Asymptotically Safe Starobinsky Inflation

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together with
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arXiv: 1311.0881 [gr–qc]

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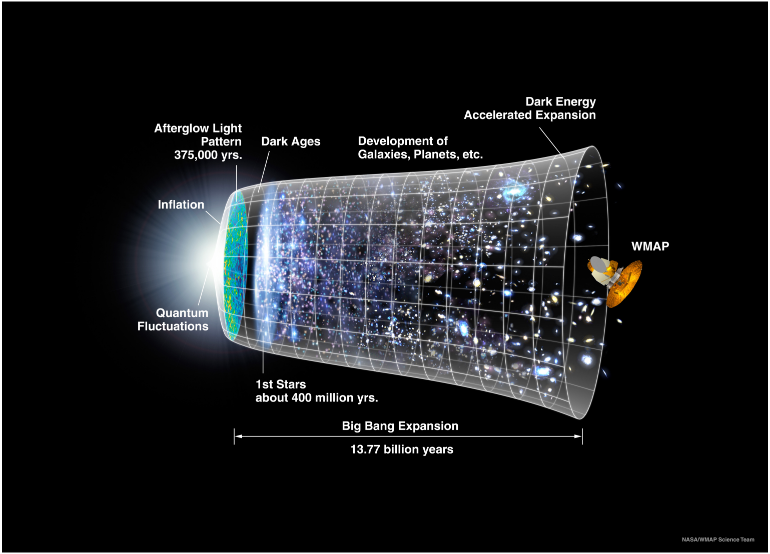
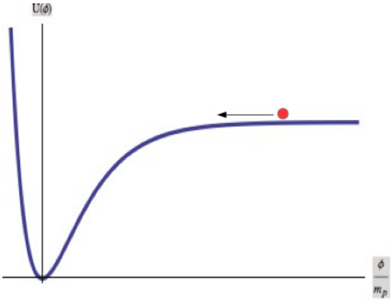
I will be explaining about ...

- ... primordial inflation and the observational success of the Starobinsky model
- ... embedding Starobinsky inflation within the scenario of Asymptotic Safety for quantum gravity
- ... asymptotic freedom as a key ingredient for an observationally viable Starobinsky inflation in this context


Dark Energy
Accelerated Expansion
Afterglow Light
Pattern
375,000 yrs.
Dark Ages
Development of
Galaxies, Planets, etc.
Inflation
WMAP
Quantum
Fluctuations
1st Stars
about 400 million yrs.
Big Bang Expansion
13.77 billion years
NASA/WMAP Science Team

Inflation and the origin of structure

- Inflation is a very rapid expansion of the Universe in the very early universe. A source producing **antigravity** effects is needed: A scalar field ϕ .



- Field ϕ slowly rolls down the potential:

$$\left(\frac{1}{a(t)}\frac{da(t)}{dt}\right)^2 \equiv H(t)^2 \simeq \frac{U(\phi)}{m_p^2} \gg \left(\frac{d\phi}{dt}\right)^2$$

- Spacetime is assumed to be homogeneous and isotropic:

$$ds^2 = -dt^2 + a(t)^2 d\mathbf{x}^2$$

- Universe expands in accelerating manner: $\frac{d^2}{dt^2}a(t) > 0$

- Inflation provides with an explanation of the *initial conditions* for the large scale structure we observe today, using fundamental physics

- As of now, there does not exist a fully convincing inflationary model

Primordial quantum fluctuations

- **Quantum fluctuations** during inflation acted as the seed for the large scale fluctuations we observe today (i.e galaxies, clusters of galaxies, e.t.c)

$$\phi + \delta\phi \qquad \longleftrightarrow \qquad g_{\mu\nu} + \delta g_{\mu\nu}$$

$$\ddot{\delta\phi}(t) + 3H(t)\dot{\delta\phi}(t) + c_S^2 \frac{k^2}{a(t)^2}\delta\phi(t) = 0 \qquad \ddot{h}_{ij}^{TT}(t) + 3H(t)\dot{h}_{ij}^{TT}(t) + c_T^2 \frac{k^2}{a(t)^2} h_{ij}^{TT}(t) = 0$$

- The amplitudes and spectral indices of the power of scalar (P_{scalar}) and tensor (P_{tensor}) fluctuations respectivel are **observables** on the sky (Planck satellite, BICEP2, ...)

$$P_S|_{k=aH} \sim \frac{1}{\epsilon} G \times U(\phi) \qquad P_T|_{k=aH} \sim G \times U(\phi) \qquad [\epsilon \equiv -\frac{1}{H^2}\frac{dH}{dt}]$$

- The space dependence of the power spectra is described through the corresponding spectral indices

$$n_S - 1 \equiv \left.\frac{dP_S}{dlnk}\right|_{k=aH} \qquad n_T \equiv \left.\frac{dP_T}{dlnk}\right|_{k=aH}$$

Inflation and asymptotic safety

- The **value and running of the gravitational couplings** under the RG is important for the predictions and viability of inflationary models:

  $\rightarrow$ How much inflation?

  $\rightarrow$ How strong/weak the primordial, quantum fluctuations are?

  $\rightarrow$ Does the correct IR limit exist?

- Cosmological predictions of asymptotic safety are in principle **falsifiable** through the calculated values for the corresponding fixed point(s)/eigenvalue(s)

  $\rightarrow$ Scalar field inflation has been shown to be viable for particular choices of the scalar potential [1]

  $\rightarrow$ Inflation not viable in the fixed point regime for specific (higher order) truncations due to the small amount of e–foldings [2]

  $\rightarrow$ Within the Einstein–Hilbert truncation, accelerating solutions with sufficient number of e-foldings shown to exist in the UV, **but**[3],

$$(G \times \Lambda)_{\text{fixed point}} \sim \mathcal{O}(10^{-3}) \gg 10^{-10} \rightsquigarrow \text{Amplitude of tensor waves too large!}$$

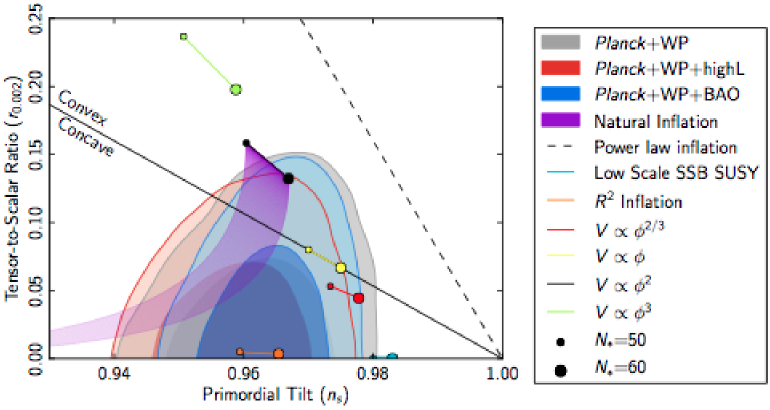
[1] A. Contillo, M. Hindmarsh, C. Rahmede (2012) & M.Hindmarsh, D. Litim, C. Rahmede (2011)

[2] M. Reuter & F. Saueressig (2005), arXiv: 0507167 [hep-th] | S. Weinberg (2010), arXiv:0911.3165 [hep-th]

[3] M. Hindmarsh & I.D.S (2013), arXiv:1203.3957 [gr-qc], Y. Cai & D. Easson (2011), arXiv:1107.5815

see also: A. Bonanno, A. Contillo, R. Percacci (2011) arXiv:1006.0192

Why Starobinsky inflation?



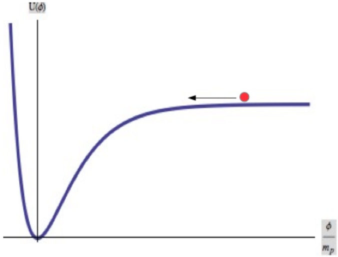
$$S = \int d^4x\sqrt{-g}\left(\frac{1}{16\pi G}R + \frac{1}{b}R^2\right)$$

- Excellent agreement with the CMB data according to the Planck satellite results!
- Recently revisited also by many authors in a a supergravity context [4]

[4] S. Ferrara, R. Kallosh, A. Van Proeyen (2013), J. Ellis, D. V. Nanopoulos, and K. A. Olive (2013), J. Alexandre N. Houston, N. Mavromatos (2013)

The mechanism behind Starobinsky inflation

$$\int\sqrt{-g}\left(\frac{1}{16\pi G}R+\frac{1}{b}R^2\right)\xleftrightarrow{\tilde{g}_{\alpha\beta}=g_{\alpha\beta}e^{\sqrt{2/3}\frac{\phi}{m_p}}}\int\sqrt{-\tilde{g}}\left(\frac{R}{16\pi G}+\frac{1}{2}(\partial\phi)^2+U(\phi)\right)$$



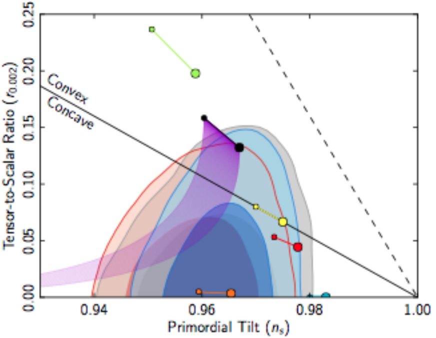
$$\frac{U(\phi)}{m_p^4}\sim\quad b\times\left(1-e^{-\sqrt{2/3}\frac{\phi}{m_p}}\right)^2$$

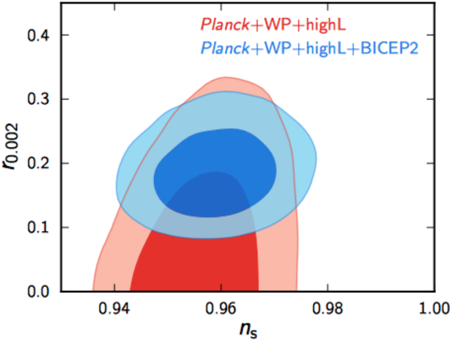
- Domination of the R^2 term inflates the universe
- The coupling b is our order parameter: Control on the magnitude of the primordial fluctuations produced during inflation
- Significance of quantum corrections for the dynamics of the model? [5]

[5] Notice that, the study of quantum corrections in this work is performed in the Jordan frame.

The rise and fall (?) of Starobinsky inflation

The Planck satellite [6] (left) and BICEP2 [7] (right) experiments put strong bounds on inflationary models





The BICEP2 results disfavour the Starobinsky model, but they still have to be confirmed with the upcoming Planck satellite CMB polarisation results

[6] P. Ade et al. (Planck Collaboration), (2013), arXiv:1303.5082 [astro-ph.CO].

[7] The BICEP2 Collaboration, (2014), arXiv: 1403.3985 [astro-ph]

The key points and questions so far

- The Starobinsky model is the most favoured one according to the Planck satellite results
- Can we embed the Starobinsky inflationary model within Asymptotic Safety?
- What is the fixed point structure and the associated RG dynamics for the Starobinsky action?
- Can we explain the initial conditions for the two couplings of the Starobinsky action needed for a viable inflation in this context?

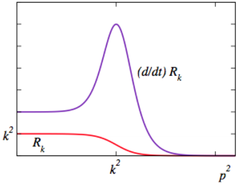
Calculating quantum corrections

- An **Exact Renormalisation Group Equation (ERGE)** for the scale dependent effective action Γ_k [8]

$$\frac{\partial}{\partial \ln k}\Gamma_k = \frac{1}{2}\mathrm{Trace}\left[\left(\Gamma_k^{(2)} + R_k\right)^{-1} \cdot k\partial_k R_k\right]$$

$$\delta\Gamma_k^{(2)} := \int \delta\phi \cdot \Gamma_k^{(2)} \cdot \delta\phi$$

R_k := k-dependent regulator suppressing momenta with $p^2 < k^2$



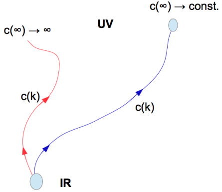
- **An effective action ansatz**,

$$\Gamma_k = \int d^4x\sqrt{g}\sum_i c_i(k) \cdot \mathcal{O}_i[g_{\alpha\beta}, \partial g_{\alpha\beta}, \partial^2 g_{\alpha\beta}, \ldots]$$

- **Solution:** A *family of effective actions* Γ_k smoothly connected from UV ($k \to \infty$) to IR ($k \to 0$)

$$\frac{d}{d\ln k}c_i(k) = \beta_i(c_1, c_2, \ldots, c_j)$$

Existence of an **attractive, non–trivial UV fixed point** under the RG, ensures that the couplings do not diverge as $k \to \infty$: Theory is **"Asymptotically Safe"** (S. Weinberg, 1979)



[8] C. Wetterich Phys. Lett. **B 301**, 90 (1993) | M. Bonini, M. D Attanasio and G. Marchesini, Nucl. Phys. B 409 (1993) 441 T. R. Morris, Int. J. Mod. Phys. A 9 (1994) 2411.

The RG flow of the Starobinsky action

- The **starting point** is the exact RG equation for the effective action:

$$\frac{\partial}{\partial \ln k}\Gamma_k[g_{\mu\nu}] = \frac{1}{2}\mathrm{Tr}\left[\left(\Gamma_k^{(2)} + R_k\right)^{-1} k\partial_k R_k\right]$$

$$\Gamma_k = \int_{S^4} d^4x\sqrt{g}\left(-\frac{1}{16\pi G_k}R + \frac{1}{b_k}R^2\right) \equiv \int_{S^4} d^4x\sqrt{g}\, f(R,k)$$

- The exact RG equation is evaluated under the following **assumptions**:
  - Background space-time: a Euclidean sphere S^4
  - A choice of gauge: Landau type gauge
  - A choice for the form of the regulator R_k: Opitimised (Litim's) cut–off [9]

- Evaluation of the trace on the r.h.s of the exact RG equation yields the flow equation [10]

$$384\pi^2\left(\partial_\mu\tilde{f} + 4\tilde{f} - 2\tilde{R}\tilde{f}_{\tilde{R}}\right) = \underbrace{\frac{d\Gamma_k}{d\mu}\left[\tilde{f}, \tilde{f}_{\tilde{R}}, \tilde{f}_{\tilde{R}\tilde{R}}, \partial_\mu\widetilde{G}_k, \partial_\mu b_k\right]}_{\text{non–trivial function of couplings and their RG derivatives}}$$

$$\partial_\mu \equiv \frac{\partial}{\partial \ln k}$$

$$\tilde{f} \equiv \frac{f}{k^4}, \tilde{R} \equiv \frac{R}{k^4}, \widetilde{G} \equiv k^2 G$$

- The beta functions are extracted by comparing the two sides of the flow equation

$$\frac{d}{d\ln k}\widetilde{G}(k) = \beta_{\widetilde{G}}(\widetilde{G}, b), \qquad \frac{d}{d\ln k}b(k) = \beta_b(\widetilde{G}, b)$$

[9] D. Litim (2000) arXiv: 0005245 [hep–th]

[10] for an explicit calculation see P.F. Machado & F. Saueressig arXiv:0712.0445 [hep-th] | A. Codello, R. Percacci & C. Rahmede (2008) arXiv:0805.2909 [hep-th]

The (not so inspiring) form of the beta functions

The explicit form of the beta functions is quite complex ...

$$\frac{d}{d\ln k}\widetilde{G} = \beta_{\widetilde{G}}(\widetilde{G}, b) \equiv \frac{A_0(1-B_2)+A_2B_0}{1-A_1-B_2+A_1B_2-A_2B_1}$$

$$\frac{d}{d\ln k}b = \beta_{\widetilde{G}}(\widetilde{G}, b) \equiv \frac{B_0(1-A_1)+A_0B_1}{1-A_1-B_2+A_1B_2-A_2B_1}$$

$$A_0 \equiv \frac{\widetilde{G}\left(b^3(144\pi-301\widetilde{G})+3456\pi b^2(17\widetilde{G}-8\pi)\widetilde{G}+9216\pi^2 b(144\pi-323\widetilde{G})\widetilde{G}^2+17694720\pi^3\widetilde{G}^4\right)}{72\pi b(b-96\pi\widetilde{G})^2}$$

$$A_1 \equiv \frac{4\widetilde{G}\left(b^3-225\pi b^2\widetilde{G}+15840\pi^2 b\widetilde{G}^2-276480\pi^3\widetilde{G}^3\right)}{9\pi b(b-96\pi\widetilde{G})^2}$$

$$A_2 \equiv \frac{16\widetilde{G}^3\left(b^2-200\pi b\widetilde{G}+7680\pi^2\widetilde{G}^2\right)}{b^2(b-96\pi\widetilde{G})^2}$$

$$B_0 \equiv -\frac{491b^5-157088\pi b^4\widetilde{G}+18275328\pi^2 b^3\widetilde{G}^2-916586496\pi^3 b^2\widetilde{G}^3+17694720000\pi^4 b\widetilde{G}^4-135895449600\pi^5\widetilde{G}^5}{2880\pi^2(b-96\pi\widetilde{G})^3}$$

$$B_1 \equiv -\frac{-89b^5+31818\pi b^4\widetilde{G}-4328064\pi^2 b^3\widetilde{G}^2+276203520\pi^3 b^2\widetilde{G}^3-8493465600\pi^4 b\widetilde{G}^4+101921587200\pi^5\widetilde{G}^5}{4320\pi^2\widetilde{G}(b-96\pi\widetilde{G})^3}$$

$$B_2 \equiv \frac{\widetilde{G}\left(731b^4-222912\pi b^3\widetilde{G}+24247296\pi^2 b^2\widetilde{G}^2-1150156800\pi^3 b\widetilde{G}^3+16986931200\pi^4\widetilde{G}^4\right)}{720\pi b(b-96\pi\widetilde{G})^3}$$

The fixed point structure of the Starobinsky action

The theory can be asymptotically safe/free only if the appropriate fixed point(s) of the corresponding RG flow exists

- On a fixed point, the beta functions equal to zero

$$\beta_{\widetilde{G}}(\widetilde{G}, b) = 0, \quad \beta_b(\widetilde{G}, b) = 0$$

- For the Starobinsky action, we find a Gaussian ("free") and two non–trivial UV fixed points

	Fixed Points	Associated eigenvalues
Gaussian fixed point	$(\widetilde{G}, b) = (0, 0)$	$(\lambda_{\widetilde{G}}, \lambda_b) = (2, 0)$
Asympt. Safe UV fixed point	$(\widetilde{G}, b)_1 = (2.451, 914.57)$	$(\lambda_{\widetilde{G}}, \lambda_b)_1 = (-39.79, -2.71)$
Asympt. Safe/Free UV fixed point (!)	$(\widetilde{G}, b)_2 = (24\pi/17, 0)$	$(\lambda_{\widetilde{G}}, \lambda_b)_2 = (-102/41, -2)$

- The Gaussian, ("free") fixed point: the regime where perturbation theory is usually applied ("small coupling" regime). The associated eigenvalues correspond to the canonical mass dimension of the coupling, implying that quantum corrections are turned off.

- The two UV fixed points: non–trivial, UV-attractive fixed points of the RG flow, ensuring the absence of divergences as the infinite cut–off limit is taken ($k \to \infty$).

- The asymptotically safe and free fixed point will be crucial for the realisation of Starobinsky inflation. The **large eigenvalue ($= -2$)** for the coupling b declares that the fixed point is of a non–perturbative nature.

1–loop beta functions and analytic solutions

- The asymptotically safe/free fixed point provides us with a small parameter at high energies (the coupling b). We can expand the RG flow in the Planck regime,

$$\text{At} \quad k \sim m_p : \quad b \ll 1, \quad \widetilde{G} \equiv k^2 G \lesssim 1$$

- The beta functions acquire an 1–loop form which captures all the essential features of the RG flow needed to study inflation,

$$\frac{d}{d\ln k}\widetilde{G} \simeq 2\widetilde{G} - \frac{41}{36\pi}\widetilde{G}^2 + \mathcal{O}(\widetilde{G}^3, b) \qquad \widetilde{G}(k) \simeq \frac{\widetilde{G}_0}{1 + \frac{41\widetilde{G}_0}{72\pi}\left(\frac{k}{k_0}\right)^2}\left(\frac{k}{k_0}\right)^2$$

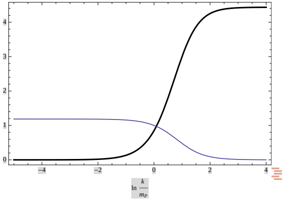
$$\frac{d}{d\ln k}b \simeq -\frac{41}{36\pi}\widetilde{G}b + \mathcal{O}(\widetilde{G}^2, b^2) \qquad b(k) \simeq \frac{b_0}{1 + \frac{41\widetilde{G}_0}{72\pi}\left(\frac{k}{k_0}\right)^2}$$

  - k_0 := an arbitrary reference scale
  - $\widetilde{G}_0, b_0$:= integration contstants to be fixed through appropriate renormalisation conditions
  - The high/low energy limits:

As $k/k_0 \to \infty$, $b \to 0$, and $\widetilde{G} \to 72\pi/41$ [i.e Asymptotically free and safe respectively]

As $k/k_0 \ll 1$, $b \to b_0 = \text{const.}$, and $\widetilde{G} \simeq \widetilde{G}_0 (k/k_0)^2$ or $G \simeq \widetilde{G}_0/k_0^2 = \text{const.}$ [Classical regime]

RG evolution from the UV to IR: The numerical solution for $\widetilde{G}(k)$ and $b(k) \times 10^8$ is in very good agreement with the analytic 1–loop one



Setting the renormalisation conditions

- We can conveniently choose in the 1–loop solutions, for the reference scale $k_0 = m_p$:

$$\widetilde{G}(k) \simeq \frac{\widetilde{G}_0}{1+\frac{41\widetilde{G}_0}{72\pi}\left(\frac{k}{m_p}\right)^2}\left(\frac{k}{m_p}\right)^2 \qquad b(k) \simeq \frac{b_0}{1+\frac{41\widetilde{G}_0}{72\pi}\left(\frac{k}{m_p}\right)^2},$$

$b_0 \ll 1$ (solutions are derived around $b = 0$)

- Determining the value of the initial conditions $\tilde{G}_0, b_0$ requires matching with observations, i.e setting appropriate renormalisation conditions at the scale k where the corresponding measurement of the coupling is performed

- Each of the two couplings has to be matched with the observations at different scales:
  - Newton's coupling G is measured at earth/solar scales

    At scales $k \sim k_{solar} \ll k_0 = m_p\ :\ G = \widetilde{G}(k)k^{-2} = 1/m_p^2 \rightarrow \quad \widetilde{G}_0 \simeq 1$.

  - The R^2 coupling $b(k)$ is responsible for setting the scale of inflation: b_0 will be determined from CMB observations

    At scales $k \sim k_{inflation} \ll k_0 = m_p\ :\quad b = b_{CMB} \simeq b_0$

A coarse grained view of the cosmological evolution

- The scale dependent, Lorentzian Starobinsky action,

$$S = \int d^4x \sqrt{-g} \left(\frac{1}{16\pi G(k)} R + \frac{1}{b(k)} R^2 \right)$$

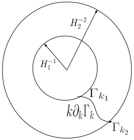
- The RG–improved action defines a family of actions dependent on the cut–off k

What is the value of the cut–off scale during inflation? [11]

Think of the universe's horizon as defining the typical scale of correlations between different degrees of freedom

$$k^2 \sim H^2 \sim R$$

⇝ Integrating out degrees of freedom proceeds along with the expansion of the universe



- We expect inflation to occur at scales well below the Planck mass, $k^2/m_p^2 \sim R/m_p^2 \ll 1$

$$k^2 G(k) \equiv \widetilde{G}(k) \simeq \underbrace{\frac{1}{1 + 41\widetilde{G}_0/72\pi \left(k/m_p\right)^2} \left(k/m_p\right)^2}_{\Rightarrow\ G \,\simeq\, 1/m_p^2}, \qquad b(k) \simeq \underbrace{\frac{b_0}{1 + \frac{41\widetilde{G}_0}{72\pi} \left(k/m_p\right)^2}}_{\Rightarrow\ b \,\simeq\, b_0 \,=\, const.}$$

⇝ The RG-improved Starobinsky action then takes the standard form

$$S \simeq \int d^4x \sqrt{-g} \left[\frac{m_p^2}{16\pi} R + \frac{1}{b_0} R^2 + \mathcal{O}\left(\frac{1}{b_0} \frac{R^3}{m_p^2} \right) \right]$$

[11] A. Bonanno, A. Contillo, R. Percacci (2011) arXiv:1006.0192 | V. Frolov, J. Guo (2011), arXiv:1101.4995 | A. Bonanno (2012), arXiv:1203.1962 [hep-th] | M.Hindmarsh & IDS (2012), arXiv: 203.3957 M. Reuter & F. Saueressig (2005), arXiv: 0507167 | Y. Cai & D. Easson (2011), arXiv:1107.5815

Slow-roll inflationary dynamics

$$S \simeq \int d^4x\sqrt{-g}\left(\frac{m_p^2}{16\pi}R + \frac{1}{b_0}R^2\right) \equiv \int d^4x\sqrt{-g}f(R), \qquad ds^2 = -dt^2 + a(t)^2dx^2$$

- The Friedmann equation for a generic $f(R)$ model takes the form

$$\frac{f_R}{H^2}(1-\epsilon) + 6f_{RR}\left(4\epsilon + \frac{\dot\epsilon}{H} - 2\epsilon^2\right) - \frac{1}{6}\frac{f}{H^4} = 0, \qquad [\dot{} \equiv d/dt, \quad f_R \equiv df(R)/dR, \quad \epsilon \equiv -\dot H/H^2]$$

  - We are interested in slow–roll, (quasi-) de Sitter solutions of the background dynamics,

$$\dot\epsilon, \epsilon^2 \ll \epsilon \ll 1$$

  - In the slow–roll regime, the Hubble parameter evolves as

$$H(t) \simeq H_0 - \frac{1}{576\pi}b_0 m_p^2(t - t_0) \quad \left(\rightsquigarrow \epsilon \simeq \frac{1}{36}\frac{m_p^2/16\pi}{1/b_0}\frac{1}{H(t)^2}\right)$$

- The amount of inflation is quantified by the number of e-foldings N

$$N \equiv -\int_H^{H_{\mathrm{end}}} \frac{d\log H}{\epsilon} \simeq \frac{288\pi}{b_0 m_p^2}(H^2 - H^2_{\mathrm{end}}) \simeq \frac{1}{2\epsilon} \quad \left(H^2 \gg H_{end}\right)$$

Generation of linear primordial fluctuations

- The small fluctuations of the metric field during inflation are expected to produce the observed temperature anisotropies in the CMB sky [12]

  During slow-roll: $\rightsquigarrow\ f_R \simeq 24/b_0 H^2,\ \epsilon \simeq 1/2N$

Theory	*Planck* data at the pivot scale $0.002\mathrm{Mpc}^{-1}$
$P_{scalar} \simeq \frac{1}{48\pi^2}\frac{H^2}{f_R}\frac{1}{\epsilon^2} \simeq \frac{N^2}{288\pi^2} b_0$	$\ln(10^{10} P_{scalar}) = 3.089^{+0.024}_{-0.027}$
$P_{tensor} \simeq \frac{1}{\pi^2}\frac{H^2}{f_R} \simeq \frac{1}{24\pi^2} b_0$	—
$r \equiv \frac{P_{tensor}}{P_{scalar}} \simeq 48\epsilon^2 \simeq \frac{12}{N^2}$	$r_{0.002} < 0.12\quad (\rightsquigarrow \epsilon < 0.009)$
$n_{scalar} - 1 \simeq -4\epsilon, \quad n_{tensor} \simeq 0$	$n_{scalar} = (0.9603 \pm 0.0073)$

- The data place a bound on the Hubble parameter H during inflation which in turn places a bound on the cut–off scale k

$$\frac{H}{m_p} \sim \frac{k}{m_p} \lesssim \mathcal{O}(1) \times 10^{-5}$$

- For $N = 55$ e-foldings and the *Planck* bound for P_S, we extract the **renormalisation condition for the coupling** b,

$$b_0 \simeq b\left(k \sim 10^{-5}\right) \simeq \mathcal{O}(1) \times 10^{-9}$$

- The approximate constancy of the coupling b for scales well below the Planck mass ($k/m_p \ll 1$), is crucial for the approximate constancy of the spectral index

$$\text{1–loop beta function for b} \rightsquigarrow \left| k\frac{db}{dk} \right| \simeq 10^{-9}\widetilde{G} \ll 1$$

[12] see for example: V.F. Mukhanov, H.A. Feldman, R.H. Brandenberger (1992)

Putting all pieces together

- The **scale of inflation** in the Starobinsky model is set by the R^2 coupling b: The coupling has to be tuned to be small for small primordial fluctuations to remain small.

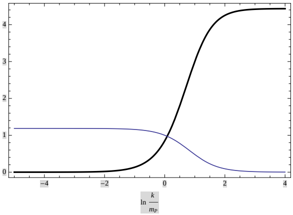
- The **asymptotic freedom** of the R^2 coupling in the UV provides us with a natural way of realising Starobinsky inflation:
$\rightsquigarrow$ The coupling can be made naturally small for a wide range of energy scales. What is more, its constancy for energies below m_p ensures negligible running of the spectral indices

- Newton's coupling is **asymptotically safe** in the UV, and its running correctly recovers GR in the IR

- The **bound** on the R^2 coupling at solar scales based only on classical considerations is very weak: $b > 10^{-115}$.
$\rightsquigarrow$ The renormalisation condition of the R^2 coupling for a viable inflation, combined with the coupling's constancy in the IR, provides with a much stronger bound on the coupling b at classical scales

$$b_{IR} \sim 10^{-9}$$

- *As long as in the UV the couplings start close to $b \ll 1$ and $\widetilde{G} \simeq \mathcal{O}(1)$, the coupling b remains small, and the RG flow is stable and attracted towards the IR. The CMB observations specify the precise order of the coupling b for a viable inflation, which is consistent for a wide range of scales along the RG flow.*



What if the BICEP2 results are confirmed?

- It will be the first experimental confirmation of quantum gravity!
- Starobinsky inflation disfavoured, but a scale of inflation $\sim 10^{-2} m_p$ implies that the effect higher order curvature operators should not be naively ignored

$$\Gamma_k = \int d^4x\sqrt{g}\left(f(R,\phi)R + \frac{1}{b(k)}R^2 + \ldots + C_{\alpha\beta\gamma\delta}C^{\alpha\beta\gamma\delta} + \frac{1}{2}(\partial\phi)^2 + U(\phi)\right)$$

Some of the challenges for Asymptotic Safety:

- Are there viable RG trajectories connecting the UV fixed point with the IR, yielding on the same time observationally viable inflationary periods at the scale predicted by BICEP2, for higher order truncations?
- How important are the quantum gravity corrections for inflationary observables for energies close to the Planck scale?
- Can inflation be realised purely in the curvature sector beyond the Starobinsky truncation?

Conclusions

- For the Starobinsky action, an **attractive, UV fixed point** exists under the RG, where Newton's coupling is asymptotically safe, and **the R^2 coupling asymptotically free**.

- The **asymptotic freedom** of the R^2 coupling, $b(k \to \infty) = 0$, ensures that the universe enters into a de Sitter-like expansion at some sufficiently high cut–off scale, as the R^2 term comes to dominate the action $(1/b(k) \gg 1)$

- The fixed point further provides us with a mechanism of naturally producing **small primordial fluctuations**, and inflation can occur for a wide range of cut–off scales: from the GUT scale down to the electroweak scale. The CMB data provide us with the appropriate renormalisation condition for the coupling b along the RG flow

- The RG evolution is **stable** and connects smoothly the UV with the IR regime

Thank you!