Discussing the UV structure of quantum unimodular gravity

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based on arXiv:1410.6163 [hep-th]



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Outline of the talk

• The idea behind unimodular gravity, its implementation, and a diffeomorphism invariant formulation of it at the level of the action

• The study of the unimodular theory's UV completion within the Exact Renormalisation Group and Asymptotic Safety

• The similarity between its UV completion with that of General Relativity (GR), as well as the main differences of the two theories in this context

Introducing unimodular gravity

• General Relativity: A Diffeomorphism (Diff) invariant, and successful theory of gravity at solar and cosmological (?) scales

$$S=\int d^4x\sqrt{-g}rac{R-2\Lambda}{16\pi G}+S_{
m matter}$$

- Motivation: The cosmological constant problem, $|\Lambda + 8\pi G < \rho >_{matter}| \sim H_0^2$
- The idea of unimodular gravity: Change the status of the coupling Λ by decoupling it from the gravitational dynamics 1

$$\frac{\delta}{\delta g_{\mu\nu}}\sqrt{-g}=0$$
 Unimodularity condition

• The classical dynamics of unimodular gravity:

Bianchi identities: $R - 8\pi G T^{\mu}{}_{\mu} = \text{const.} \equiv 4\lambda_0$

Field equations: $G_{\mu\nu} + \lambda_0 g_{\mu\nu} = 8\pi G T_{\mu\nu}$

• Bianchi identities are no longer identically satisfied due to the enforcement of the unimodularity condition

Classical equations are equivalent with those of GR, but now the cosmological constant appears as a constant of integration

Unimodular gravity and diffeomorphism symmetry

• The gauge symmetry of the metric field in GR is the symmery under diffeomorphisms

$$g_{\alpha\beta} \to g_{\alpha\beta} + \delta g_{\alpha\beta} \equiv g_{\alpha\beta} + \nabla_{\alpha}\xi_{\beta} + \nabla_{\beta}\xi_{\alpha} \tag{1}$$

 Imposing the unimodularity condition, classically or quantum-mechanically, restricts the allowed variations of the metric field

$$\frac{\delta}{\delta g_{\mu\nu}} \sqrt{-g} = 0 \qquad \rightsquigarrow \quad \nabla_{\mu} \xi^{\mu} = 0 \quad (\text{restricted symmetry:TDiff}) \tag{2}$$

- Classically, the unimodularity condition implies a modification of the Bianchi identities
- Different ways to impose unimodularity: Quantum mechanically, they lead to different quantisation procedures
- **This talk**: A quantisation of a fully-diffeomorphism unimodular action for gravity within the exact renormalisation group and asymptotic safety

A fully diffeomorphism-invariant action for unimodular gravity

• Unimodularity in action: The unimodularity condition can be implemented as an on-shell condition through a lagrange multiplier $\lambda(x)^2$

$$S = \int d^{4}x \left[\sqrt{-g} \frac{R}{16\pi G} - \lambda \left(\sqrt{-g} - \epsilon_{0} \right) \right]$$
(3)

• Stückelberg-ing the action: Introduce four Stückelberg fields $\phi^{\alpha}(x)$, following the pattern of a general coordinate transformation: $x^{\alpha} \rightarrow \phi^{\alpha}(x)$, $\alpha = 0, \dots 3^{3}$

$$\int d^4 x \lambda \left(\sqrt{-g} - \epsilon_0 \right) \rightarrow \int d^4 x \lambda \left(\sqrt{-g} - \epsilon_0 \left| J^{\alpha}{}_{\beta} \right| \right) \equiv \int d^4 x \sqrt{-g} \lambda \left(1 - \epsilon_0 \psi \right)$$
(4)

The Stückelberg Jacobian:
$$|J^{\alpha}{}_{\beta}| \equiv \left|\frac{\partial \phi^{\alpha}(x)}{\partial x^{\beta}}\right|$$
 with $\psi \equiv \frac{|J^{\alpha}{}_{\beta}|}{\sqrt{-g}}, \quad \alpha, \beta = 0, \dots, 3$
(5)

• A generalised and Diff-invariant unimodular formulation of GR ⁴

$$S = \int d^{4}x \sqrt{-g} \left[\frac{R}{16\pi G} - \lambda f(\psi) - q(\psi) \right]$$
(6)

 \leadsto Its easy to see that the equations of motion for the fields λ and ψ ensure the classical dynamics are the same as those of GR

²W. Buchmuller and N. Dragon, Phys.Lett., vol. B223, p. 313 (1989).

³See also K. V. Kuchar PRD43, 3332?3344 (1991)

⁴A. Paddila and I. D. Saltas, arXiv:1409.3573 [gr-qc]

A fully diffeomorphism-invariant action for unimodular gravity

$$S = \int d^{4}x \sqrt{-g} \left[\frac{R}{16\pi G} - \lambda f(\psi) - q(\psi) \right] + S_{\text{matter}}$$
(7)

• The classical equations result from variation with respect to the metric $(g_{\mu\nu})$, Lagrange multipler (λ) and Stückelberg fields $(\phi^{\alpha})^{5}$

$$G_{\mu\nu} = 8\pi G \left[T_{\mu\nu} + g_{\mu\nu} \left(\lambda V(\psi) + U(\psi) \right) \right]$$
(8)

$$f(\psi) = 0, \quad \partial_{\alpha} \left(\lambda f'(\psi) + q'(\psi) \right) = 0, \tag{9}$$

$$\psi \equiv \frac{\left|\partial_{\alpha}\phi^{\beta}\right|}{\sqrt{-g}}, \quad V(\psi) \equiv \psi f'(\psi) - f(\psi), \quad U(\psi) \equiv \psi q'(\psi) - q(\psi)$$

- Classical dynamics of the theory are the same as in GR with a cosmological constant
- The new constraint equations ensure the Bianchi identities are satisfied

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⁵A. Paddila and I. D. Saltas, arXiv:1409.3573 [gr-qc]

Unimodular gravity and asymptotic safety: A brief review of previous work

- A conceptually different approach has been followed before in the literature ⁶
- The main key of the approach: Define a TDiff-invariant quantisation of the Einstein-Hilbert truncation (without a cosmological constant)

$$Z[J] = \int Dg_{\mu\nu} \ e^{iS[g_{\mu\nu}] + i \int J^{\alpha\beta}g_{\alpha\beta} + \Delta S_k} \ , \tag{10}$$

$$S[g_{\mu\nu}] = \int d^4x \sqrt{-g} \frac{R}{16\pi G} + S_{\text{TDiff-gauge fixing}} + S_{\text{TDiff-ghost}}$$
(11)

• Usual gauge fixing condition F_{μ} acted upon with a transverse projector

$$F_{\mu} \rightarrow F_{\mu}^{\text{TDiff}} \equiv P_{\mu}{}^{\kappa}F_{\kappa}, \qquad P_{\mu\nu} = \frac{1}{\overline{\nabla}^2} (g_{\mu\nu}\overline{\nabla}^2 - \overline{\nabla}_{\mu}\overline{\nabla}_{\nu})$$
(12)

• Unimodularity condition imposed up to second order in expansion of the metric:

$$\delta_g^{(1)}\sqrt{g} = 0, \quad \delta_g^{(2)}\sqrt{g} = 0 \quad \rightarrow \quad \Gamma_{\text{TDiff}}^{(2)}$$
(13)

• Theory was found to be asymptotically safe with a UV fixed point $G_* = 2.65$ and eigenvalue = -2.341

⁶A. Eichhorn Class.Quant.Grav., vol. 30, p. 115016 (2013)

Setting up the calculation: Generating functional and gauge fixing

• The starting point is the generating functional, where all fields of the theory are coupled to external sources

$$Z[J] = \int Dg_{\mu\nu} D\phi^{\alpha} D\lambda \ e^{iS[\Phi_A] + i \int J_A \Phi_A + \Delta S_k} \ , \qquad \Phi_A = \{g_{\mu\nu}, \lambda, \phi^{\alpha}\}$$
(14)

$$S[\Phi_A] = \int d^4x \sqrt{-g} \left[\frac{R - 2\Lambda}{16\pi G} + \lambda(x)f(\psi) + q(\psi) \right] + S_{\text{gauge fixing}} + S_{\text{ghost}}, \qquad \psi \equiv \frac{1}{\sqrt{g}} \left| \frac{\partial \phi^{\alpha}(x)}{\partial x^{\beta}} \right|$$
(15)

$$\Delta S_k = \frac{1}{2} \int d^4 x \sqrt{-g} \, \Phi_A \, R_k^{AB}(-\Box) \, \Phi_B, \tag{16}$$

• The theory is Diff invariant, and the gauge fixing sector defines the de Donder gauge

$$S_{\text{gauge fixing}} = Z_G \int d^4 x \sqrt{\bar{g}} \bar{g}^{\mu\nu} h_{\alpha\beta} \mathcal{F}^{\alpha\beta}_{\mu} \mathcal{F}^{\gamma\delta}_{\nu} h_{\gamma\delta}, \qquad \mathcal{F}^{\alpha\beta}_{\mu} \equiv \delta^{\beta}_{\mu} \bar{g}^{\alpha\gamma} \bar{\nabla}_{\gamma} - \frac{1}{2} \bar{g}^{\alpha\beta} \bar{\nabla}_{\mu}$$
(17)

 There are two ghost contributions: One from the gauge fixing and another from the Stückelberg sector

$$S^{\text{GF ghost}} = -\int d^4 x \sqrt{\bar{g}} \ \bar{C}_{\mu} \left(-\delta^{\mu}_{\nu} \Box - \frac{1}{4} \delta^{\mu}_{\nu} R \right) C^{\nu} \qquad \mathsf{Z}^{\text{GF ghost}} = 1$$
(18)

$$S^{\text{Stück. ghost}} \equiv -\frac{1}{2} \int d^4 x \sqrt{\bar{g}} \ \bar{\eta}(-\Box)\eta \qquad Z^{\text{Stück. ghost}} = 1$$
(19)

Why introduce the cosmological constant term in the action?
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Setting up the calculation: Field expansion and background choice

• Our tool: The Exact Renormalisation Group Equation (ERGE) ⁷

$$\partial_t \Gamma_k = \frac{1}{2} \operatorname{Tr} \left[\left(\Gamma_k^{(2)} + \mathbf{R}_k \right)^{-1} k \partial_k \mathbf{R}_k \right],$$

$$\Gamma_{k} = -\int d^{4}x \sqrt{g} \left[Z_{G}(R - 2\Lambda) - \lambda f(\psi) - q(\psi) \right] + S_{\text{ghosts}} + S_{\text{gauge fixing}}$$
(20)

• The metric, Lagrange multiplier and Stückelberg fields fluctuate as

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + G_0^{1/2} \hat{h}_{\mu\nu} + \frac{1}{4} G_0^{1/2} \bar{g}_{\mu\nu} h$$
$$\lambda = \bar{\lambda} + G_0^{-1/2} \delta \lambda$$
$$\phi^{\alpha} = \bar{\phi}^{\alpha} + G_0^{1/2} \delta \hat{\phi}^{\alpha} + G_0 \bar{\nabla}^{\alpha} \delta \phi$$
(21)

Background choice:

$$\bar{R}_{\alpha\beta\gamma\delta} = \frac{1}{12}\bar{R}\left(\bar{g}_{\alpha\gamma}\bar{g}_{\beta\delta} - \bar{g}_{\beta\gamma}\bar{g}_{\alpha\delta}\right), \qquad \bar{\phi}^{\alpha} = \epsilon x^{\alpha}, \qquad \psi \equiv \frac{|J^{\alpha}_{\beta}|}{\sqrt{-g}} = \frac{\epsilon^{4}}{\sqrt{\bar{g}}}$$
$$\bar{R} = const., \quad \bar{\lambda} = const. \tag{22}$$

7C. Wetterich Phys. Lett. B 301, 90 (1993) |T. R. Morris, Int. J. Mod. Phys. A 9 (1994) 2411. < 🗇 > < 🗄 > < 🗄 > 👘 🐑 🖉

Evaluating the exact RG equation

$$\Gamma_{k}[g_{\mu\nu},\lambda,\psi] = -\int d^{4}x \sqrt{g} \left[\frac{R - 2\Lambda_{k}}{16\pi G_{k}} - \lambda f_{k}(\psi) - q_{k}(\psi) \right] + S_{\text{ghosts}} + S_{\text{gauge fixing}}$$
(23)

 A cosmological-constant type modification to the "Einstein-Hilbert" Hessians

$$\begin{bmatrix} \Gamma_{(\hat{h}\hat{h})}^{(2)} \end{bmatrix}_{\rho\sigma}^{\mu\nu} = Z_{\hat{h}\hat{h}} \begin{bmatrix} -\Box - 2\Lambda + \frac{2}{3}R - Z_{G}^{-1}(\lambda V(\psi) + U(\psi)) \end{bmatrix} \begin{pmatrix} \delta_{\rho\sigma}^{\mu\nu} - \frac{1}{4}g^{\mu\nu}g_{\rho\sigma} \end{pmatrix},$$
(24)

$$\Gamma_{(hh)}^{(2)} = Z_{hh} \begin{bmatrix} -\Box - 2\Lambda - Z_{G}^{-1}(\lambda V(\psi) + U(\psi)) - 2Z_{G}^{-1}\psi^{2}(\lambda f''(\psi) + q''(\psi)) \end{bmatrix}$$
(25)

$$Z_{\hat{h}\hat{h}} \equiv \frac{1}{32\pi} \frac{G_0}{G_k}, \quad Z_{hh} \equiv -\frac{1}{128\pi} \frac{G_0}{G_k}, \quad V(\psi) \equiv f(\psi) - \psi f'(\psi)$$
(26)

New, non-trivial interactions in the scalar sector

$$\Gamma^{(2)}_{\phi\phi}[-\Box], \ \Gamma^{(2)}_{\phi\lambda}[-\Box], \ \Gamma^{(2)}_{h\phi}[-\Box]$$
(27)

• Stückelberg sector: Only longitudinal mode (ϕ) propagates, transverse one $(\hat{\phi}^{\alpha})$ is integrated out $(R_{\hat{\phi}\hat{\phi}} = 0)$

Solving the flow equation

$$\Gamma_{k}[g_{\mu\nu},\lambda,\psi] = -\int d^{4}x \sqrt{g} \left[Z_{G}(R-2\Lambda) - \lambda f(\psi) - q(\psi) \right] + S_{\text{ghosts}} + S_{\text{gauge fixing}}$$

- We choose a Type 1 cut-off, with $-\Box \rightarrow -\Box + R_k(-\Box)$, together with the optimised regulator R_k ⁸
- The trace over momenta in the ERGE is evaluated asymptotically on S₄ using a heat kernel expansion. The flow equation takes schematically the form

$$\left(\tilde{\text{Vol}}\right)^{-1} \partial_t \Gamma_k = \partial_t \Gamma_k|_{\mathsf{T}} + \partial_t \Gamma_k|_{\mathsf{Scalar}} + \partial_t \Gamma_k|_{\mathsf{Ghosts}} \equiv \partial_t \Gamma_k[G_k^{(0,1)}, \Lambda_k^{(0,1)}; f_k'^{(0,1)}, f_k''^{(0,1)}; f \leftrightarrow q],$$
(28)

 We solve the flow equation approximately through a polynomial ansatz in the Stückelberg sector

$$f(\psi) = \sum_{i=0}^{N_f} \frac{1}{i!} \rho_i \psi^i \qquad q(\psi) = \sum_{i=1}^{N_q} \frac{1}{i!} \sigma_i \psi^i$$
(29)

• The beta functions are derived by expanding w.r.t $\tilde{R}, \bar{\psi}, \, \tilde{\lambda}$ and projecting out similar operators on both sides

Einstein-Hilbert sector:
$$k\partial_k \tilde{\Lambda} = (-2 + \eta_{\tilde{\Lambda}})\tilde{\Lambda}, \quad k\partial_k \tilde{G} = (2 + \eta_{\tilde{G}})\tilde{G}$$

Stückelberg sector: $k\partial_k \tilde{\rho}_i = (-2 + \eta_{\tilde{\rho}_i})\tilde{\rho}_i, \quad k\partial_k \tilde{\sigma}_i = (-4 + \eta_{\tilde{\sigma}_i})\tilde{\sigma}_i$

$$[\tilde{c} \equiv c/k^d, \eta_c \equiv \frac{k\partial_k c}{c}]$$

The beta functions for G̃, Λ̃ receive contributions from ρ̃, σ̃ and vice versa:
 η_i = η_i(G̃, Λ̃; ρ̃_j, σ̃_j)

⁸D. F. Litim, Phys.Lett., B486, 92-99 (2000)

Fixed points and critical exponents: The minimal unimodular case

Fixed points:
$$eta_{ ilde{c}_i}(\widetilde{G},\widetilde{\Lambda};\widetilde{
ho}_j,\widetilde{\sigma}_m)=0$$

Eigenvalues:
$$\partial_t \tilde{c}_i = \sum_j \left. \frac{\partial \beta_i(\tilde{c}_n)}{\partial \tilde{c}_j} \right|_{\tilde{c}_j = \tilde{c}_{j*}} imes (\tilde{c}_j - \tilde{c}_{j*})$$

• The GR truncation: $f(\psi) = 0$, $q(\psi) = 0$, $\mathcal{L} = \sqrt{g}Z_G(R - 2\Lambda)$ Fixed points: $(\tilde{\Lambda}, \tilde{G}) = (0.193, 0.707)$

The minimal unimodular case:

 $\begin{aligned} f(\psi) &= \rho_0 + \rho_1 \psi, \quad q(\psi) = 0, \quad \mathcal{L} = \sqrt{g} Z_G(R - 2\Lambda) - \lambda(\sqrt{g} - |J^{\alpha}{}_{\beta}|) \\ \text{Fixed points:} \ (\tilde{\Lambda}, \tilde{G}, \tilde{\rho}_0, \tilde{\rho}_1) = (0.206, 0.661, 0, 0) \end{aligned}$

Higher order Stückelberg interactions

$$\mathcal{L} = \sqrt{g} \left[Z_G(R - 2\Lambda) - \lambda f(\psi) - q(\psi) \right]$$

• Beyond the minimal unimodular ansatz

$$f(\psi) = \sum_{i=0}^{4} \frac{1}{i!} \rho_i \psi^i , \qquad q(\psi) = \sum_{i=1}^{4} \frac{1}{i!} \sigma_i \psi^i$$
(32)

 Fixed point and attractive, complex eigen-values of (Λ, G) persist and show good quantitative stability as we increase the interations in the Stückelberg sector

- The Stückelberg couplings ρ̃_i, σ̃_j remain trivial in the UV, while the associated eigenvalues remain negative as we increase the truncation order
- $\bullet\,$ The effective actions for GR and the unimodular theory look similar in the UV

$$\Gamma^{\text{Unim.}}\Big|_{k/k_0\gg 1}\simeq \left.\Gamma^{\text{GR}}\right|_{k/k_0\gg 1} \tag{33}$$

$$\left. \tilde{G}_* \tilde{\Lambda}_* \right|_{\mathsf{Unimod.}} \simeq \left. \tilde{G}_* \tilde{\Lambda}_* \right|_{\mathsf{GR}} \simeq 0.13$$
 (34)

• The Diff-invariant unimodular theory and GR share similar UV completions

The full results: Fixed points and eigenvalues for up to fourth order Stückelberg sector

$\tilde{\Lambda}_*$	\tilde{G}_*	$\tilde{\rho}_{0*}$	$\tilde{\rho}_{1*}$	$\tilde{\rho}_{2*}$	$\tilde{\rho}_{3*}$	$\tilde{\rho}_{4*}$	$\tilde{\sigma}_{1*}$	$\tilde{\sigma}_{2*}$	$\tilde{\sigma}_{3*}$	$\tilde{\sigma}_{4*}$
0.193	0.707	-	—	_	_	—	_	—	—	-
0.206	0.661	0	0	_	_	_	_	_	_	_
0.206	0.661	0	0	0	_	_	_	_	_	_
0.206	0.661	0	0	0	0	—	_	_	—	—
0.206	0.661	0	0	0	0	0	-	—	—	—
0.201	0.674	_	_	_	_	_	_	0	_	_
0.202	0.670	-	-	_	-	_	_	0	0	-
0.203	0.666	-	-	—	-	_	-	0	0	0
0.206	0.661	0	0	_	_	_	0	_	_	—
0.206	0.661	0	0	_	-	_	0	0	_	-
$\gamma_{\tilde{\Lambda}}$	$\gamma_{\tilde{G}}$	$\gamma_{\tilde{\rho}_0}$	$\gamma_{\tilde{\rho}_1}$	$\gamma_{\tilde{\rho}_2}$	$\gamma_{\tilde{\rho}_3}$	$\gamma_{\tilde{\rho}_4}$	$\gamma_{\tilde{\sigma}_1}$	$\gamma_{\tilde{\sigma}_2}$	$\gamma_{\tilde{\sigma}_3}$	$\gamma_{\tilde{\sigma}_4}$
$\boxed{ \begin{array}{c} \gamma_{\tilde{\Lambda}} \\ -1.475 + 3.043i \end{array} }$	$\frac{\gamma_{\tilde{G}}}{-1.475-3.043i}$	$\gamma_{\tilde{\rho}_0}$ —	$\gamma_{\tilde{\rho}_1}$ -	$\gamma_{\tilde{\rho}_2}$ —	$\gamma_{\tilde{\rho}_3}$ —	$\gamma_{\tilde{\rho}_4}$ —	$\gamma_{\tilde{\sigma}_1}$	$\gamma_{\tilde{\sigma}_2}$ -	$\gamma_{\tilde{\sigma}_3}$ -	$\gamma_{\tilde{\sigma}_4}$
$ \begin{array}{ c c c c }\hline & & & \\ \hline & & & \\ \hline & & -1.475 + 3.043i \\ \hline & & -1.611 + 3.244i \end{array} $	$\begin{array}{c} \gamma_{\tilde{G}} \\ -1.475 - 3.043 i \\ -1.611 - 3.244 i \end{array}$	$\gamma_{\tilde{ ho}_0}$ 6.066	$\frac{\gamma_{\tilde{\rho}_1}}{-2}$	$\gamma_{\tilde{\rho}_2}$ -	$\gamma_{\tilde{\rho}_3}$	$\gamma_{\tilde{\rho}_4}$	$\gamma_{\tilde{\sigma}_1}$ -	$\gamma_{\tilde{\sigma}_2}$ - -	$\gamma_{\tilde{\sigma}_3}$ - -	$\gamma_{\tilde{\sigma}_4}$
$\begin{array}{ c c c c c }\hline & & & & \\ \hline & & & & \\ \hline & & & & \\ \hline & & & &$	$\begin{array}{c} \gamma_{\tilde{G}} \\ -1.475 - 3.043i \\ -1.611 - 3.244i \\ -1.611 - 3.244i \end{array}$	$\frac{\gamma_{\tilde{ ho}_0}}{-}$ -6.066 -6.066	$\frac{\gamma_{\tilde{\rho}_1}}{-}$ -2 -2	$\frac{\gamma_{\tilde{\rho}_2}}{-}$ 0.780	$\gamma_{\tilde{\rho}_3}$	$\gamma_{\tilde{\rho}_4}$	$\gamma_{\tilde{\sigma}_1}$	$\gamma_{\tilde{\sigma}_2}$	$\gamma_{\tilde{\sigma}_3}$ - - -	$\gamma_{\tilde{\sigma}_4}$
$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	$\begin{array}{c} \gamma_{\tilde{G}} \\ -1.475 - 3.043i \\ -1.611 - 3.244i \\ -1.611 - 3.244i \\ -1.611 - 3.244i \end{array}$	$\gamma_{\tilde{ ho}_0}$ - -6.066 -6.066 -6.066	$\gamma_{\tilde{\rho}_1}$ -2 -2 -2	$\frac{\gamma_{\tilde{\rho}_2}}{-}$ -0.780 -0.780	$\frac{\gamma_{\tilde{\rho}_3}}{-}$ - -1.458	γ _{ρ̃4} - - -	$\gamma_{\tilde{\sigma}_1}$	$\gamma_{\tilde{\sigma}_2}$ - - - - -	γ _{σ̃3} — — — —	$\gamma_{\tilde{\sigma}_4}$
$\begin{array}{c} \hline \gamma_{\bar{\Lambda}} \\ \hline -1.475 + 3.043i \\ \hline -1.611 + 3.244i \\ \hline \end{array}$	$\begin{array}{c} \gamma_{\bar{G}} \\ -1.475 - 3.043i \\ -1.611 - 3.244i \\ -1.611 - 3.244i \\ -1.611 - 3.244i \\ -1.611 - 3.244i \end{array}$	$\gamma_{\tilde{ ho}_0}$ -6.066 -6.066 -6.066 -6.066	$\frac{\gamma_{\tilde{\rho}_1}}{-2} \\ -2 \\ -2 \\ -2 \\ -2 \\ -2 \\ -2 \\ -2 \\$	$\gamma_{\tilde{\rho}_2}$ 0.780 - 0.780 - 0.660	$\frac{\gamma_{\tilde{\rho}_3}}{-}$ - - -1.458 -1.458	$\gamma_{\tilde{\rho}_4}$	γ _{õ1} 	γ _{õ2} - - - -	$\gamma_{\tilde{\sigma}_3}$ - - - - - - - - -	
$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	$\begin{array}{c} \gamma_{\bar{G}} \\ -1.475 - 3.043i \\ -1.611 - 3.244i \\ -1.611 - 3.244i \\ -1.611 - 3.244i \\ -1.611 - 3.244i \\ -1.644 - 3.119i \end{array}$	$\gamma_{\tilde{\rho}_0}$ 6.066 - 6.066 - 6.066 - 6.066 6.06 6.06 6.06	$\gamma_{\tilde{\rho}_1}$ -2 -2 -2 -2 -2 -2 -2	$\gamma_{\tilde{\rho}_2}$ 0.780 - 0.780 - 0.660 -	$\gamma_{\tilde{\rho}_3}$	$\gamma_{\tilde{\rho}_4}$	γ _{õ1} 	$\gamma_{\tilde{\sigma}_2}$	γ _{σ̃3} - - - - - - -	γ _{õ4}
$\begin{array}{c} \hline \gamma_{\bar{\Lambda}} \\ \hline -1.475 + 3.043i \\ \hline -1.611 + 3.244i \\ \hline -1.644 + 3.119i \\ \hline -1.624 + 3.139i \end{array}$	$\begin{array}{c} \gamma_{\bar{G}} \\ -1.475 - 3.043i \\ -1.611 - 3.244i \\ -1.611 - 3.244i \\ -1.611 - 3.244i \\ -1.611 - 3.244i \\ -1.644 - 3.119i \\ -1.624 - 3.139i \end{array}$	$\gamma_{\tilde{ ho}_0}$ 6.066 - 6.066 - 6.066 6.066	$\gamma_{\tilde{\rho}_1}$ -2 -2 -2 -2 -2 -2 -2 -2 -2 -2 -2 -2 -2	$\gamma_{\tilde{P}_2}$ 0.780 - 0.780 - 0.660	$\gamma_{\tilde{\rho}_3}$	$\gamma_{\tilde{\rho}_4}$	$\gamma_{\tilde{\sigma}_1}$	$\begin{array}{c c} \gamma_{\tilde{\sigma}_2} & & \\ & - & \\ & - & \\ & - & \\ & - & \\ & - & \\ & - & \\ & - & \\ & -3.131 \end{array}$	$\gamma_{\tilde{\sigma}_3}$	γ _{õ4}
$\begin{array}{c} \hline \gamma_{\bar{\Lambda}} \\ \hline -1.475 + 3.043i \\ \hline -1.611 + 3.244i \\ \hline -1.644 + 3.119i \\ \hline -1.624 + 3.139i \\ \hline -1.630 + 3.159i \end{array}$	$\begin{array}{c} \gamma_{\bar{G}} \\ -1.475 - 3.043i \\ -1.611 - 3.244i \\ -1.624 - 3.119i \\ -1.624 - 3.139i \\ -1.630 - 3.159i \end{array}$	$\gamma_{\tilde{ ho}_0}$ -6.066 -6.066 -6.066 -6.066 - - - - - - - - -	$\gamma_{\tilde{\rho}_1}$ -2 -2 -2 -2 -2 -2 -2 -2 -2 -2 -2 -2 -2	$\gamma_{\tilde{\rho}_2}$	$\gamma_{\tilde{\rho}_3}$	$\gamma_{\tilde{\rho}_4}$	$\gamma_{\tilde{\sigma}_1}$	$\begin{array}{c c} \gamma_{\tilde{\sigma}_2} & & \\ & - & \\ & - & \\ & - & \\ & - & \\ & - & \\ & - & \\ & - & \\ & - & 3.131 \\ & - & 3.465 \end{array}$	$\begin{array}{c} \gamma_{\tilde{\sigma}_{3}} & & \\ - & & \\ - & & \\ - & & \\ - & & \\ - & & \\ - & & \\ -3.131 & \\ -3.465 & \end{array}$	$\gamma_{\tilde{\sigma}_4}$ - - -3.465
$\begin{array}{c} \hline \gamma_{\bar{\Lambda}} \\ \hline -1.475 + 3.043i \\ \hline -1.611 + 3.244i \\ \hline -1.624 + 3.119i \\ \hline -1.624 + 3.139i \\ \hline -1.630 + 3.159i \\ \hline \hline -1.611 + 3.244i \end{array}$	$\begin{array}{c} \gamma_{\bar{G}} \\ -1.475 - 3.043i \\ -1.611 - 3.244i \\ -1.624 - 3.119i \\ -1.624 - 3.139i \\ -1.630 - 3.159i \\ -1.611 - 3.244i \end{array}$	$\begin{array}{c} \gamma_{ar{ ho}_0} & - & - & - & - & - & - & - & - & - & $	$\gamma_{\tilde{\rho}_1}$ -2 -2 -2 -2 -2 -2 -2 -2 -2 -2 -2 -2 -2	$\frac{\gamma_{\tilde{\rho}_2}}{-}$	$\frac{\gamma_{\bar{\rho}_3}}{-}$	$\frac{\gamma_{\bar{\rho}_4}}{-}$	$\gamma_{\tilde{\sigma}_1}$	$\begin{array}{c} \gamma_{\tilde{\sigma}_2} & - & - & - & - & - & - & - & - & - & $	$\frac{\gamma_{\bar{\sigma}_3}}{-}$	γ _{õ4} - - -3.465 -

RG dynamics away from the UV fixed point

• Away from the fixed point regime, the RG flow will drive the Stückelberg sector of the Diff-invariant unimodular theory away from triviality: $f_k(\psi) \neq 0$, $q_k(\psi) \neq 0$

• The non-trivial Stückelberg interactions generated by the RG flow at lower energies imply that in principle

 $\Gamma^{\text{Unim.}} \neq \Gamma^{\text{GR}}$, away from the fixed point regime (35)

 \rightarrow The two theories are in principle not equivalent to each other quantum-mechanically (*Reminder*: The Lagrange multiplier and Stückelberg fields were coupled to external sources in the path integral!)

Predictivity of the theory in the UV

- An apparent problem: The number of relevant couplings in the Stückelberg sector increases with the order of truncation in ψ , as the anomalous dimension remains much smaller than the canonical one for $\tilde{\rho} (=-2)$ and $\tilde{\sigma} (=-4)$
- Root of the problem: The dimensionless nature of the field $\psi \equiv \frac{\left|\partial_{\alpha}\phi^{\beta}\right|}{\sqrt{-g}}$ (In contrast with usual scalar field theories)
- Canonically normalising the field ψ ,

$$\psi \to \psi / Z_{\psi}^{1/2}, \quad [Z_{\psi}] = 2$$
 (36)

shifts all Stückelberg eigenvalues as

$$\gamma_{\tilde{\rho}_n} \to \gamma_{\tilde{\rho}_n} + n \tag{37}$$

 Shift produces a 'critical' eigenvalue after which couplings become irrelevant, while fixed point and eigenvalues for G̃, Λ̃ remain unaffected

The number of relevant eigenvalues appears to become finite, and the theory predictive

Summary

- A fully diffeomorphism-invariant unimodular theory was studied within the Wilsonian approach of the exact renormalisation group and asymptotic safety
- The effective action of the unimodular theory was shown to share a similar structure with that of GR in the UV, and the two theories shared similar UV completions within Asymptotic Safety
- The UV fixed-point and corresponding eigen-values for Newton's G and A were found to be similar with those of the Einstein-Hilbert truncation, and the unimodular theory appeared to be predictive
- Away from the UV-fixed point regime, the non-trivial (Stückelberg) interactions generated along the RG flow in principle make the unimodular theory to differ from its GR counterpart

Thank you!

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