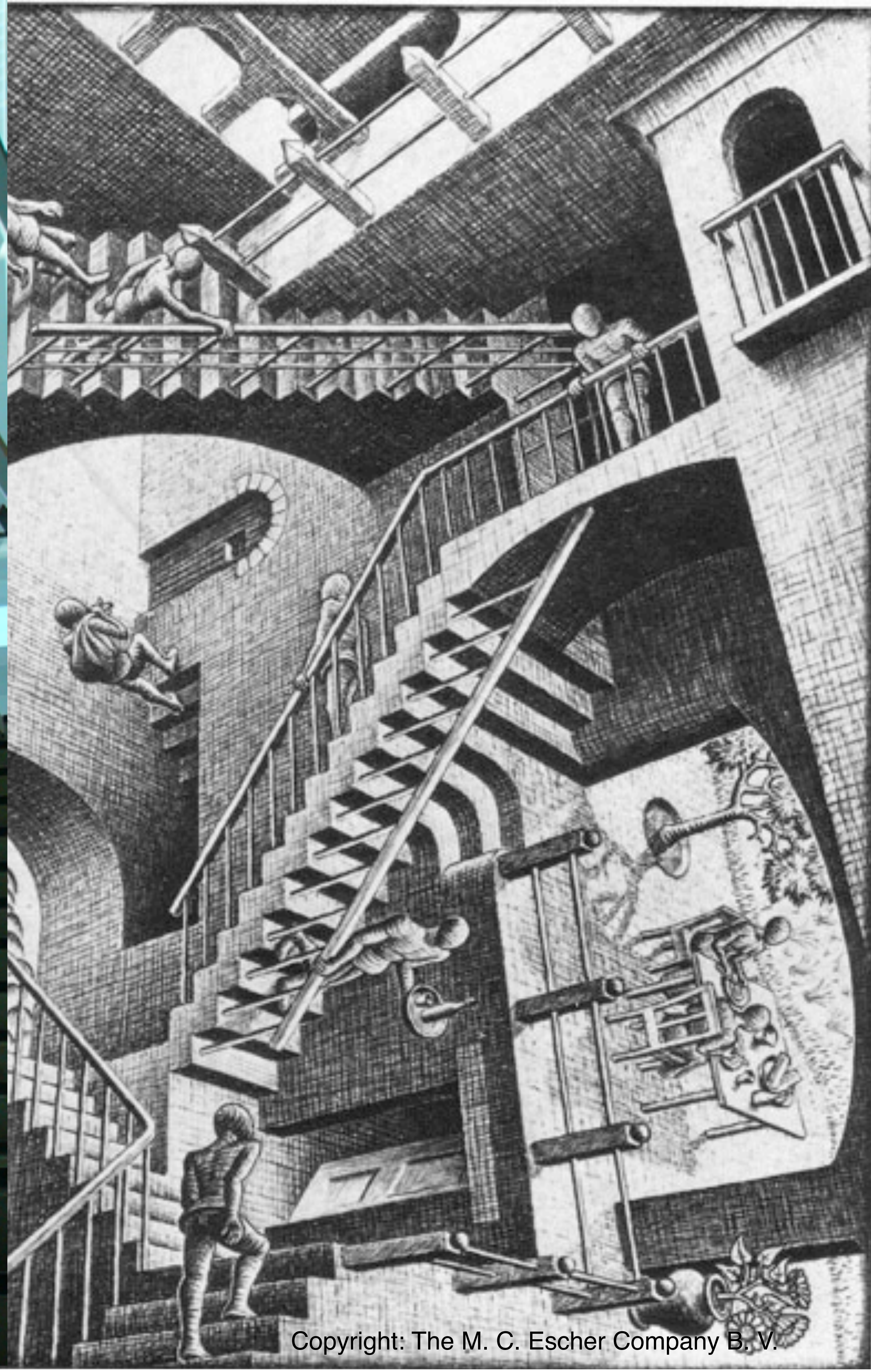
Raussendorf



Literature

Conceptual.

“Time evolution as refining, coarse graining and entangling,”
B. Dittrich, S. Steinhaus,
arXiv:1311.7565[gr-qc]

“From the discrete to the continuous – towards a cylindrically consistent dynamics,”
B. Dittrich,
New J. Phys. 14 (2012) 123004 (28pp)
arXiv:1205.6127 [gr-qc]

(Latest) Coarse graining results for spin foams / spin nets.

“Quantum group spin net models: refinement limit and relation to spin foams,”
B. Dittrich, M. Martin-Benito, S. Steinhaus,
arXiv:1312.0905[gr-qc]

New representation for LQG.

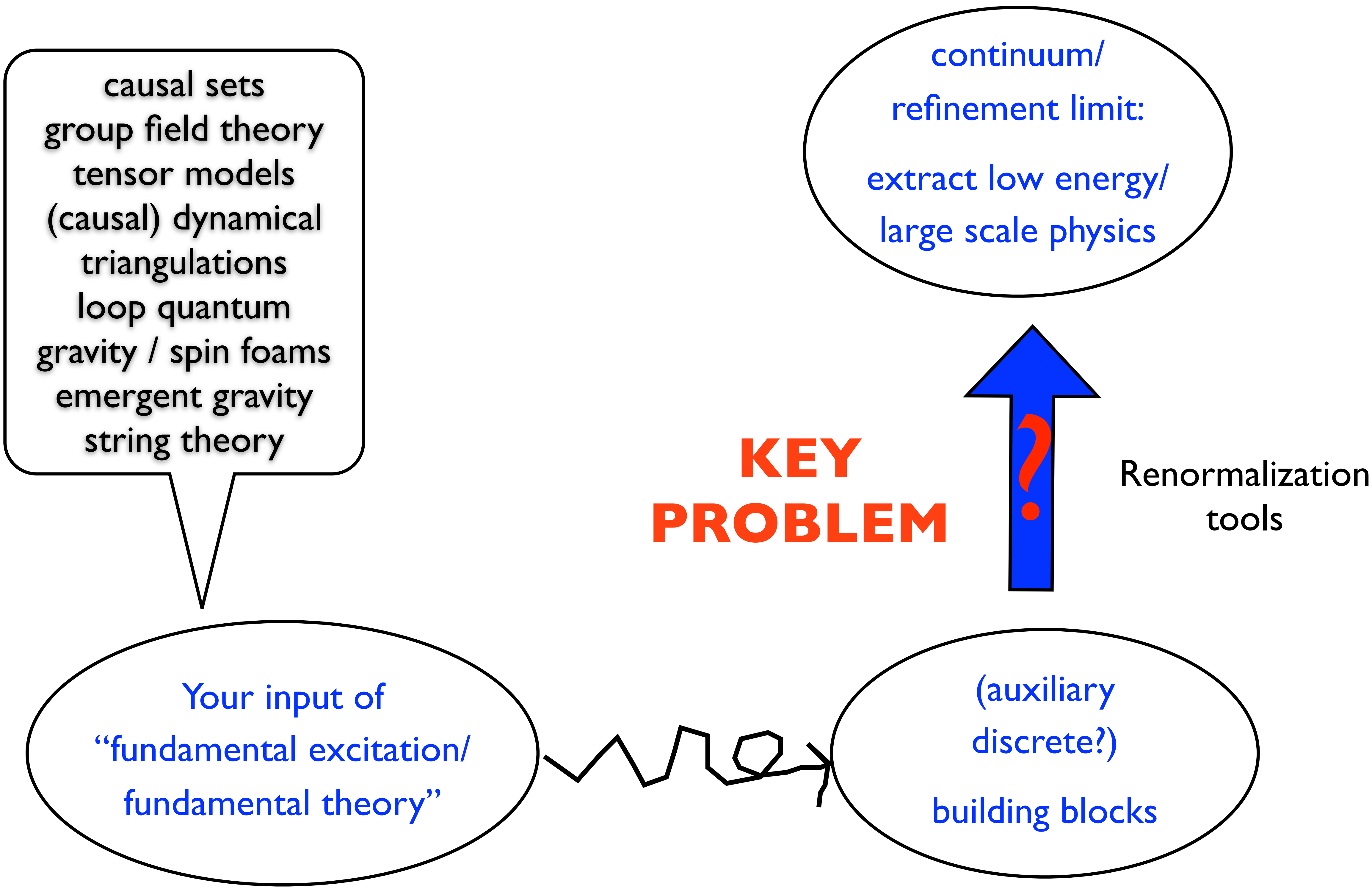
“A new vacuum for loop quantum gravity,”
B. Dittrich, M. Geiller,
arXiv:1401.6441[gr-qc]

Tensor network method in condensed matter. (plus many many more)

M. Levin, C. P. Nave, “Tensor renormalization group approach to 2D classical lattice models,” Phys. Rev. Lett. **99** (2007) 120601, [arXiv:cond-mat/0611687 [cond-mat.stat-mech]].

Z. -C. Gu and X. -G. Wen, “Tensor-Entanglement-Filtering Renormalization Approach and Symmetry Protected Topological Order,” Phys. Rev. B **80** (2009) 155131 [arXiv:0903.1069 [cond-mat.str-el]].

Bottom-up design



Overview

Motivation.

Spin foams and space time atoms.

Coarse graining without a scale.

The importance of refining states. Tensor network coarse graining algorithms.

Application to spin foams / spin nets.

Classifying (symmetry protected) phases.

New representations and vacua in loop quantum gravity

Expanding the theory around different vacua corresponding to different ways of refinement.

Why loop quantum gravity / spin foams?

(My personal short list of reasons)

• "minimal assumptions": theories result from quantizing gravity (as a geometric theory), taking background independence seriously

[gives you a higher probability to obtain a quantum theory of gravity]

• choice of (connection) variables: originally motivated by "convenience",

  • allowed first rigorous quantization of gravity (kinematical)

  • so far leads to the most advanced inclusion of diffeomorphism symmetry

[kinematics: for instance LOST uniqueness theorem, but also BD, Geiller 14]

[conceptually very important in particular for coarse graining / renormalization, key role in regaining gravity]

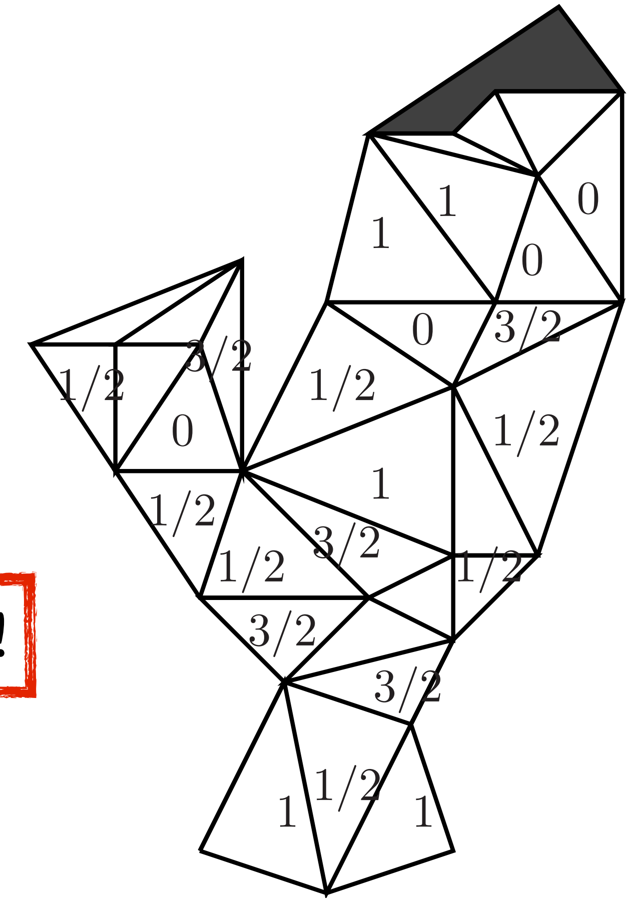
[dynamics: we know what we are looking for! [for instance in the formulation BD, Steinhaus 13]]

• (in particular spin foams): deep connection to topological field theories, (because there is) no Wick rotation involved

[topological field theories are fixed points of renormalization flow]

[Wick rotation leads to action unbounded from below: prevents useful continuum limit]

What are spin foams?



$$\int \exp(iS(\text{geom}))\ \mathcal{D}\text{geom}$$

We keep the i !

years of research: Reisenberger, Rovelli, Barrett-Crane, Barbieri, Freidel-Krasnov, Engle-Pereira-Rovelli, Livine, ...

→ discretization / quantization/ reformulation into generalized lattice gauge theory

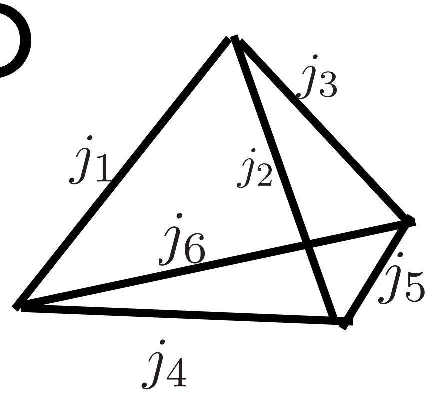
$$\sum_{\text{geom labels}} \mathcal{A}(\text{ geom labels})$$

with local (i.e. factorizing) amplitude:

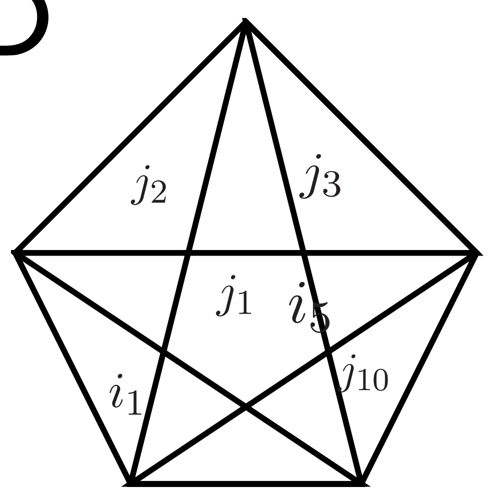
$$\mathcal{A}(\text{ geom labels}) = \prod_{\text{fund building blocks}} \mathcal{A}_{fbb}(\text{ geom labels})$$

Space time atoms: relation to gravity action

3D



4D



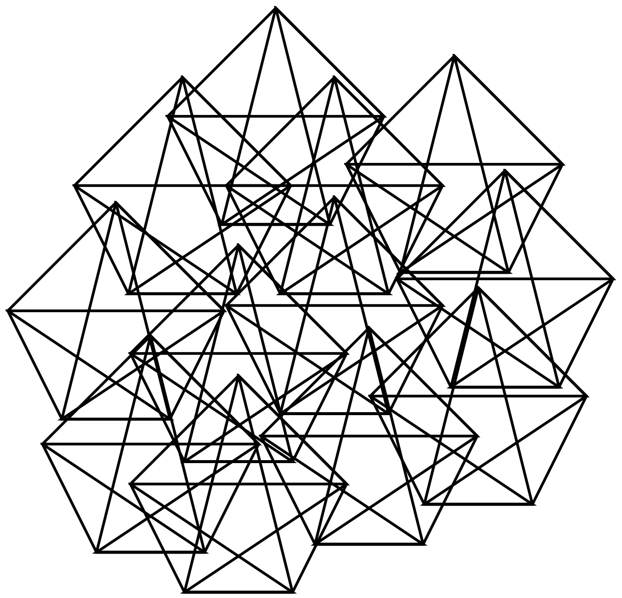
sum over orientations of space time atoms

$$\mathcal{A}(j) \underset{j>>1}{\sim} \exp(iS_{\text{discr grav}}) + \exp(-iS_{\text{discr grav}})$$

Large j (semi-classical) limit for single building blocks gives discrete GR action (for flat building blocks)!

[Ponzano-Regge, Barrett et al, Conrady-Freidel, ...]

Many space time atoms?



Is there a phase describing smooth space time?

Do we get General Relativity at "large scales"?

What are the phases of spin foam theories
(and in loop quantum gravity)?

Spin foams as lattice theories

- there is an underlying (auxiliary) lattice: refinement limit with respect to this lattice
- use tools developed in condensed matter, in particular tensor network renormalization.

  [Cirac,Levin, Nave, Gu, Wen, Verstraete, Vidal, ...] [‘Emergent gravity’]
- Can deal with complex amplitudes: (we do not Wick rotate!)

- However many conceptional differences:
  - result should be independent on choice of lattice (discretization independence)
  - diffeomorphism symmetry should emerge (at fixed points of renormalization flow)
  - (well supported) conjecture: discrete notion of diffeomorphism symmetry equivalent to discretization independence - should emerge in refinement limit

    [BD 08, Bahr, BD 09, Bahr, BD, Steinhaus 11, BD 12] .
  - There is no lattice scale, instead notion of scale included in dynamical variables.
  - Hope to flow to perfect discretization, mirroring exactly continuum theory at all scales at once.

Coarse graining without a scale

Fixed point model encodes all scales!

- There is no lattice scale, instead notion of scale included in dynamical variables.
- Hope to flow to perfect discretization, mirroring exactly continuum theory at all scales at once.

View point:

- Scale encoded in boundary state - not in renormalization step.
- The initial model is an approximation (via discretization), which we hope to be valid in a small curvature regime [determined by choice of boundary states and number of building blocks].
- Models obtained by coarse graining improve this approximation and increase domain of validity.
- Refinement limit corresponds to a ``perfect discretization''.

Fixed point as a continuum theory

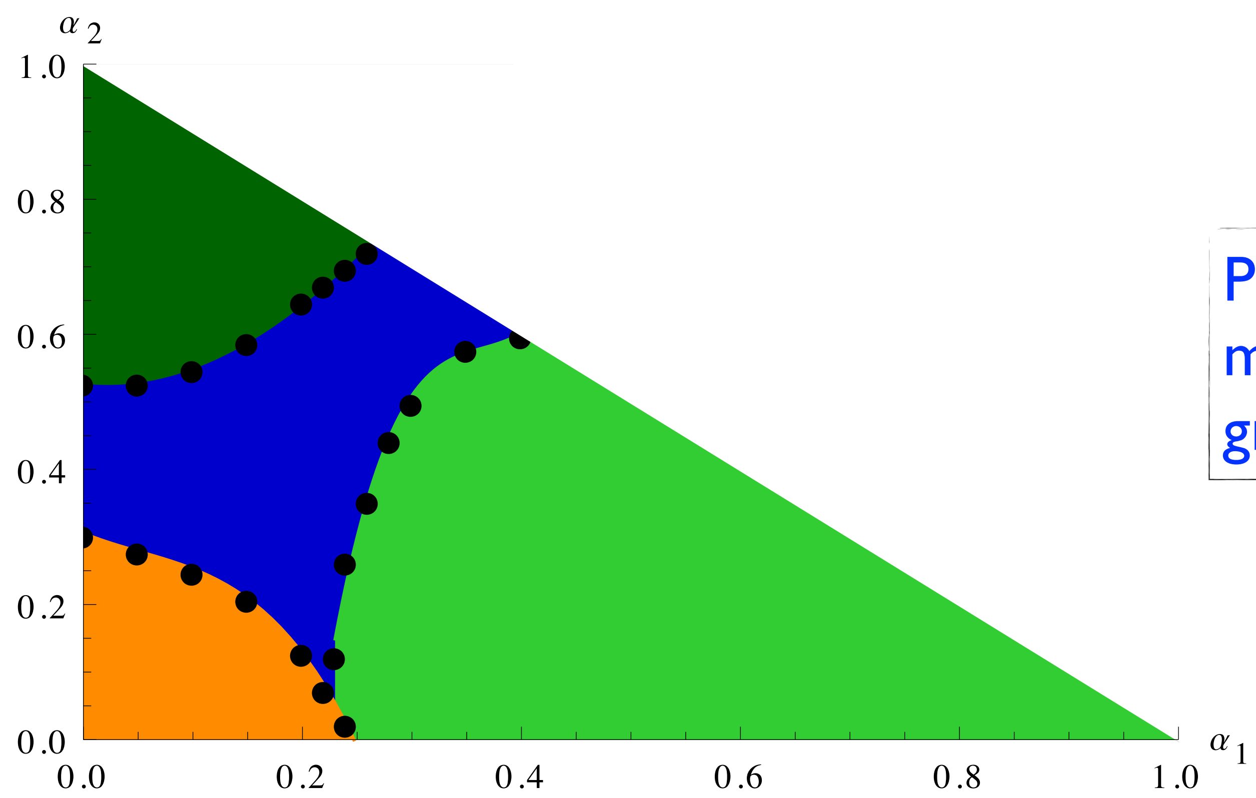
If we do not have a lattice scale, how do we know that we reached the continuum limit?

Answer:

- Fixed points of renormalization flow correspond to continuum limit.

  (a) "local" amplitudes: topological (discretization independent) theories correspond to phases

  (b) "non-local" amplitudes (infinite bond dimension) correspond to phase transitions



Phase space: (some) parameters in initial model determine end points of coarse graining flow encoded in different colours

Fixed point as a continuum theory

Fixed point model still looks discrete. How to reconstruct continuum theory?

Answer (here a short summary, expanded upon in the next slides):

- Build continuum theory as inductive limit: standard technique of loop quantum gravity [Ashtekar, Isham, Lewandowski 92+]
- Based on embedding (or refining) coarser (boundary) states into Hilbert space describing finer boundary states

- In this way any coarse (discrete looking) state can be understood as a state of the continuum Hilbert space.
- (Cylindrically) consistency requirements on observables, amplitudes, inner product.

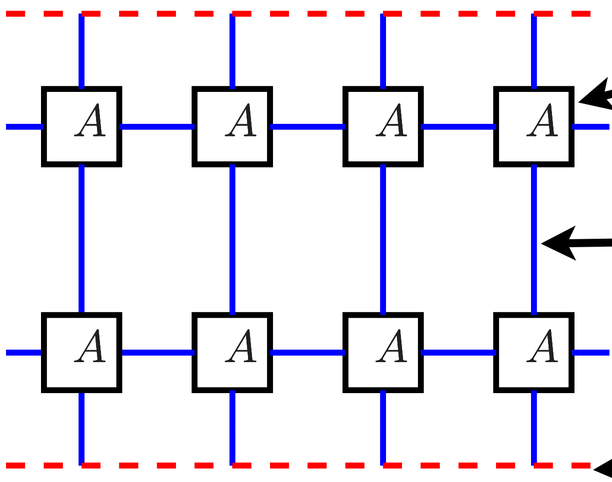
- Surprisingly (condensed matter) tensor network renormalization methods are well adapted to this technique and can provide embedding maps - determined by dynamics of the system. [BD 12]
- Allows to formulate continuum theory of spin foams based on discrete boundary states

[BD, Steinhaus 13]

Inductive limit techniques and

Tensor network coarse graining algorithms.

Boundary Hilbert spaces and transition amplitudes

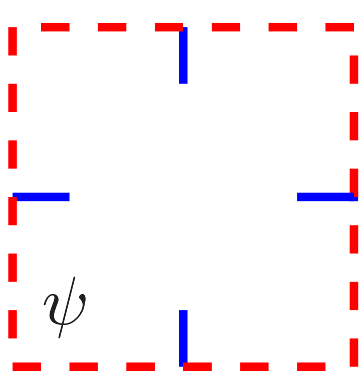


(local) amplitudes depending on (boundary) variables, represented by blue edges

summing over (bulk) variables in path integral

boundary Hilbert space (tensor product over local sites)

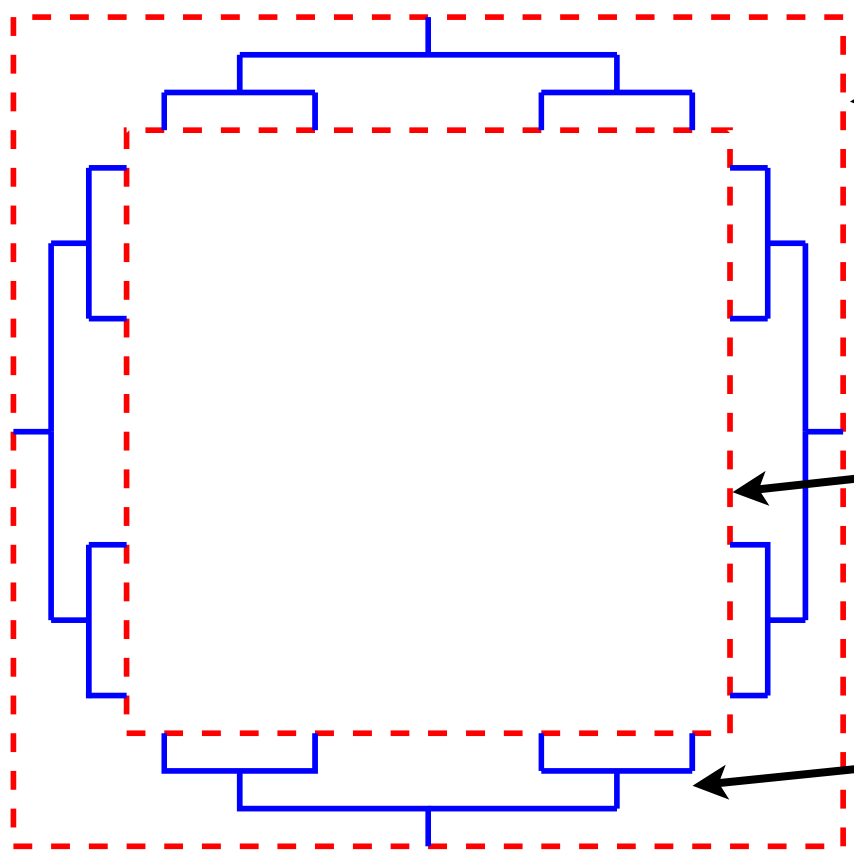
This represents a transition amplitude built from local amplitudes.
The boundary Hilbert space has two components.



This represents a one-component boundary Hilbert space.

[generalized boundary formalism: Oeckl 03]

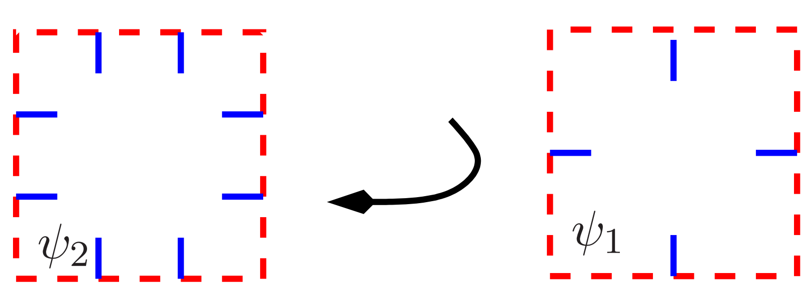
Embedding Hilbert spaces



coarser boundary (with four sites) with
attached boundary Hilbert space
(tensor product over four sites)

finer boundary (16 sites) with
attached boundary Hilbert space

(local) embedding map



embedding map identifies coarse states with states
in finer boundary Hilbert space

Good embedding maps?

What are the embedding maps good for?

Answer:

- efficiency!
- do computation on the most coarse grained level possible
- can represent continuum theory by discrete (boundary) data

➡ To be really efficient, embedding maps have to be adjusted to dynamics of the system:

➡Coarse Hilbert space should represent ``most typical states''

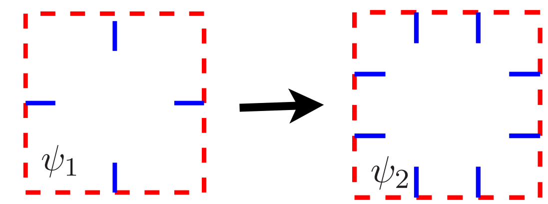
(state describing smooth geometry / low energy)

Whereas initial boundary states may rather describe fundamental excitations, and one expects a large number of these to describe smooth geometry.

Amplitudes encode the dynamics of the system

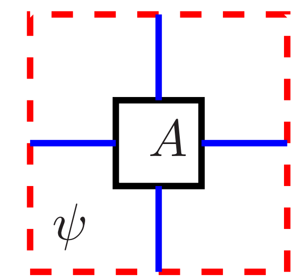
•embedding maps: $\iota_{bb'}: \quad \mathcal{H}_b \rightarrow \mathcal{H}_{b'}$

•order boundaries into coarser and finer



•(transition) amplitudes encode dynamics: $A_b : \mathcal{H}_b \mapsto \mathbb{C}$

[transition amplitudes if there are two boundary components]



•(transition) amplitudes for instance defined by path integral
(blue edges represent sum over variables)

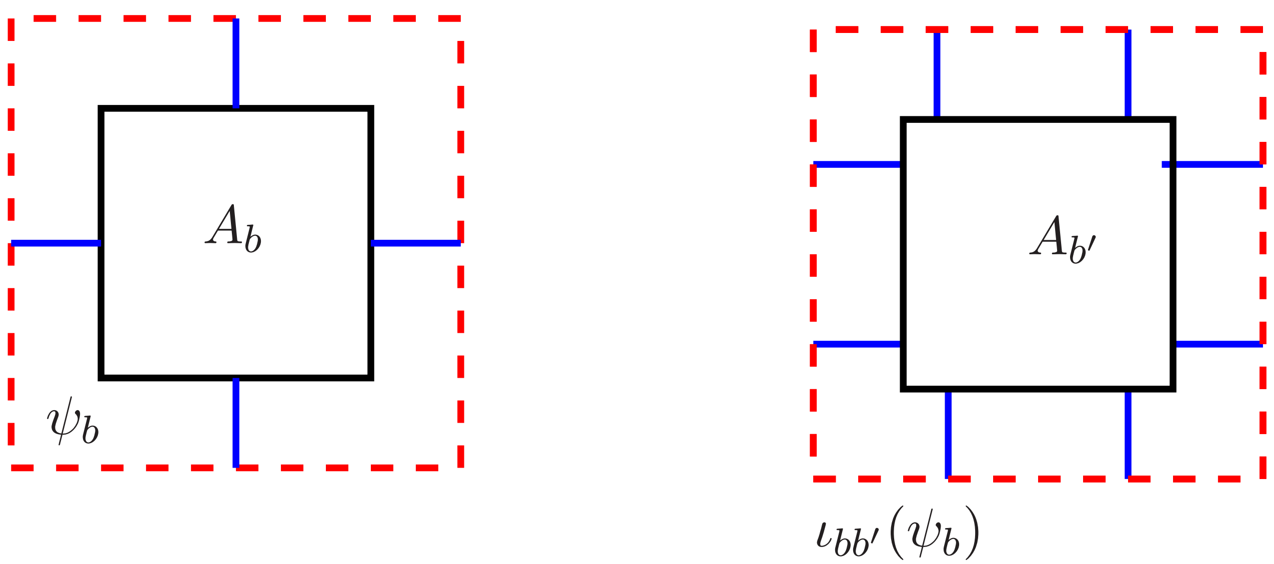
•in fact we have a family of (transition) amplitudes labelled by boundaries b

•Cylindrical consistency conditions relate amplitudes for different boundaries b and b'.

Cylindrically consistent amplitudes

$$A_b(\psi_b) \;=\; A_{b'}(\iota_{bb'}(\psi_b))$$

Computing the amplitude for a coarse state should give the same result, as
- embedding coarse state to a finer state
- using the amplitude map for finer states.

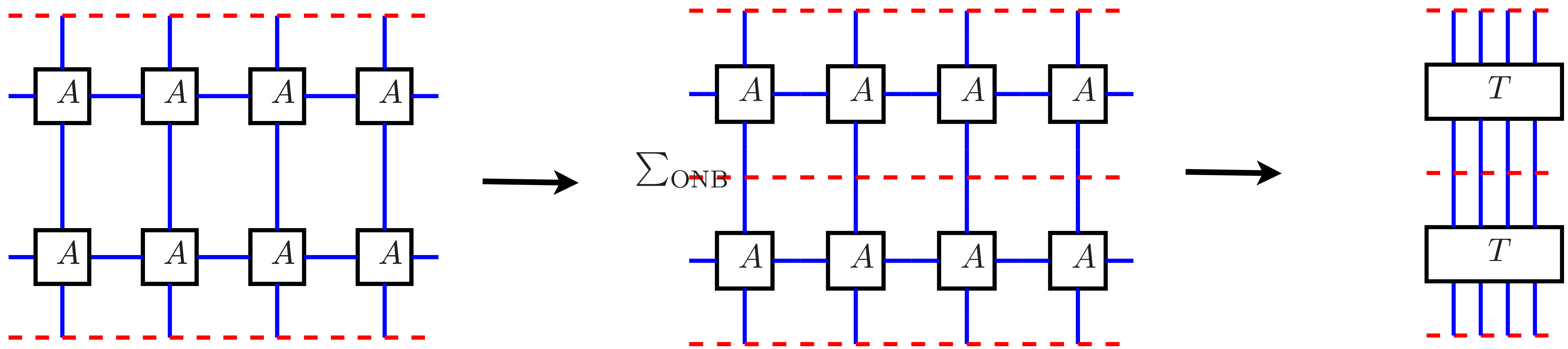


Thus, computation for coarse states give already continuum results.
Cylindrically consistent amplitudes define the continuum limit of the theory.

[BD 12, BD, Steinhaus 13]

What are good choices for the embedding maps
(for the basis of boundary states)?

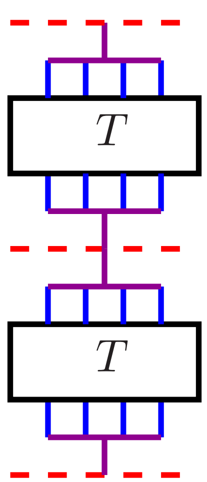
Motivation: transfer operator technique



Transition amplitude between two states $\langle\psi_1|\mathcal{A}|\psi_2\rangle$

insert id $= \sum_{\mathrm{ONB}} |\psi\rangle\langle\psi|$

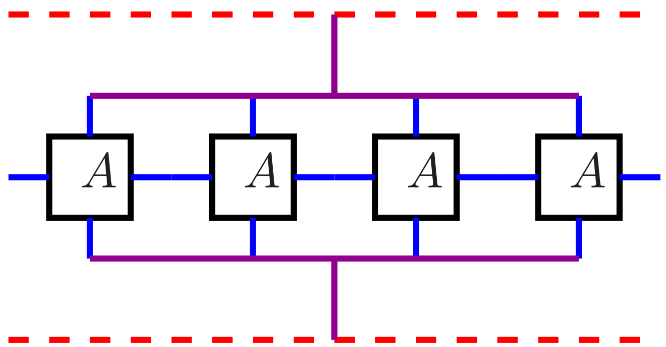
$\mathcal{A} = T^N$



Truncate by restricting $\sum_{\mathrm{ONB}}$ to the eigenvectors of T with the χ largest (in mod) eigenvalues.

Here coarse states correspond to low energy states.
Results (typically) in non-local embedding maps (example: Fourier transform for free theories).
Explicit diagonalization difficult for large systems.

Localized embedding maps



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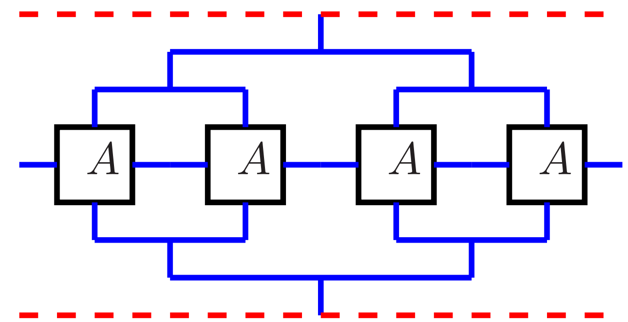
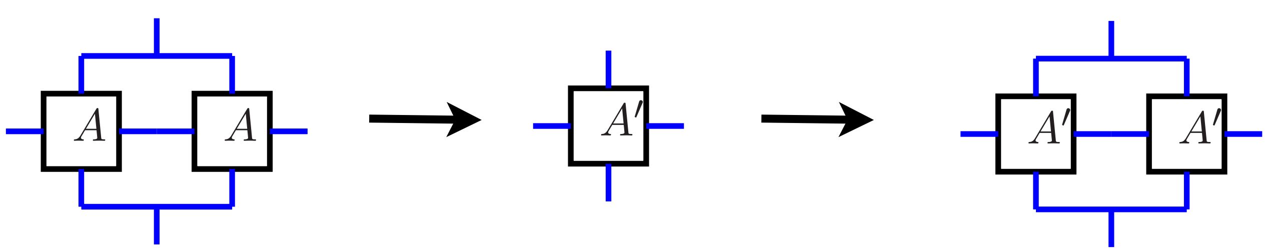
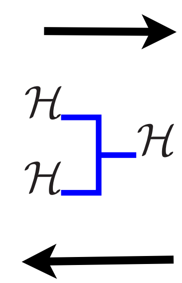


Localize truncations, diagonalize only subparts of transfer operator

iteration procedure



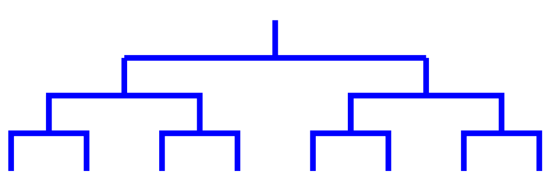
blocking



Determined by (generalized) EV-decomposition.

embedding

embedding map after 3 iterations



Relation to vacuum

Here coarse states correspond to low energy states / states with few excitations from vacuum.

Embedding maps depend on the dynamics of the system.

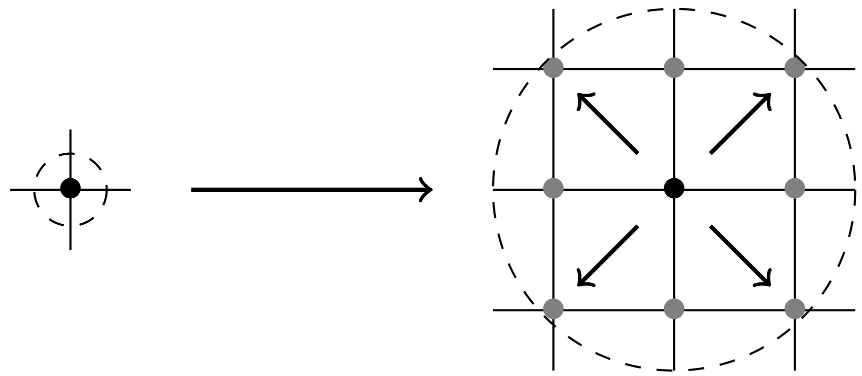
However in General Relativity:
What is vacuum?
Hamiltonian is a constraint and hence zero (and we do not Wick rotate).
Transfer matrix should be a projector if diffeomorphism symmetry is realized
(with eigenvalues one and zero).
NB: In 4D diffeomorphism symmetry is broken, eigenvalues take other values too.

So we can hope that even for GR, unphysical states (eigenvalue zero) are truncated away.

There is however a further mechanism that differentiates states even in GR: radial evolution.

[BD, Steinhaus 13: Time evolution as refining, coarse graining and entangling]

Radial time evolution and coarse states



Consider a "radial time evolution" from a ``smaller" to a "larger" Hilbert space.

(We can easily construct such evolutions for systems based on simplicial discretizations, such as spin foams).

This time evolution itself defines an embedding map:
The image of the time evolution defines coarser states in the larger Hilbert space.

Conjecture: This definition leads to states describing coarse grained excitations.
(Radial) time evolution can be used to define useful embedding maps.

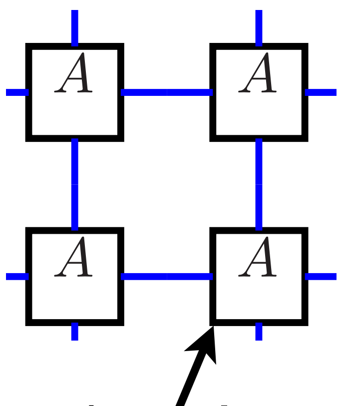
[BD, Steinhaus 13]

[also BD, Hoehn 11,13, Hoehn 14, BD, Hoehn, Jacobson wip]

NB: Again this definition would lead in general to non-local embedding maps.

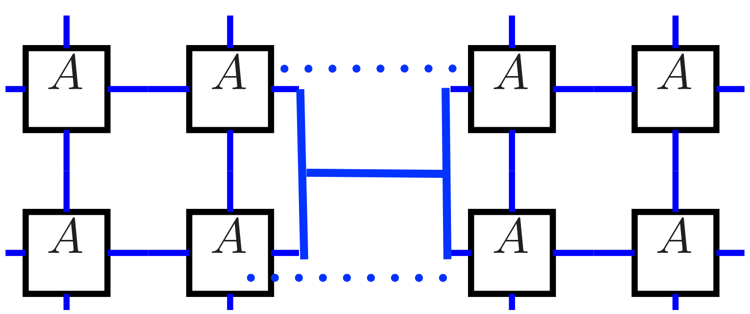
Tensor network renormalization methods

... rather using local embedding maps (to keep algorithms efficient)
but different versions are possible.

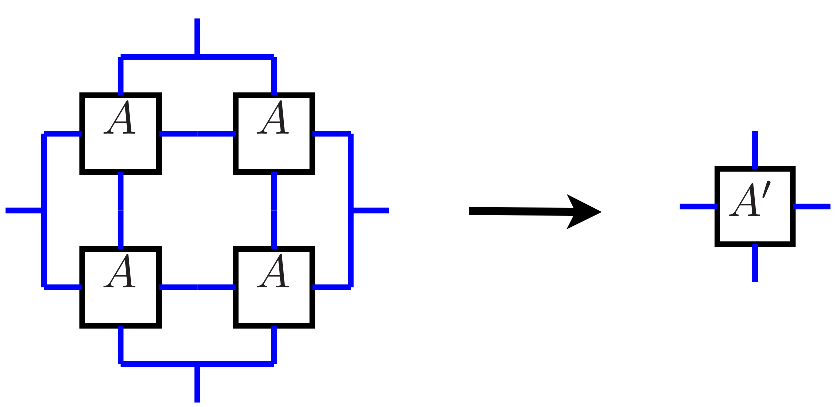


bare/initial amplitude
depending on four variables

Contract initial amplitudes (sum over bulk variables). Obtain "effective amplitude" with more boundary variables.

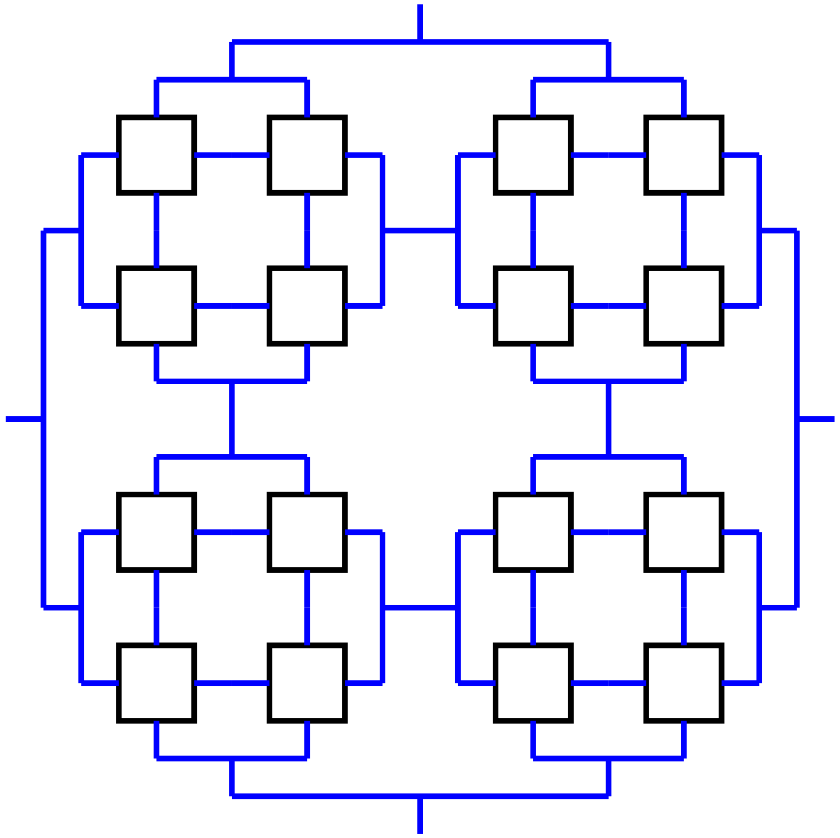


Find an approximation (embedding map) that would minimize the error as compared to full summation (dotted lines). For instance using singular value decomposition, keeping only the largest ones.
Leads to field redefinition, and ordering of fields into more and less relevant.

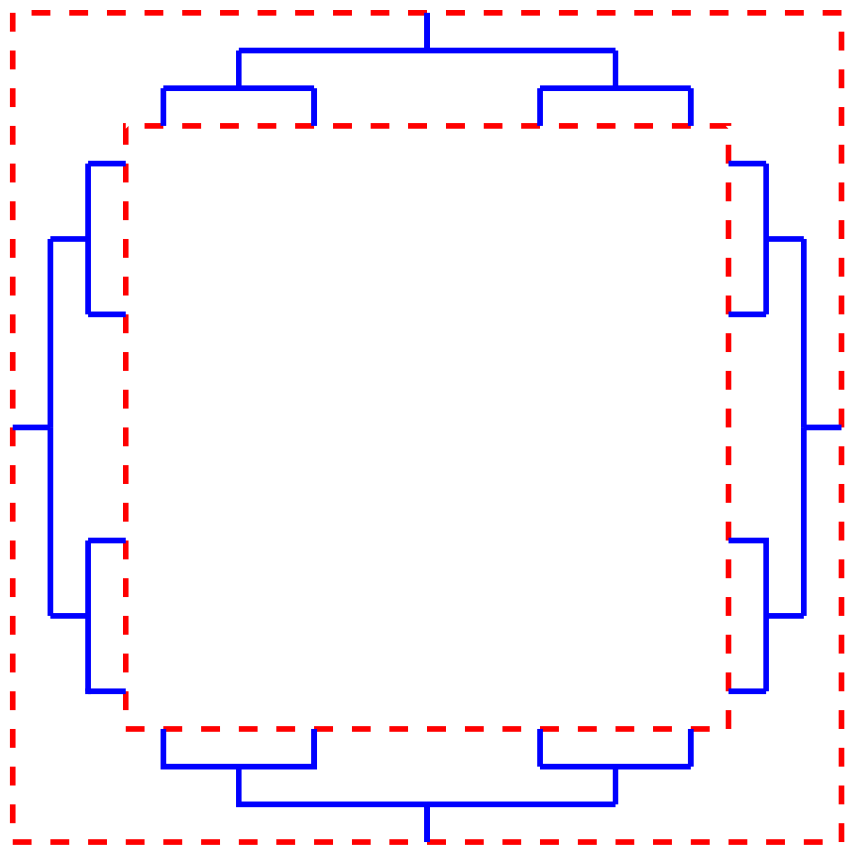


Use embedding maps to define coarse grained amplitude with the same (as initial) number of boundary variables.

Tensor network renormalization methods

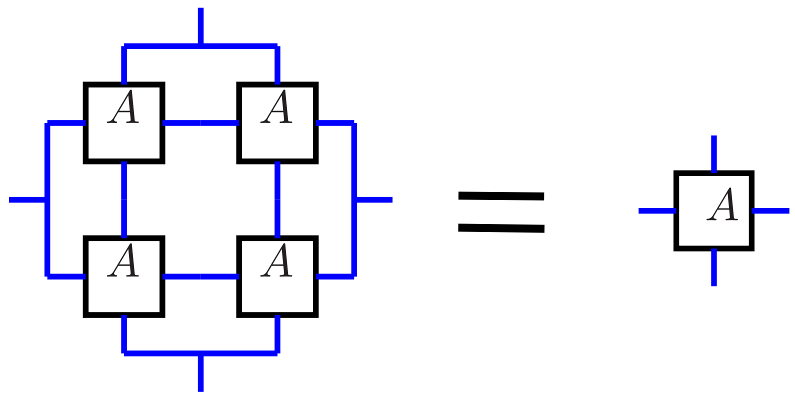


Iteration. Summation over bulk variables are truncated using the embedding maps.

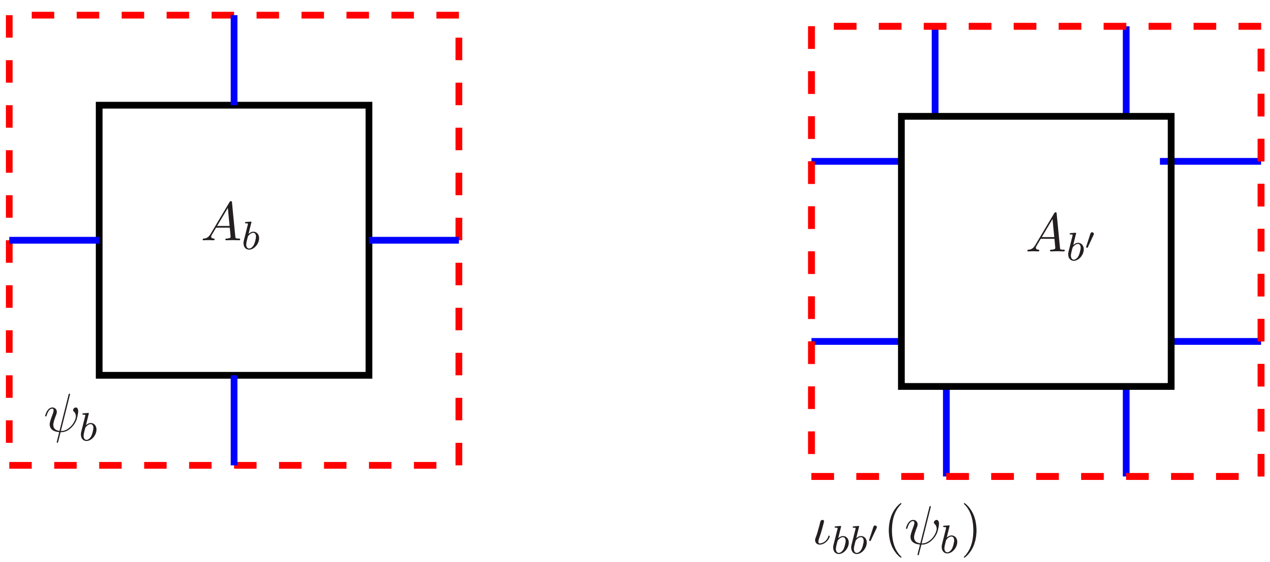


Associated embedding maps for boundary Hilbert space.

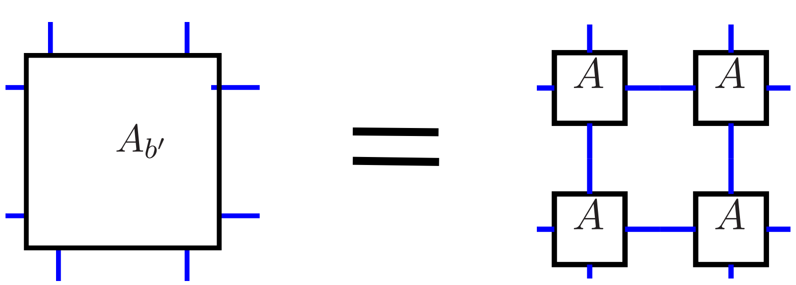
Fixed points give cylindrical consistent amplitudes



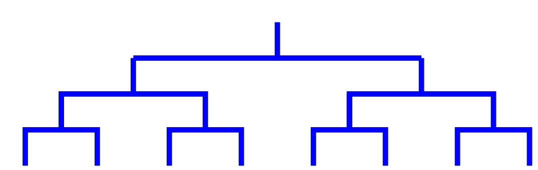
Condition for cylindrically consistent amplitudes



satisfied for



$\iota_{bb'} =$



Fixed points corresponding to topological field theories or interacting field theories

•method works particularly well for convergence to a "gapped phase"
 (lead to topological field theories)

•finitely many ground states separated from excited states: determine bond dimension
 (number of non-vanishing singular values)

•phase transition: usually involve 'infinite bond dimension'

•expect to find interacting theories / propagating degrees of freedom

•truncation can be nevertheless quite good, and provides a method to determine phase transition (and sometimes even to calculate critical exponents)

•However argument about radial time evolution suggest that better truncations (near phase transitions) might be possible if non-local embedding maps are incorporated.

•Might even not need infinite bond dimension.

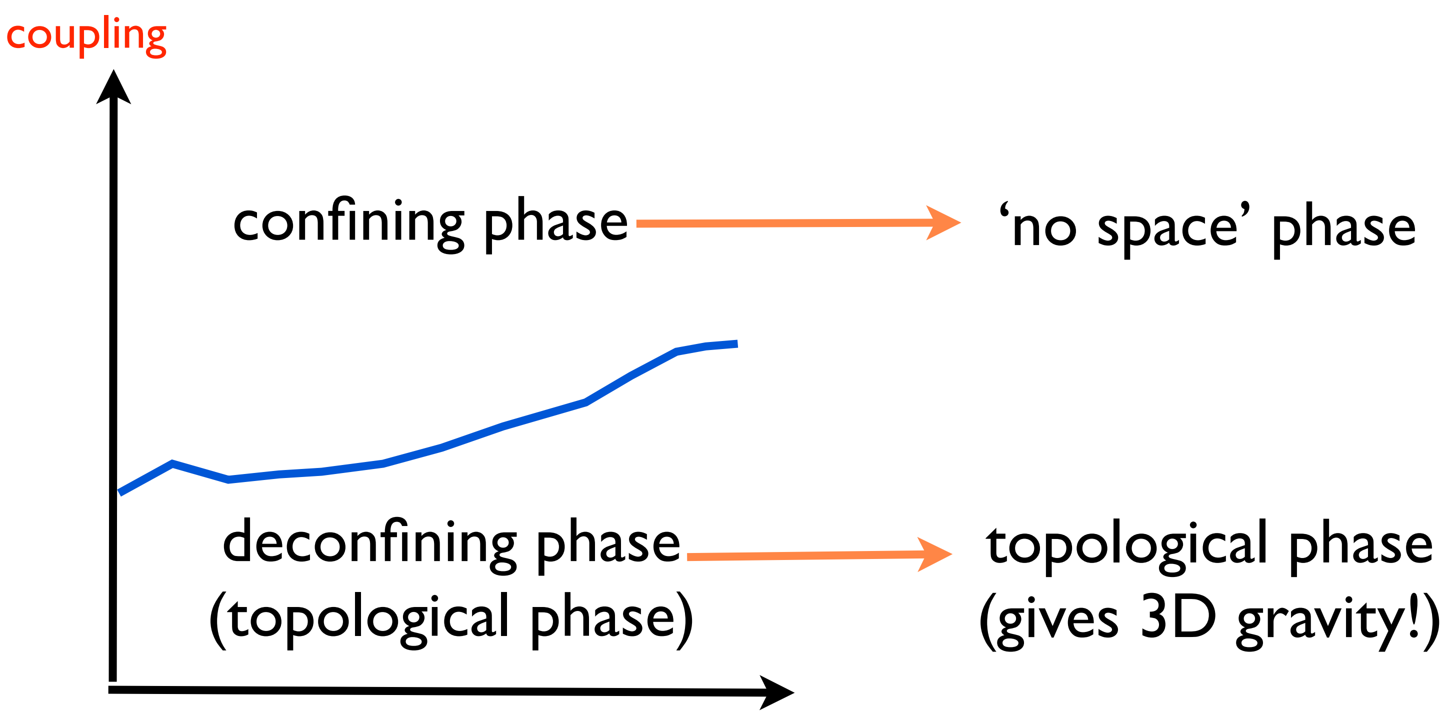
Applications: (analogue) spin foam models

- 4D spin foam models are very very complicated
- devised analogue models capturing the essential dynamical ingredients of spin foams
  [similar to 4D lattice gauge and 2D spin system duality]
- these spin nets can actually be interpreted as spin foams based on very special discretizations
- unlike spin foams the spin net models are non-trivial in 2D
- investigated the phase diagrams for such models (with quantum group structure)
- these phase diagrams have a very rich structure

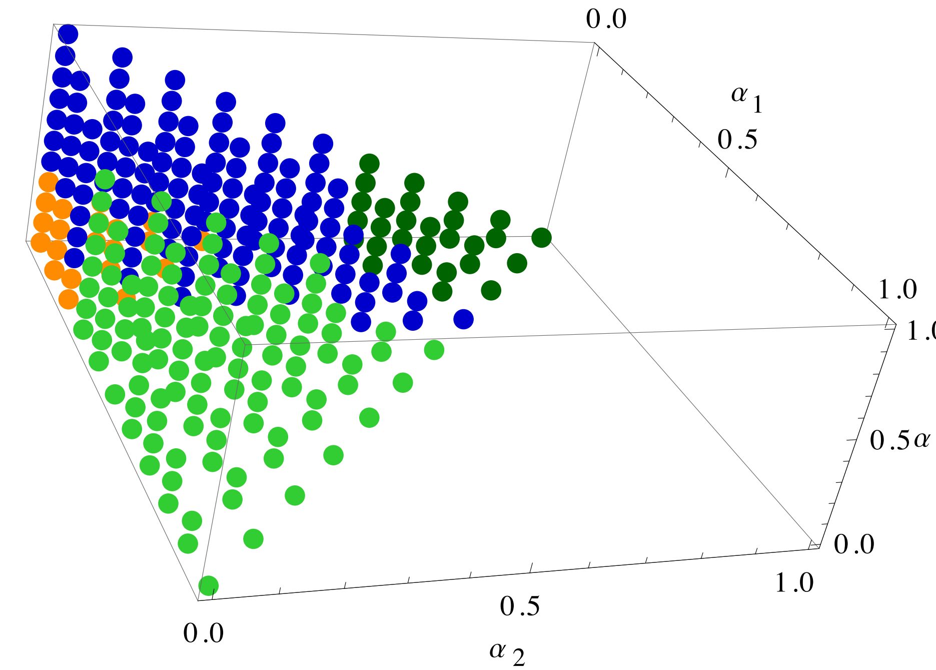
(The details of this and the adaption of the tensor network methods take easily up another talk.)

Applications: (analogue) spin foam models

Lattice gauge theory/ Ising like systems



Spin nets



Much richer phase structure!

[BD, Martin-Benito, Schnetter 2013, BD, Martin-Benito, Steinhaus 2013]

Interpretation: different phases describe uncoupled space time atoms (green) and coupled space time atoms (orange,blue).

Towards spin foam coarse graining

•So far encouraging results for ‘spin foam analogue’ models.

Conclusions:

- •Relevant parameters related to (SU(2)) intertwiners (also appearing in spin/anyon chains) leading to rich phase space structure. This is expected from the gravity dynamics.
- •Need to implement a weak version of discretization independence to uncover these rich phase spaces (escape the two lattice gauge theory phases).
- •Positive indication for finding a geometric phase in spin foams.

•Are now looking at spin foam analogue and actual spin foam models in higher dimensions.

•Need to extend tensor network tools (applicable to lattice gauge theories) to this end. [wip]

Each phase corresponds to a toplogical field theory.

Defines embedding maps and associated vacua.

New representations (Hilbert spaces) for

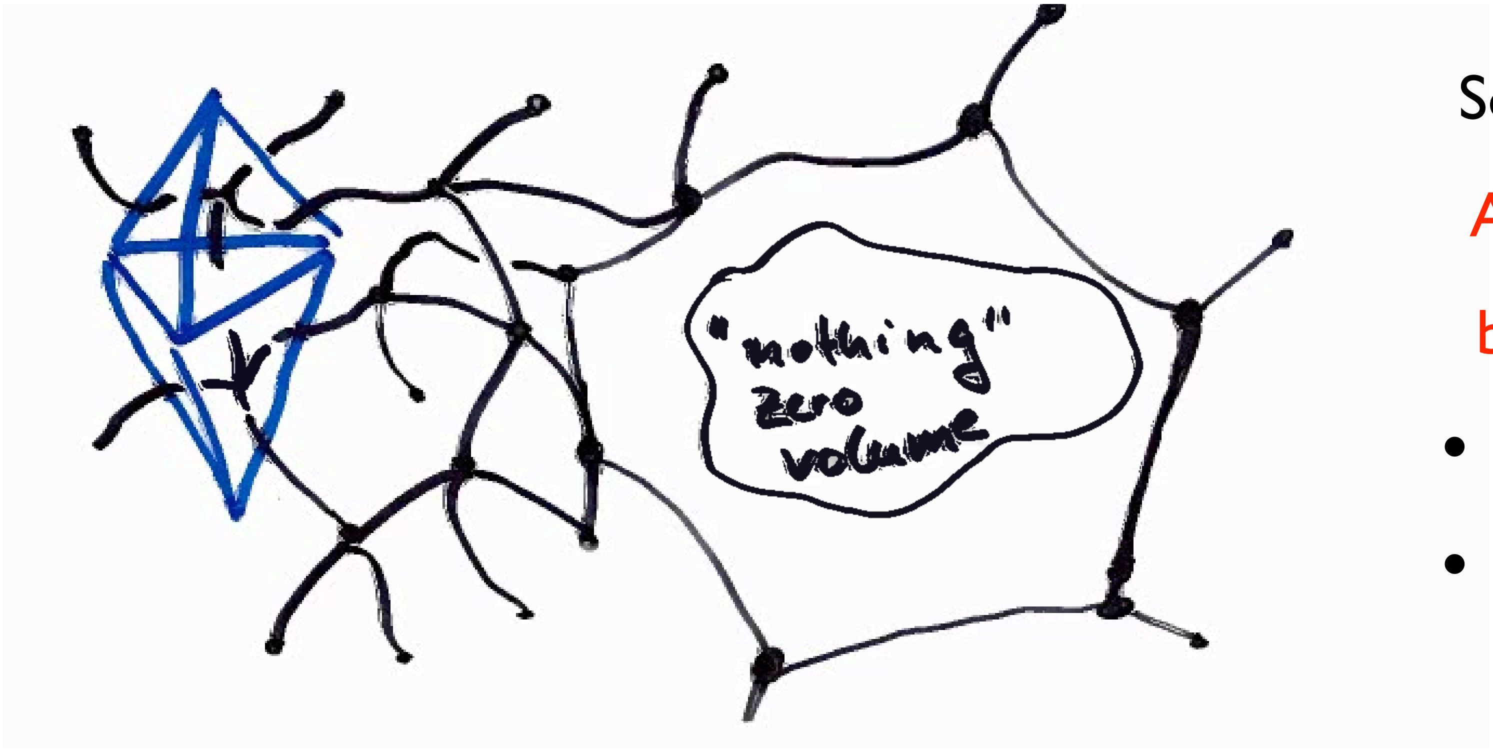
Loop quantum gravity.

Loop quantum gravity

- succeeded in constructing a quantum representation of geometric operators

  [Ashtekar, Lewandowski, Isham, Rovelli, Smolin, Corichi, ... 90's]

- spatial geometric operators (area, volume operators) have discrete eigenvalues
- spin networks based on (finite) graphs give a basis in the (non-separable) LQG Hilbert space
- despite this (discrete) basis the theory is defined in the continuum: via embedding maps
- dynamics based on Hamiltonian constraints [Thiemann '96]



So far the theory is based on the

Ashtekar-Lewandowski representation

based on a (AL) vacuum describing

- zero volume spatial geometry
- maximal uncertainty in conjugated variable.

What is vacuum?

- Ashtekar-Lewandowski representation /vacuum corresponds to confining phase for lattice gauge theories (i.e. embedding maps coincide).

Question: Can we base the representation on a different vacuum corresponding to one of the other phases?

Loop quantum gravity vacua

geometric variables: $\{A\,,\,E\} = \delta$

connection

flux: spatial geometry

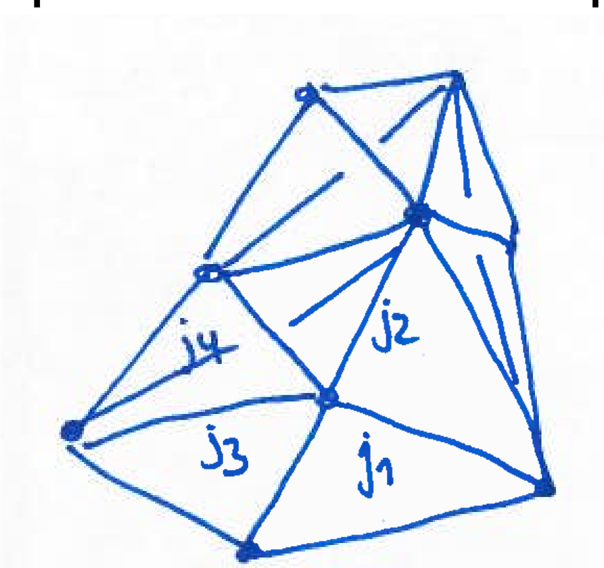
[BD, Geiller 2014,
BD, Geiller to appear]

Ashtekar - Lewandowski representation (90's)

$$\psi_{vac}(A) \equiv 1 \quad , \qquad E \equiv 0$$

peaked on degenerate (spatial) geometry
maximal uncertainty in connection

excitations:
spin network states supported on graphs



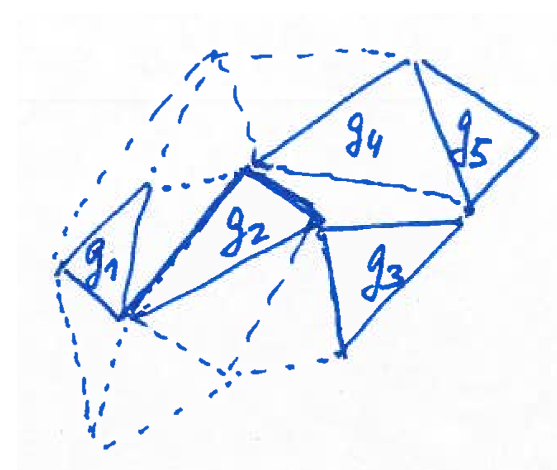
(representation)
labels for edges

BF (topological) theory representation

$$\psi_{vac}(E_{Gauss}) \equiv 1 \quad , \qquad F(A) \equiv 0$$

peaked on flat connections
maximal uncertainty in spatial geometry

excitations:
flux states supported on (d-1) D-surfaces

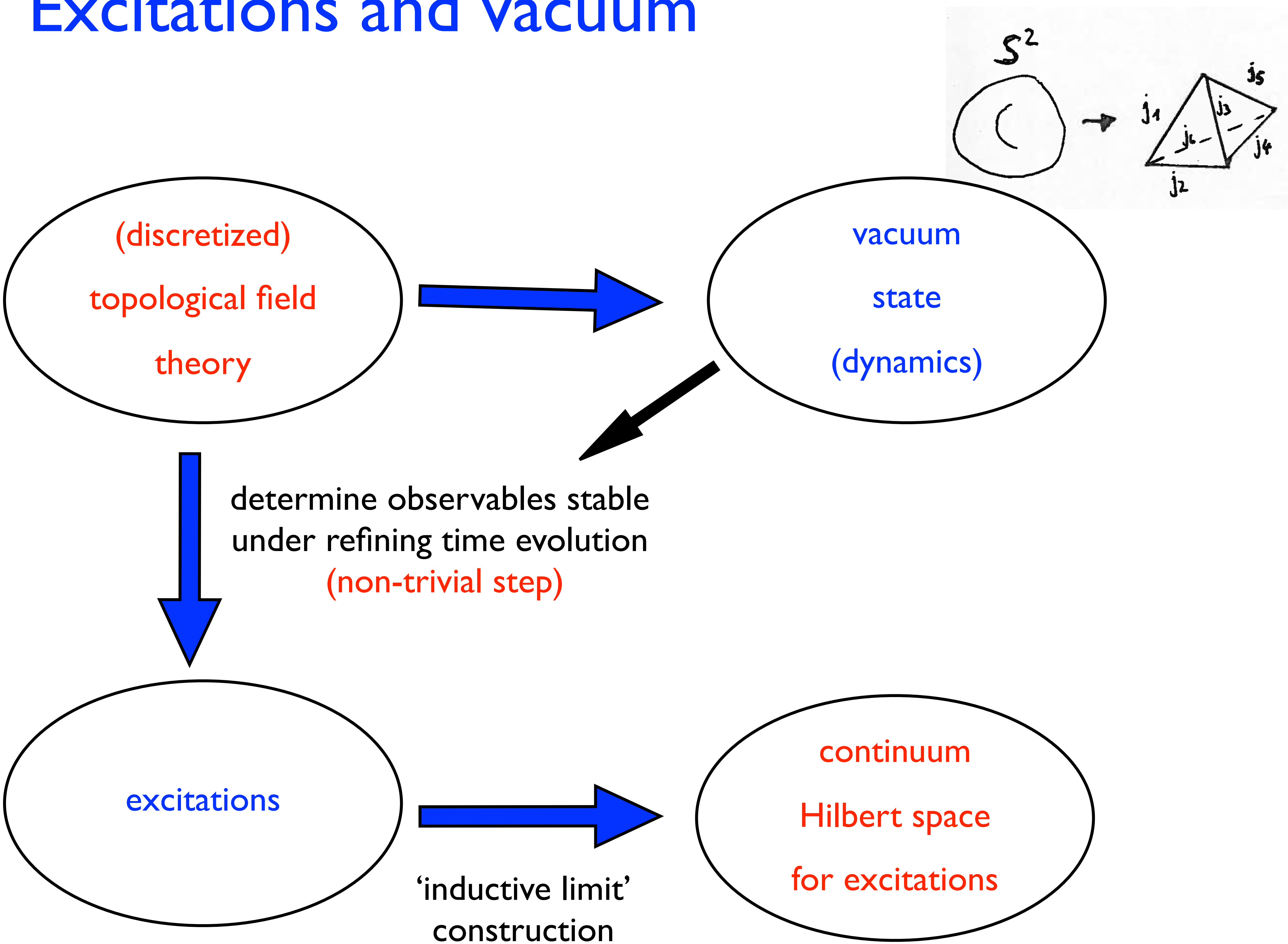


(group) labels
for faces

We applied this to BF (topological field theory), corresponding to unconfined phase. [BD, Geiller 14]

But construction can be generalized to other topological field theories:

Excitations and vacuum



Advantages

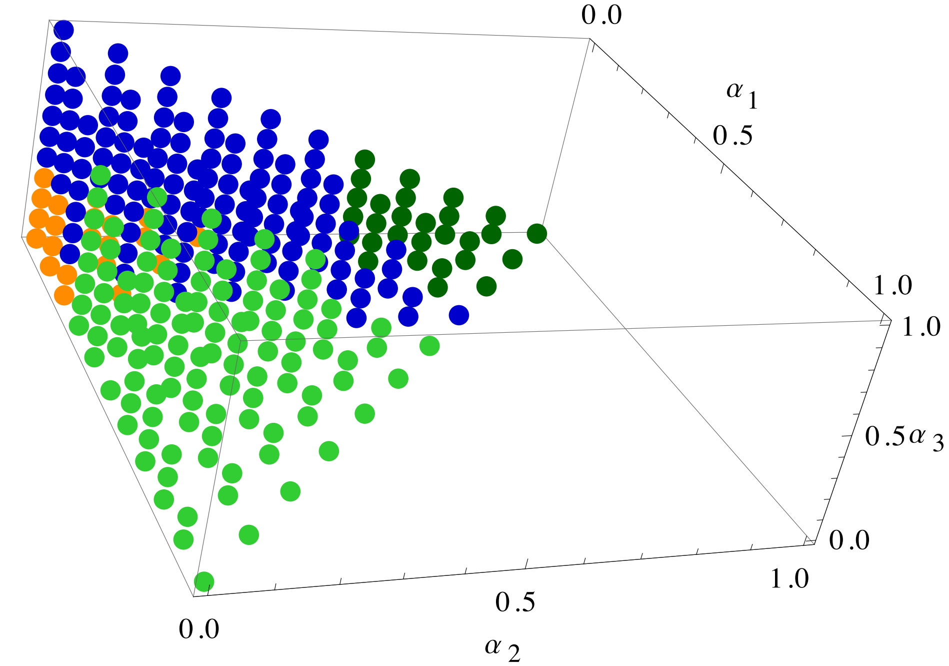
[BD, Geiller 2014, BD, Geiller to appear]

- This new construction allows to expand loop quantum gravity around different vacua corresponding to the different phases (fixed points) for spin foams.

- Ashtekar-Lewandowski representation: peaked on completely degenerate spatial geometry, maximal fluctuation in conjugated variable: difficult to built up states corresponding to smooth geometry
- New representation [BD, Geiller 2014] : peaked on flat connection, maximal fluctuation in spatial geometry. Corresponds to the physical state in (2+1)D gravity and in general to condensate states with respect to Ashtekar-Lewandowski representation.

- Facilitates construction of states corresponding to smooth geometries and should be useful for discussions of i.e. black hole entropy in loop quantum gravity [Sahlmann 2011].
- New representation much easier to interpret geometrically: new handle on the dynamics of the theory. [need substitute of Thiemanns (1996) quantization of Hamiltonian constraints]

Phase transitions?



- This new construction allows to expand loop quantum gravity around different vacua corresponding to the different phases (fixed points) for spin foams.

- What about phase transitions ('non-trivial' fixed points)?
- Instead of topological theory expect conformal theory / propagating degrees of freedom.

- Expect such fixed point amplitudes to be non-local ⟶ non-local embedding maps

  (also given by time evolution?)

- Do we regain triangulation independence / diffeomorphism symmetry there?

Summary

Coarse graining is about refining boundary states.

Tensor network methods construct embeddings of coarse states, representing "low energy/coarse degrees of freedom", into continuum Hilbert space.

Summary and Outlook:

Quantum Space Time Engineering

- We are on a good way to understand the 'many body physics' of spin foams and loop quantum gravity.
- Corresponds to the 'continuum limit' of these models.

- In the path integral approach (spin foams): mapping out the phase diagrams
  - connections to condensed matter physics / (new?) topological field theories and phases
  - challenge: going to higher dimensions
  - challenge: understanding symmetries of phase transition fixed points

- In the canonical approach (loop quantum gravity): expanding around different vacua
  - facilitates investigation/construction of physical states describing extended/smooth geometries
  - each vacuum comes with its own set of excitations: investigate dynamics of these.

Thank you!