Asymptotic Safety & Background Independence

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Outline

Asymptotic Safety & Background Independence

Asymptotic Safety (UV)

Non-perturbative renormalizability :$\Leftrightarrow$

- Existenz of a (non-trivial) UV-fixed point of FRGE ...

  Evidence for topology-, field-, gauge group-, constraints changes ✓

- ... with finite dimensional critical hypersurface $\mathscr{S}_{\text{UV}}$

  Tests: $f(R)$-truncations ✓

Background Independence

In quantum gravity spacetime is dynamical $\Rightarrow$ Background Independence

In practice two different approaches:

1. No background at all $\Rightarrow$ CDT, LQG, ...

   Construct spacetime out of 'Nothing'

2. Generic background $\Rightarrow$ EAA

   Choose arbitrary background $\bar{g}_{\mu\nu}$ on which dynamical field $g_{\mu\nu}$ propagate

   Physical sector, $\Gamma_{k=0}$ should be independent on specific $\bar{g}$!

   $\Rightarrow$ Split-symmetry condition: $\Gamma^{\text{grav}}_{k=0}[g, \bar{g} + \delta\bar{g}] = \Gamma^{\text{grav}}_{k=0}[g, \bar{g}]$

A Global Requirement

Requirements on the FRGE flow for quantum gravity:

- In the **UV**: Asymptotic Safety (NGFP, $\dim \mathscr{S}_{\text{UV}} < \infty$)
- In the **IR**: Restoration of split-symmetry $\Gamma_{k=0}$

Question:

Existence of RG trajectories satisfying **both** conditions (global issue!)

Single-Metric & Bi-Metric Approximations

Functional Renormalization Group Equation

Nonlinear functional differential equation:

$$\partial_t \Gamma_k[g,\bar{g}] = \frac{1}{2}\mathrm{STr}\left\{\left(\Gamma_k^{(2)}[g,\bar{g}] + R_k[\bar{g}]\right)^{-1} \partial_t R_k[\bar{g}]\right\}$$

Evaluation steps:

1. **Functional:** Reduction to ODE for couplings (LHS & RHS)
2. **Hessian** $\Gamma_k^{(2)}$**:** w.r.t dynamical fields, $g_{\mu\nu}$, ξ, $\bar{\xi}$ (RHS)
3. **Nonlinear:** Invert operator (RHS)
4. **Trace** STr**:** heat kernel techniques, . . . (RHS)

The Bi-Metric Einstein-Hilbert Truncation

1. Effective Average Action $\Gamma_k[g, \bar{g}]$:

   (Invariants up to second order in the derivatives)

$$\partial_t \Gamma_k[g, \bar{g}] = \partial_t \int \sqrt{\bar{g}} \left(\Lambda_k^{\mathsf{B}} Z_k^{\mathsf{B}} + Z_k^{\mathsf{B}} \bar{R} + Z_k^a R \right) + \sqrt{g} \left(\gamma_k^c + Z_k^c \bar{R} + Z_k^d R \right) + \partial_t \sum_{n \neq 0} \gamma_k^{(n)} \int \left(\frac{\sqrt{\bar{g}}}{\sqrt{g}} \right)^n (\cdots) + \partial_t \Gamma_k^{\mathsf{gf}}[g, \bar{g}] + \partial_t \Gamma_k^{\mathsf{gh}}[g, \bar{g}, \xi, \bar{\xi}]$$

2. Hessian $\Gamma_k^{(2)}[g, \bar{g}]$:

3. Inversion:

4. Trace STr:

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2. Hessian $\Gamma_k^{(2)}[g,\bar{g}]$:

$$\Gamma_k^{(2)}[g,\bar{g},\xi,\bar{\xi}] = \begin{pmatrix} \Gamma_{k\,gg}^{(2)} & (\cdots)\bar{\xi} \\ (\cdots)\xi & (\cdots) \end{pmatrix}$$

3. Inversion:

4. Trace STr:

The Bi-Metric Einstein-Hilbert Truncation

1. Effective Average Action $\Gamma_k[g,\bar{g}]$:

(Invariants up to second order in the derivatives)

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$$+ \partial_t \sum_{n\neq 0} \gamma_k^{(n)} \int \left(\frac{\sqrt{\bar{g}}}{\sqrt{g}}\right)^n (\cdots) + \partial_t \Gamma_k^{\mathsf{gf}}[g,\bar{g}] + \cancel{\partial_t S^{\mathsf{gh}}[g,\bar{g},\xi,\bar{\xi}]}$$

2. Hessian $\Gamma_k^{(2)}[g,\bar{g}]$:

$$\Gamma_k^{(2)}[g,\bar{g},\xi,\bar{\xi}] = \begin{pmatrix} \Gamma^{(2)}_{k\,gg} & (\cdots)\bar{\xi} \\ (\cdots)\xi & (\cdots) \end{pmatrix} \xrightarrow{\xi=0=\bar{\xi}} \begin{pmatrix} \Gamma^{(2)}_{k\,gg} & 0 \\ 0 & (\cdots) \end{pmatrix}$$

3. Inversion: $\text{FRGE}|_{\bar{\xi}=0=\xi} \Rightarrow \Gamma_k^{\mathsf{gh}} \to S^{\mathsf{gh}}$ (new truncation)

4. Trace STr:

The Bi-Metric Einstein-Hilbert Truncation

1. Effective Average Action $\Gamma_k[g,\bar{g}]$:

   (Invariants up to second order in the derivatives)

$$\partial_t \Gamma_k[g,\bar{g}] = \partial_t \int \sqrt{\bar{g}} \left(\Lambda_k^{\mathrm{B}} Z_k^{\mathrm{B}} + Z_k^{\mathrm{B}} \bar{R} + Z_k^{a} R \right) + \sqrt{g} \left(\gamma_k^{c} + Z_k^{c} \bar{R} + Z_k^{d} R \right)$$

$$+ \partial_t \sum_{n \neq 0} \gamma_k^{(n)} \int \left(\frac{\sqrt{\bar{g}}}{\sqrt{g}} \right)^n (\cdots) + \partial_t \Gamma_k^{\mathrm{gf}}[g,\bar{g}]$$

2. Hessian $\Gamma_k^{(2)}[g,\bar{g}]$:

$$\Gamma_k^{(2)}[g,\bar{g}] = (\cdots) D_\mu D_\nu + (\cdots) D_\mu \bar{D}_\nu + (\cdots) \bar{D}_\mu D_\nu + (\cdots) \bar{D}_\mu \bar{D}_\nu$$

3. Inversion: $\mathrm{FRGE}|_{\bar{\xi}=0=\xi} \Rightarrow \Gamma_k^{\mathrm{gh}} \to S^{\mathrm{gh}}$ (new truncation)
4. Trace STr: $D_\mu \to \bar{D}_\mu$ + extra informations

The Single-Metric Approximation $\text{FRGE}|_{g=\bar{g}}$

RHS: $\text{STr}\{(\cdots)\}|_{g=\bar{g}} = \text{STr}\{(\cdots)\bar{D}_\mu \bar{D}_\nu\}$

LHS:

$$\partial_t \Gamma_k[g=\bar{g},\bar{g}] = \partial_t \int \sqrt{\bar{g}}\left(\Lambda_k^{\text{B}} Z_k^{\text{B}} + Z_k^{\text{B}}\bar{R} + Z_k^a \bar{R}\right) + \sqrt{\bar{g}}\left(\gamma_k^c + Z_k^c \bar{R} + Z_k^d \bar{R}\right)$$

$$+ \partial_t \sum_{n\neq 0} \gamma_k^{(n)} \int \left(\frac{\sqrt{\bar{g}}}{\sqrt{\bar{g}}}\right)^n (\cdots) + \partial_t \Gamma_k^{\text{gf}}[\bar{g},\bar{g}]$$

The Single-Metric Approximation $\mathrm{FRGE}|_{g=\bar{g}}$

RHS: $\mathrm{STr}\{(\cdots)\}|_{g=\bar{g}} = \mathrm{STr}\{(\cdots)\bar{D}_\mu \bar{D}_\nu\}$

LHS:

$$\partial_t \Gamma_k[g=\bar{g},\bar{g}] = \partial_t \int \sqrt{\bar{g}} \underbrace{\left(\Lambda_k^{\mathrm{B}} Z_k^{\mathrm{B}} + \gamma_k^a + \sum\nolimits_n \gamma_k^{(n)}\right)}_{\equiv \Lambda_k^{\mathrm{sm}}/G_k^{\mathrm{sm}}} + \int \sqrt{\bar{g}} \bar{R} \underbrace{\left(Z_k^{\mathrm{B}} + Z_k^b + Z_k^c + Z_k^d + (\cdots)\right)}_{\equiv 1/G_k^{\mathrm{sm}}}$$

Huge loss of information:

⇒ ∞ number of couplings degenerates to only 2, Λ_k^{sm} and G_k^{sm}

⇒ Bi-metric character is lost! (split-symmetry enforced for all k)

The Conformal Projection Technique $\text{FRGE}|_{g=e^{2\Omega}\bar{g}}$

RHS: $\text{STr}\{(\cdots)\}|_{g=e^{2\Omega}\bar{g}} = \text{STr}\{(\cdots)\bar{D}_\mu \bar{D}_\nu + \Omega(\cdots)\bar{D}_\mu \bar{D}_\nu + \mathcal{O}(\Omega^2)\}$

LHS:

$$\partial_t \Gamma_k[g = e^{2\Omega}\bar{g}, \bar{g}] = \partial_t \int \sqrt{\bar{g}} \left(\Lambda_k^{\mathsf{B}} Z_k^{\mathsf{B}} + Z_k^{\mathsf{B}} \bar{R} + Z_k^{b} e^{-2\Omega} \bar{R}\right) + (\cdots)$$
$$+ \partial_t \int e^{d\Omega} \sqrt{\bar{g}} \left(\Lambda_k^{c} + Z_k^{c} \bar{R} + Z_k^{d} e^{-2\Omega} \bar{R}\right) + \cancel{\partial_t \Gamma_k^{\mathsf{gf}}[e^{2\Omega}\bar{g}, \bar{g}]}$$

The Conformal Projection Technique $\text{FRGE}\big|_{g=e^{2\Omega}\bar{g}}$

RHS: $\text{STr}\{(\cdots)\}\big|_{g=e^{2\Omega}\bar{g}} = \text{STr}\{(\cdots)\bar{D}_\mu\bar{D}_\nu + \Omega(\cdots)\bar{D}_\mu\bar{D}_\nu + \mathcal{O}(\Omega^2)\}$

LHS:

$$\partial_t\Gamma_k[g=e^{2\Omega}\bar{g},\bar{g}] = \partial_t\int\sqrt{\bar{g}}\underbrace{\left(\Lambda_k^{\text{B}}Z_k^{\text{B}}+\gamma_k^a+\sum\nolimits_n\gamma_k^{(n)}\right)}_{\equiv\frac{\Lambda_k^{(0)}}{G_k^{(0)}}=\frac{\Lambda_k^{\text{B}}}{G_k^{\text{B}}}+\frac{\Lambda_k^{\text{Dyn}}}{G_k^{\text{Dyn}}}} + \int\sqrt{\bar{g}}\bar{R}\underbrace{\left(Z_k^{\text{B}}+Z_k^b+Z_k^c+Z_k^d+(\cdots)\right)}_{\equiv\frac{1}{G_k^{(0)}}=\frac{1}{G_k^{\text{B}}}+\frac{1}{G_k^{\text{Dyn}}}}$$

$$+\,\Omega\partial_t\int\sqrt{\bar{g}}\underbrace{\left(\gamma_k^a+\sum\nolimits_n\gamma_k^{(n)}\right)}_{\equiv\Lambda_k^{\text{Dyn}}/G_k^{\text{Dyn}}} + \int\sqrt{\bar{g}}\bar{R}\underbrace{\left(Z_k^b+Z_k^c+Z_k^d+(\cdots)\right)}_{\equiv 1/G_k^{\text{Dyn}}}$$

- Absorb ∞ number of couplings in 4, Λ_k^{Dyn}, G_k^{Dyn}, $\Lambda_k^{(0)/\text{B}}$, and $G_k^{(0)/\text{B}}$
- $1/G_k^{\text{B}}$, $\Lambda_k^{\text{B}} \neq 0$ break split-symmetry ($\mathcal{O}(\Omega^0) \neq \mathcal{O}(\Omega^p)$ $p \geq 1$)

The Hessian simplified

$$
\begin{aligned}
32\pi\, G_k^{\mathsf{sm}}\, \Gamma_k^{(2)}[e^{2\Omega}\bar{g}, \bar{g}]_{gg}^{\mu\nu\rho\sigma} = & -\alpha^{-1}\Big[\bar{g}^{\rho\mu}\bar{g}^{\sigma\nu} + (2\varpi^2 - 1)\bar{g}^{\mu\nu}\bar{g}^{\rho\sigma}\Big]\bar{D}^2 \\
& - \alpha^{-1}(1 - 2\varpi)\Big[\bar{g}^{\rho\sigma}\,\bar{D}^{\mu}\,\bar{D}^{\nu} + \bar{g}^{\mu\nu}\,\bar{D}^{\sigma}\,\bar{D}^{\rho}\Big] \\
& + \Big(\alpha^{-1} - 1\Big)(\cdots)\,\bar{D}^{\mu}\,\bar{D}^{\nu} \\
& + 2\,\alpha^{-1}\Big[\bar{g}^{\nu\sigma}\,\bar{R}^{\mu\rho} + \bar{R}^{\rho\mu\nu\sigma}\Big] + \mathcal{U}^{\mu\nu\rho\sigma}[\bar{g}]
\end{aligned}
$$

The Hessian simplified

$$
\begin{aligned}
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& - \alpha^{-1}(1 - 2\varpi)\left[\bar{g}^{\rho\sigma}\,\bar{D}^\mu\,\bar{D}^\nu + \bar{g}^{\mu\nu}\,\bar{D}^\sigma\,\bar{D}^\rho\right] \\
& + \left(\alpha^{-1} - 1\right)(\cdots)\,\bar{D}^\mu\,\bar{D}^\nu \\
& + 2\,\alpha^{-1}\left[\bar{g}^{\nu\sigma}\,\bar{R}^{\mu\rho} + \bar{R}^{\rho\mu\nu\sigma}\right] + \mathcal{U}^{\mu\nu\rho\sigma}[\bar{g}]
\end{aligned}
$$

Single-Metric		
$\varpi = 1/2$		
$\alpha = 1$		
$\Rightarrow \mathsf{STr}\{(\cdots)\bar{D}^2\}$		

The Hessian simplified

$$
\begin{aligned}
32\pi\, G_k^{\mathsf{Dyn}}\, \Gamma_k^{(2)}[e^{2\Omega}\bar{g}, \bar{g}]_{gg}^{\mu\nu\rho\sigma} = &-\alpha^{-1}\Big[\bar{g}^{\rho\mu}\bar{g}^{\sigma\nu} + (2\varpi^2 - 1)\bar{g}^{\mu\nu}\bar{g}^{\rho\sigma}\Big]\bar{D}^2 \\
&-\alpha^{-1}(1-2\varpi)\Big[\bar{g}^{\rho\sigma}\bar{D}^\mu\bar{D}^\nu + \bar{g}^{\mu\nu}\bar{D}^\sigma\bar{D}^\rho\Big] \\
&+\Big(\alpha^{-1} - e^{(d-6)\Omega}\Big)(\cdots)\bar{D}^\mu\bar{D}^\nu \\
&+2\,\alpha^{-1}\Big[\bar{g}^{\nu\sigma}\bar{R}^{\mu\rho} + \bar{R}^{\rho\mu\nu\sigma}\Big] + e^{(d-6)\Omega}\,\mathcal{U}^{\mu\nu\rho\sigma}[\bar{g}]
\end{aligned}
$$

Single-Metric	Bi-Metric I	
$\varpi = 1/2$	$\varpi = 1/d,\ \alpha \to 0$	
$\alpha = 1$	$h_{\mu\nu} = h_{\mu\nu}^{\mathsf{TT}} + \bar{D}_\mu \chi_\nu + \cdots$	
$\Rightarrow \mathsf{STr}\{(\cdots)\bar{D}^2\}$	TT-decomposition	

The Hessian simplified

$$32\pi\, G_k^{\mathsf{Dyn}}\, \Gamma_k^{(2)}[e^{2\Omega}\bar{g}, \bar{g}]_{gg}^{\mu\nu\rho\sigma} = -\alpha^{-1}\left[\bar{g}^{\rho\mu}\bar{g}^{\sigma\nu} + (2\varpi^2 - 1)\bar{g}^{\mu\nu}\bar{g}^{\rho\sigma}\right]\bar{D}^2 - \alpha^{-1}(1-2\varpi)\left[\bar{g}^{\rho\sigma}\bar{D}^\mu\bar{D}^\nu + \bar{g}^{\mu\nu}\bar{D}^\sigma\bar{D}^\rho\right] + \left(\alpha^{-1} - e^{(d-6)\Omega}\right)(\cdots)\bar{D}^\mu\bar{D}^\nu + 2\,\alpha^{-1}\left[\bar{g}^{\nu\sigma}\bar{R}^{\mu\rho} + \bar{R}^{\rho\mu\nu\sigma}\right] + e^{(d-6)\Omega}\,\mathcal{U}^{\mu\nu\rho\sigma}[\bar{g}]$$

Single-Metric	Bi-Metric I	Bi-Metric II
$\varpi = 1/2$	$\varpi = 1/d$, $\alpha \to 0$	$\varpi = 1/2$, α '=' $e^{-(d-6)\Omega}$
$\alpha = 1$	$h_{\mu\nu} = h_{\mu\nu}^{\mathsf{TT}} + \bar{D}_\mu\chi_\nu + \cdots$	$\Gamma_k \to \Gamma_k + \Gamma_k^{\mathsf{simplify}}$
$\Rightarrow \mathrm{STr}\{(\cdots)\bar{D}^2\}$	TT-decomposition	$\Rightarrow \mathrm{STr}\{(\cdots)\bar{D}^2\}$

Summary: Introduction

A quantum theory of gravity is described by:

Split-symmetry $\xleftarrow{k=0}$ **RG trajectory** $\xrightarrow{k\to\infty}$ NGFP (with $\dim \mathscr{S}_{\text{UV}} < \infty$)

Evaluation of the FRGE needs approximation techniques:

- Single-metric approximation: $\text{FRGE}\big|_{g=\bar{g}}$ (Background Independence $\times$)
- Conformal projection: $\text{FRGE}\big|_{g=e^{2\Omega}\bar{g}}$ (Background Independence ($\checkmark$))
- **New bi-metric method** with $\varpi = 1/2$ and $\alpha\text{`='}e^{(6-d)\Omega}$:
    - Close analogy to single-metric calculation
    - Simplifies trace evaluation
    - *Justification*: $\Gamma_k \to \Gamma_k + \Gamma_k^{\text{simplify}}$

Bi-Metric Einstein-Hilbert Truncation

A Bi-Metric Einstein-Hilbert Truncation

$$\Gamma_k^{\text{grav}}[g,\bar{g}] = -\frac{1}{16\pi {G^{\text{B}}}_k}\int\sqrt{\bar{g}}\,\left(\bar{R}-2{\Lambda^{\text{B}}}_k\right) - \frac{1}{16\pi {G^{\text{Dyn}}}_k}\int\sqrt{g}\,\left(R-2{\Lambda^{\text{Dyn}}}_k\right)$$

- Asymptotic Safety condition
- Split-symmetry, i.e. $\Gamma_k^{\text{grav}}[g,\bar{g}] \stackrel{!}{=} \Gamma_k^{\text{grav}}[g,\bar{g}+\delta\bar{g}]$

$$\Rightarrow \qquad \frac{1}{G_k^{\text{B}}} \stackrel{!}{=} 0\,, \qquad \frac{\Lambda_k^{\text{B}}}{G_k^{\text{B}}} \stackrel{!}{=} 0$$

- Explore Background Independence $k \to 0$
- Test single-metric reliability $\forall\, k$

Structure of the Beta-Functions

$$\partial_t G_k^{\text{Dyn}} = \eta^{\text{Dyn}}(G_k^{\text{Dyn}}, \Lambda_k^{\text{Dyn}})\, G_k^{\text{Dyn}}\,, \qquad \partial_t \Lambda_k^{\text{Dyn}} = \beta_\Lambda^{\text{Dyn}}(G_k^{\text{Dyn}}, \Lambda_k^{\text{Dyn}})$$

$$\Downarrow$$

$$\partial_t G_k^{\text{B}} = \eta^{\text{B}}(G_k^{\text{Dyn}}, \Lambda_k^{\text{Dyn}}, G_k^{\text{B}})\, G_k^{\text{B}} \qquad = \eta^{\text{B}}(G_k^{\text{B}}; k)\, G_k^{\text{B}}$$

$$\Downarrow$$

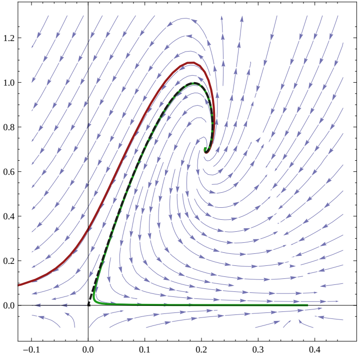
$$\partial_t \Lambda_k^{\text{B}} = \beta_\Lambda^{\text{B}}(G_k^{\text{Dyn}}, \Lambda_k^{\text{Dyn}}, G_k^{\text{B}}, \Lambda_k^{\text{B}}) \qquad = \beta_\Lambda^{\text{B}}(G_k^{\text{B}}, \Lambda_k^{\text{B}}; k)$$

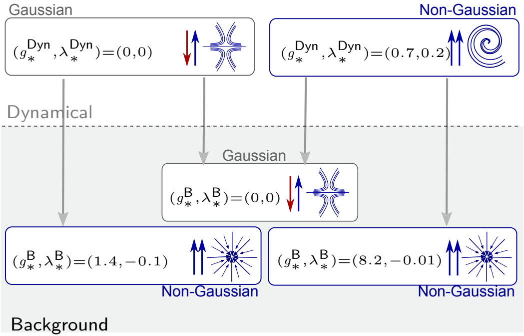
For fixed $k \mapsto \left(G_k^{\text{Dyn}},\, \Lambda_k^{\text{Dyn}}\right)$:

- **Non-autonomous** system: $\beta_g^{\text{B}}(g_k^{\text{B}}; k)$ and $\beta_\lambda^{\text{B}}(g_k^{\text{B}}, \lambda_k^{\text{B}}; k)$
- **Fixed points**: $\beta_g^{\text{B}}(g_\bullet^{\text{B}}(k); k) = 0 = \beta_\lambda^{\text{B}}(g_\bullet^{\text{B}}(k), \lambda_\bullet^{\text{B}}(k); k)$

$\Rightarrow$ $\left(G_\bullet^{\text{B}}(k), \Lambda_\bullet^{\text{B}}(k)\right)$ defines **running UV-attractor** in B-sector

Asymptotic Safety Condition UV





UV-condition fulfilled: $\exists$ NGFP with $\dim(\mathscr{S}_{\text{UV}}) = 4$

Split-Symmetry in the IR

$$1/G_k^{\mathrm{B}} = 1/G_{k_0}^{\mathrm{B}} - \int_{k_0}^{k} \mathrm{d}k'\, k^2 B_1^{\mathrm{B}}(k') \overset{!}{=} 0$$

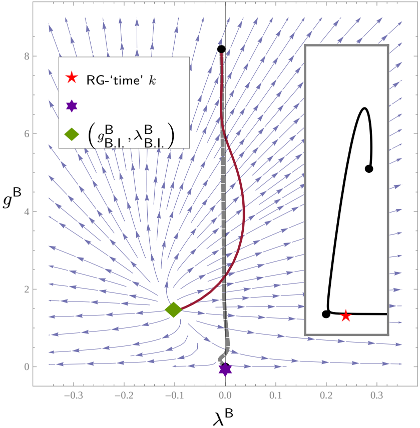
For every $(G_k^{\mathrm{Dyn}}, \Lambda_k^{\mathrm{Dyn}})$ trajectory exists one, and only one, full RG trajectory satisfying the 'final condition',

$$1/G_{\mathrm{B.I.}}^{\mathrm{B}}(k=0) = \lim_{k\to 0} 1/G_{\bullet}^{\mathrm{B}}(G_k^{\mathrm{Dyn}}, \Lambda_k^{\mathrm{Dyn}}; k) = 0$$

⇒ Fulfills **global** requirement: $\lim_{k\to\infty} \Gamma_k = \Gamma_*^{\mathrm{NGFP}}$ and $G_{\mathrm{B.I.}}^{\mathrm{B}} > 0$

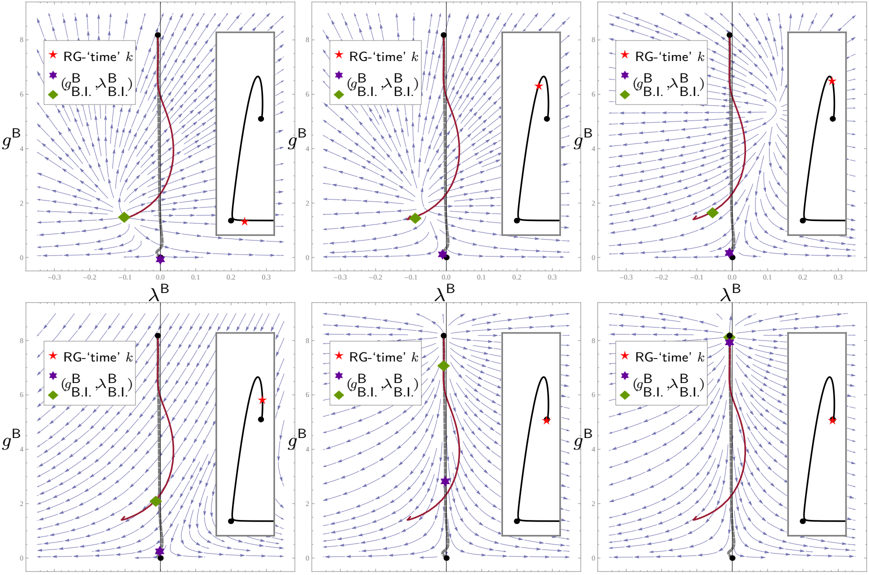
$$\Big\{ \underbrace{\left(G_k^{\mathrm{Dyn}}, \Lambda_k^{\mathrm{Dyn}}, G_k^{\mathrm{B}}, \Lambda_k^{\mathrm{B}}\right)}_{\in \mathcal{T}_{\mathrm{trunc}}:\ 4\text{-dim.}} \;\Big|\; \underbrace{\left(G_k^{\mathrm{Dyn/B}} > 0\right)}_{\text{Asym. Safety: } 4\text{-dim.}} ,\ \underbrace{\left(G_k^{\mathrm{B}} = G_{\mathrm{B.I.}}^{\mathrm{B}}(k) \wedge \Lambda_k^{\mathrm{B}} = \Lambda_{\mathrm{B.I.}}^{\mathrm{B}}(k)\right)}_{\text{Back. Ind.: } 2\text{-dim}} \Big\}$$

The Background Sector (here $k = 0$)



$k \mapsto \left(G_k^{\mathrm{Dyn}}, \Lambda_k^{\mathrm{Dyn}}\right)$ fixed $\quad \Rightarrow \quad$ k-dependent background vector field!

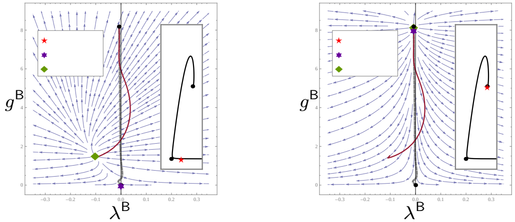
The running UV-attractor



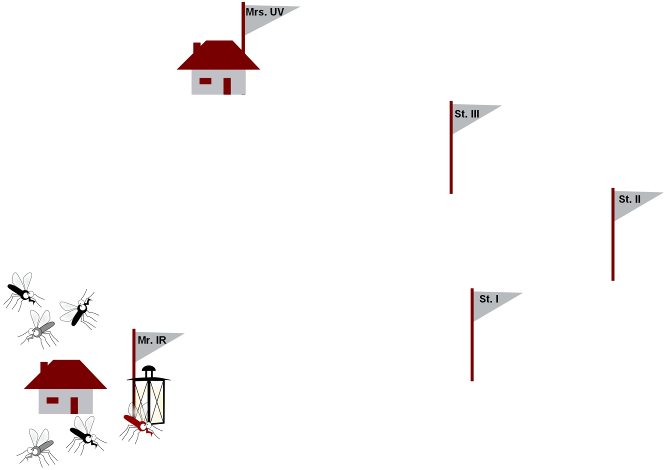
The running UV-attractor

The dual role of the running UV-attractor:

- guarantees non-perturbative renormalizability for $k \to \infty$
- guarantees Background Independence for $k \to 0$



The running UV attractor $(\mathbf{Attr})^{\mathrm{B}}{}_{k}$



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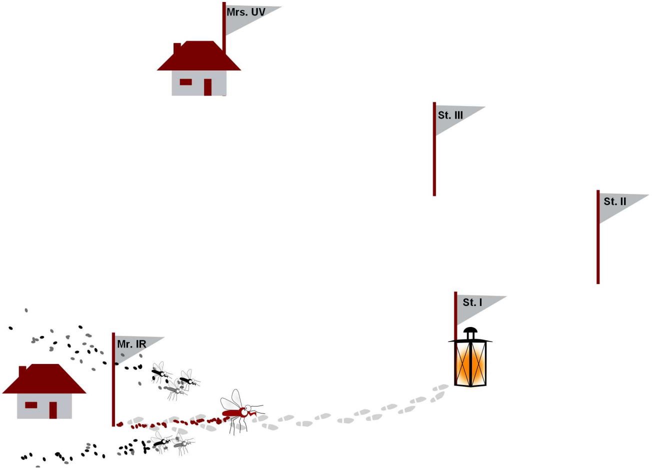
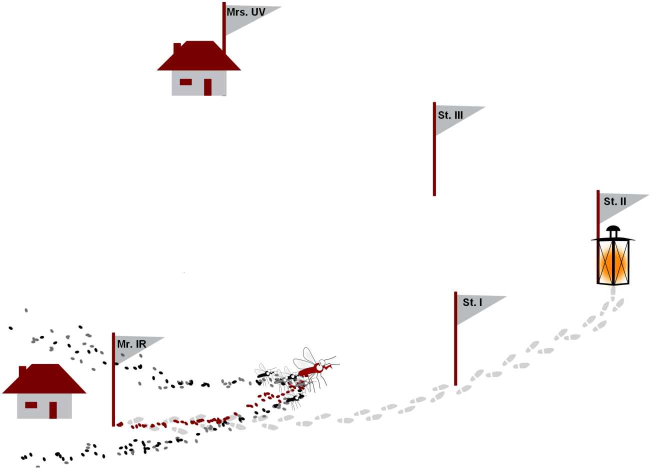
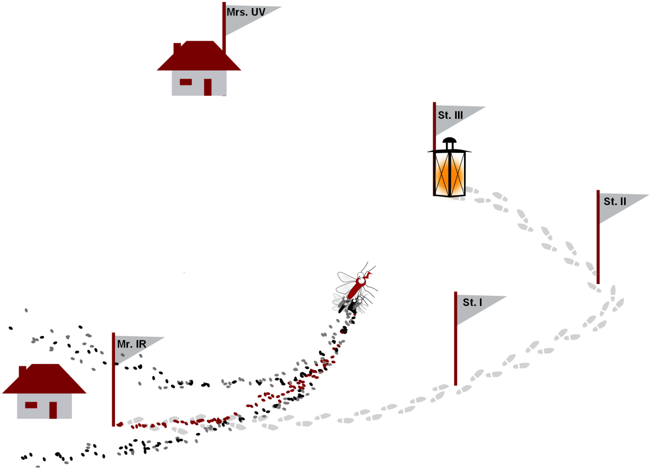


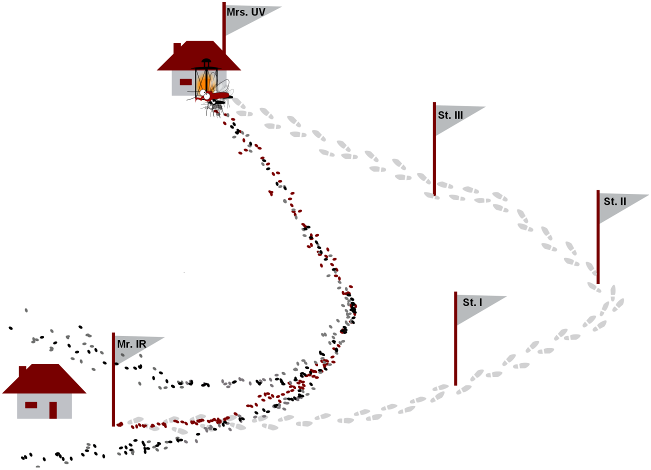
The running UV attractor $\mathbf{(Attr)}^{\mathbf{B}}{}_{k}$



The running UV attractor $\mathbf{(Attr)}^{\mathbf{B}}{}_{k}$



The running UV attractor $(\mathbf{Attr})^{\mathrm{B}}{}_{k}$



Single-Metric vs. Bi-Metric: a comparison

Test of (intermediate) Split-symmetry

$\forall$ trajectories $\exists$ only **isolated points** of intact split-symmetry

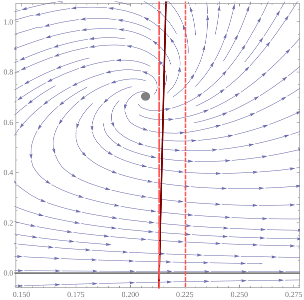
$\Rightarrow$ Single and bi-metric results differ significantly for almost all k
(especially in the semi-classical regime)

The miraculous reliability near the NGFP:

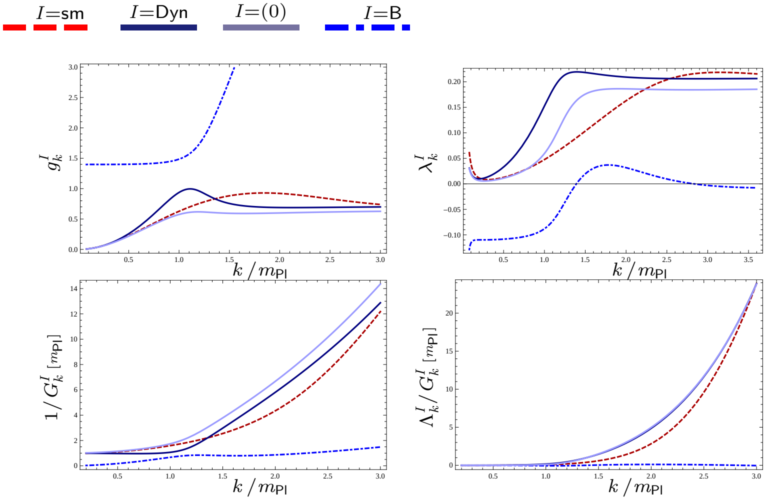
Fixed point region *approximately* split-symmetric

Single-metric reliability

IR by construction and UV (✓)
In between (semi-classical, ...) ×



A suitable RG trajectory



Summary

Asymptotic Safety & Background Independence

- A *global* requirement that needs bi-metric truncations
  Bi-EH-truncation: trajectories *exists* satisfying both conditions
  B.I. $\xleftarrow{\text{IR}}$ **running UV-attractor** $\xrightarrow{\text{UV}}$ A.S.
  (Number of free parameter reduces from 4 to 2)
- Split-symmetry broken for $0 < k < \infty$
  $\Rightarrow$ Single-metric trajectories **at best** qualitatively correct
- **UV Miracle:** Split-symmetry *approximately* intact near the NGFP
  $\Rightarrow$ Single-metric truncation correspondingly reliable

Conclusion

- Background Independence and A. S. simultaneously satisfied
- Generalize truncations to bi-metric ones (single-metric check)

Thank You!