Global Completeness of Quantum Gravity

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Introduction

- Quantum Gravity and Asymptotic Safety
- Earlier results

Our setup

- Present truncation
- Anomalous dimensions, gravitational coupling and cosmological constant
- Constant parts of vertices

Results

Summary

"Ah, gravity - thou art a heartless bitch."

- well-known: GR is perturbatively **non-renormalizable**
- how do we know?

$$S_{EH} = \frac{1}{16\pi G_N} \int \sqrt{-\det g}(R - 2\Lambda)$$

$$\Rightarrow [G_N] = -2$$

- what now?
- $\longrightarrow$ **non-perturbative techniques**

FRG

- use effective average action and the Wetterich equation

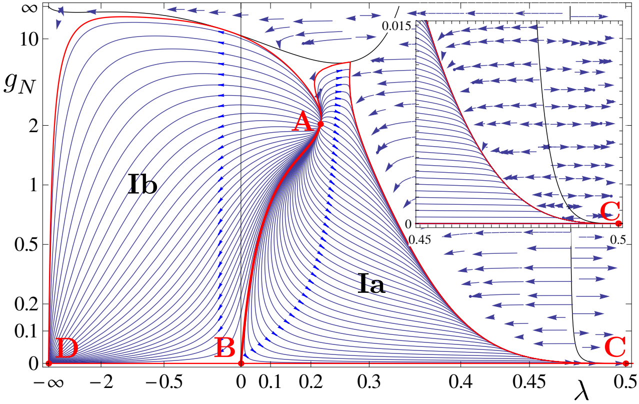
$$\dot{\Gamma}_k = \frac{1}{2}\mathrm{STr}\frac{\dot{R}_k}{\Gamma_k^{(2)} + R_k}$$

- look for UV fixed point which renders theory finite, check stability by improving truncation

Earlier results

- UVFP indeed exists in Einstein-Hilbert truncation
- several authors confirmed this result within larger truncations, e.g.
  - higher powers of R
  - f(R) gravity
  - coupling to matter (fermions, scalars, gauge fields, dilaton)
  - arbitrary dimensions
  - ...

Earlier results



[N.C., D. F. Litim, J.M.P., A.R. '12]

We are not done yet!

- some points of worry:
    - up to now: no higher order derivatives, i.e. momentum dependence $\rightarrow$ crucial in the UV!
    - in standard approach: split metric g in background $\overline{g}$ and fluctuation h, then evaluate flow equation $\rightarrow$ unphysical background dependence

$$\left.\frac{\delta^2\Gamma_k[\overline{g},h]}{\delta h^2}\right|_{h=0} \neq \frac{\delta^2\Gamma_k[\overline{g},0]}{\delta\overline{g}^2}$$

      (no split symmetry due to Nielsen identity)
    - n-graviton vertices
    - IR completeness (aka: do we exist?)

Key features of our setup

- vertex expansion of the effective action:

$$\Gamma_k[\bar{g},\phi] = \sum \frac{1}{n!}\Gamma_k^{(n)}[\bar{g},0]\phi^n$$

- classical tensor structures (i.e. from EH action with gauge fixing and ghosts)
- rescalings of the fields as

$$h \to \sqrt{G_k Z_{h,k}}\,h,\ (c,\bar{c}) \to \sqrt{Z_{c,k}}(c,\bar{c})$$

reason: canonical kinetic term, correct RG scaling properties of vertices

Key features of our setup

- form of vertex functions:

$$\Gamma_k^{(\phi_1\ldots\phi_n)} = \left(\prod_{i=1}^{n} \sqrt{Z_{\phi_i,k}}\right) G_{n,k}^{\frac{n}{2}-1} T_k^{(n)}(\Lambda_{n,k})$$

in particular, two-point function:

$$\Gamma^{(2h)}_{\mu\nu\rho\sigma} = Z_{h,k}(p^2 - 2\Lambda_{2,k})\mathcal{T}_{\mu\nu\rho\sigma}$$

$\rightarrow M_k^2 \equiv -2\Lambda_{2,k}$ is effective graviton mass parameter

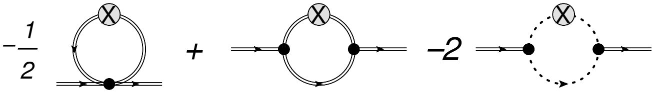
- momentum-dependent wave function renormalizations:

$$Z_{\phi,k} \rightarrow Z_{\phi,k}(p^2)$$

Key features of our setup

- to avoid background dependence of standard flow: use flow equation for two-point function → schematically,

$$\dot{\Gamma}^{(2h)} = -\frac{1}{2}\Gamma^{(4h)} G_h \dot{R}_h G_h + \Gamma^{(3h)} G_h \Gamma^{(3h)} G_h \dot{R}_h G_h \\ - 2\Gamma^{(h\bar{c}c)} G_c \Gamma^{(h\bar{c}c)} G_c \dot{R}_c G_c$$



- identify cosmological constant $\Lambda \equiv \Lambda_{1,k}$ from one-point function
- close the system with running of gravitational coupling from geometric approach: $\forall n$, $G_{n,k} \equiv G_{geo,k}$

Key features of our setup - Summary

- parameters of our theory:
  - gravitational coupling $g = G_N\, k^2$
  - effective graviton mass $\mu = M^2/k^2$ ‖ NEW
  - momentum-dependent anomalous dimensions $\eta_\phi(p^2) = -\frac{\dot{Z}_\phi(p^2)}{Z_\phi(p^2)}$ for h and $c/\bar{c}$ ‖ NEW
  - cosmological constant $\lambda = \Lambda/k^2$
  - higher order vertex constant parts $\lambda_n = \Lambda_n/k^2$ (need ansatz for these) ‖ NEW
- using flow of $\Gamma^{(2h)} = Z_h(p^2)(p^2 + \mu), \Gamma^{(\bar{c}c)} = Z_c(p^2)p^2$, can employ flat metric as expansion point

Anomalous dimensions

- general structure of flow equations of two-point functions:

$$-\eta_h(p^2)(p^2+\mu)+\dot{\mu}+2\mu = \frac{\dot{\Gamma}^{(2h)}(p^2)}{Z_h(p^2)}$$
$$-\eta_c(p^2) = \frac{\dot{\Gamma}^{(\bar{c}c)}(p^2)}{Z_c(p^2)p^2}$$

- RHS only depend on η_ϕ, not on Z_ϕ

Anomalous dimensions

- resulting flow equation for mass parameter:

$$\dot{\mu} = -2\mu + \frac{\dot{\Gamma}^{(2h)}(-\mu)}{Z_h(-\mu)}$$

- coupled set of integral equations for anomalous dimensions:

$$\eta_h(p^2) = -\frac{\frac{\dot{\Gamma}^{(2h)}(p^2)}{Z_h(p^2)} - \frac{\dot{\Gamma}^{(2h)}(-\mu)}{Z_h(-\mu)}}{p^2 + \mu}[\eta_h, \eta_c]$$

$$\eta_c(p^2) = -\frac{\dot{\Gamma}^{(\bar{c}c)}(p^2)}{Z_c(p^2)p^2}[\eta_h, \eta_c]$$

Geometric flow equations

- geometric approach allows for diff-invariant flow equations for gravitational coupling directly from the effective action, without unphysical background dependence
- distinguishes background and dynamical coupling
- dynamical coupling enters other flow equations
- physical observables (S-matrix) are constructed from background coupling
- incorporation: solve system with dynamical coupling, calculate flow of background coupling on the solution
- details: [Donkin, J.M.P. '12]

Cosmological constant

- running cosmological constant from one-point function $\Gamma^{(h)}$
- $\Gamma^{(h)}$ does not enter RHS of any flow equation $\rightarrow$ flow completely decouples
  $\rightarrow$ calculate flow of cosmological constant on the solution of other flow equations

λ_n - a divergence analysis

$$\Gamma^{(nh)} \sim Z_h^{n/2}(p^2) g^{n/2-1}(p^2 - 2\lambda_n)$$

- crucial point: behaviour of λ_n near $\mu = -1$
- EH truncation is no consistent truncation near this point
- ansatz: $\lambda_n \propto (1+\mu)^{\alpha_n}$
- compare/match degree of divergence on both sides of flow equations for λ_n

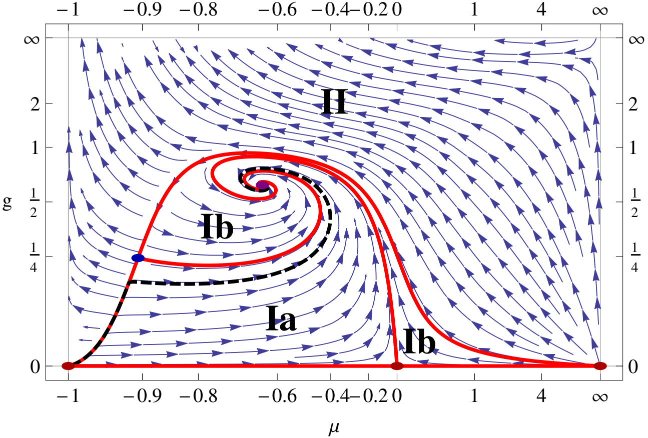
λ_n - a divergence analysis

- find recursion relation with undetermined α_3 and α_4:

$$\alpha_{2n} = (n-1)\alpha_4 - (n-2)$$
$$\alpha_{2n+1} = \alpha_3 + (n-1)\alpha_4 - (n-1)$$
$$\alpha_4 \leq 2\alpha_3 - 1$$

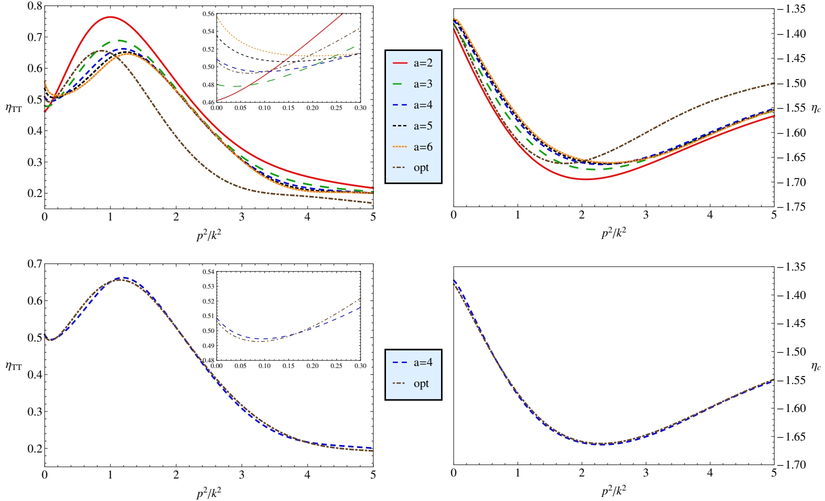
- in particular, $\alpha_n < 0$
- results here: restrict to equality and $\alpha_3 \sim -0.1$

Phase diagram

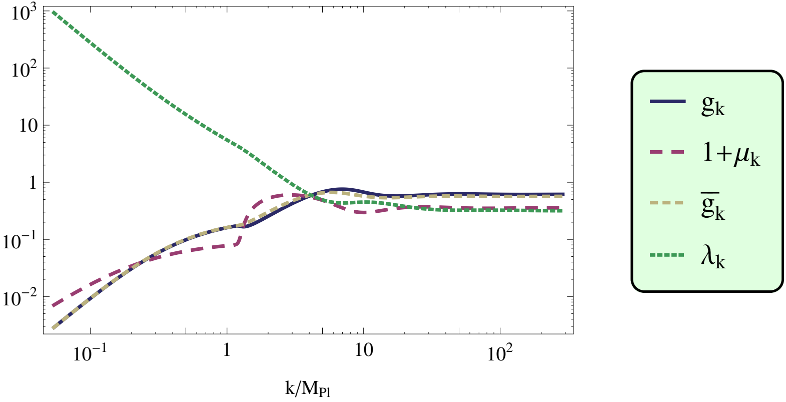


regulator used: $R(x) = (2e^{x^a} - 1)^{-1}$ with $a = 4$

Anomalous dimensions at UVFP



Running of couplings



$g, \overline{g} \propto k^2$, $\lambda \propto k^{-2} \Rightarrow G_N, \Lambda$ are constants!

Summary

- globally complete phase diagram
- UVFP, physical couplings have real critical exponents
- EH truncation is inconsistent near divergent line $\rightarrow$ consistent scaling of vertices necessary for IRFP
- classical scaling at IRFP
- fully momentum-dependent anomalous dimensions