Asymptotic safety and scalar-tensor theories

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Asymptotic Safety Seminar

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Outline

- Motivations
- Gravitational sector: parameterization and gauge fixing
- Flow equations and fixed point analysis
- Extension to O(N) models: some results

With R. Percacci and P. Labus

- Conclusions and outlook

Motivations

We consider here a scalar field interacting with gravity.
This general theory is interesting at pure theoretical level, but also for the implications in cosmology

In a QFT formulation with metric description,
one expects a general non local effective action: $\Gamma[g_{\mu\nu}, \phi]$

Fundamental problems to be solved: UV completion and UV-IR flows.
Asymptotic safety paradigm and FRG techniques. (Reuter)

In this framework we cannot avoid the use of a background field formalism.
In a metric formulation (euclidean).: $g_{\mu\nu}\left(\bar{g}_{\mu\nu}, h_{\mu\nu}\right)$

- Issue of the double metric description / modified splitting Ward Identities.
- Issue of choosing truncations as well as coarse-graining schemes. Simple enough, but able to keep some features of the full theory.

Many degrees of approximations in the covariant description:
Single metric (field) descriptions can be non local and complicated: $\Gamma_k = \int d^d x \sqrt{g}\ \mathcal{L}[\phi, R_{\mu\nu\lambda\sigma}]$

On maximally symmetric background (e.g. a sphere), for a local "LPA" truncation, still not so trivial!

$$\Gamma[\phi, g_{\mu\nu}] = \int \mathrm{d}^d x \sqrt{g} \left[F(\phi, R) + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \right]$$

(Narain, Rahmede)

… motivations

Simplest approximation: expand $F(\phi, R)$ around R=0 and keep only terms up to single power
LPA truncation:

$$\Gamma_k[\phi, g] = \int d^d x \sqrt{g} \left(V(\phi) - F(\phi) R + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \right) + S_{GF} + S_{gh}$$

This theory contains the E-H action with a cosmological constant if we remove the scalar.

Previously studied (Narain, Percacci) (single metric) but difficult to find fixed point solutions: singular structure induced by the power expansion in the background scalar curvature R around the origin. Too far off-shell!!!

Scalar tensor (ST)

$$\dot{v} = \frac{1}{3\pi^2} \left[\frac{f}{f - v} + \dots \right]$$

E-H

$$\dot{\lambda} \sim \frac{1}{1 - 2\lambda}$$

$$\varphi = \phi\, k^{-(d-2)/2}$$
$$\lambda = \Lambda / k^2$$
$$v \sim \varphi^d \qquad f \sim \varphi^2$$

Pole in the denominator —> IR singularity (d>2), for ST problem also in fixed point equation.

Why? Spin 2 fluctuations, for $g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$, couple to the scalar potential via the metric determinant

$$\sqrt{g} = \sqrt{\bar{g}} \left(1 + \frac{h}{2} + \frac{h^2}{8} - \frac{1}{4} h^{\mu\nu} h_{\mu\nu} + \cdots \right) \longrightarrow \left(\Gamma^{(2,TT)} \right)^{-1} \sim F \left(-\nabla^2 + \frac{d^2 - 3d + 4}{d(d-1)} R \right) - V$$

Gravity sector: the metric

A way to avoid this problem: use an exponential parameterization of the metric:

$$g_{\mu\nu} = \bar{g}_{\mu\rho}(e^h)^\rho{}_\nu$$

Indeed $\det e^h = e^{\mathrm{tr} h}$ so that $\sqrt{g} = e^{h/2}\sqrt{\bar{g}} = \sqrt{\bar{g}}\left(1 + \frac{h}{2} + \frac{h^2}{8} + \cdots\right)$ $\quad tr h = h = 2d\omega$

In this way the potential V couples only to the trace of the metric fluctuations.

We take the attitude that the metric could be seen as a non linear object naturally preferring the exponential parametrization.

Think about the non linear parametrization based on frames and vielbeins…

Remark: at quantum level in general the off shell effective action which can be constructed with the exponential parametrization is not equivalent to the one with the linear parameterization if

- a Jacobian is not taken into account (but in our single metric truncation does not contribute)
- the geometric formulation a la Vilkowisky-De Witt in not considered (known at one loop), e.g. the sources couple to different objects, expectations values are not trivially related, …

Not also that this change of variables is never singular and the Jacobian is well defined.

Gravity sector: gauge transformations

Gravity is a gauge theory: physics does not change under diffeomorphisms.

$$\delta_\epsilon g_{\mu\nu} = \mathcal{L}_\epsilon g_{\mu\nu} \equiv \epsilon^\rho \partial_\rho g_{\mu\nu} + g_{\mu\rho}\partial_\nu \epsilon^\rho + g_{\nu\rho}\partial_\mu \epsilon^\rho$$

The quantum gauge transformation for the fluctuations defined in the exponential parametrization:

$$\delta_\epsilon^{(Q)} h^\mu{}_\nu = (\mathcal{L}_\epsilon \bar{g})^\mu{}_\nu + \mathcal{L}_\epsilon h^\mu{}_\nu + [\mathcal{L}_\epsilon \bar{g}, h]^\mu{}_\nu + O(\epsilon h^2) \ .$$

For a single metric truncation, to define the gauge fixing and ghost terms, it is enough to keep:

$$\delta_\epsilon^{(Q)} h_{\mu\nu} = \bar{\nabla}_\mu \epsilon_\nu + \bar{\nabla}_\nu \epsilon_\mu + O(h)$$

York decomposition of the metric:

$$h_{\mu\nu} = h^{TT}{}_{\mu\nu} + \bar{\nabla}_\mu \xi_\nu + \bar{\nabla}_\nu \xi_\mu + \bar{\nabla}_\mu \bar{\nabla}_\nu \sigma - \frac{1}{d}\bar{g}_{\mu\nu}\bar{\nabla}^2\sigma + \frac{h}{d}\bar{g}_{\mu\nu}$$

and similarly for the diffeomorphism generator

$$\epsilon^\mu = \epsilon^{T\mu} + \bar{\nabla}^\mu \frac{1}{\sqrt{-\bar{\nabla}^2}}\psi \ ; \qquad \bar{\nabla}_\mu \epsilon^{T\mu} = 0$$

Transformations:

$$\delta_{\epsilon^T}\xi^\mu = \epsilon^{T\mu} \qquad \delta_\psi \sigma = \frac{2}{\sqrt{-\bar{\nabla}^2}}\psi \qquad \delta_\psi h = -2\sqrt{-\bar{\nabla}^2}\psi$$

Gauge invariant quantities

$$s = h - \bar{\nabla}^2 \sigma \qquad h^{TT}_{\mu\nu}$$

To adsorbe some Jacobians one can redefine:

$$\xi'_\mu = \sqrt{-\bar{\nabla}^2 - \frac{\bar{R}}{d}}\,\xi_\mu \ ; \qquad \sigma' = \sqrt{-\bar{\nabla}^2}\sqrt{-\bar{\nabla}^2 - \frac{\bar{R}}{d-1}}\,\sigma$$

Gravity sector: gauge fixing and ghosts

We shall use the so called physical gauge fixing: set to zero the gauge dependent fluctuations.
Therefore in the path integral there remain two kind of gauge invariant fluctuations: s and $h_{\mu\nu}^{TT}$

We face two possible GF choices:

I: $\xi'_\mu = 0 \quad , \quad h = \text{const.}$ II: $\xi'_\mu = 0 \quad , \quad \sigma' = 0$

Remark: for compact spaces (e.g. a sphere) no diffeomorphism can change the constant mode of h. This mode contributes to order R^2 in the effective action so for our linear truncation we can set it to zero as well.

Faddeev Popov determinants: varying the GF conditions.

$$\delta(\xi'_\mu) \qquad \det\left(\sqrt{-\bar{\nabla}^2 - \frac{\bar{R}}{d}}\right)$$

$$\delta(h-\text{const}) \qquad \det(\sqrt{-\bar{\nabla}^2})$$

$$\delta(\sigma') \qquad \det\left(\sqrt{-\bar{\nabla}^2 - \frac{\bar{R}}{d-1}}\right)$$

Gravitational hessian

The pure E-H action has a very simple Hessian using gauge invariant variables. For example:

$$\frac{1}{2}\int dx\sqrt{\bar{g}}\left[\frac{1}{2}h^{TT}{}_{\mu\nu}\left(-\bar{\nabla}^2+\frac{2\bar{R}}{d(d-1)}\right)h^{TT\mu\nu}-\frac{(d-1)(d-2)}{2d^2}s\left(-\bar{\nabla}^2-\frac{\bar{R}}{d-1}\right)s-\frac{d-2}{4d}R\,h^2\right]$$

E-H truncation with type I cutoff and gauge fixing I (h=0).
Flow equations for the dimensionless couplings: $\tilde{\Lambda}=\Lambda/k^2$ and $\tilde{G}=Gk^{d-2}$

$$\partial_t\tilde{G} = (d-2)\tilde{G}+B\tilde{G}^2 \ ,$$
$$\partial_t\tilde{\Lambda} = -2\tilde{\Lambda}+\frac{1}{2}AG+B\tilde{G}\tilde{\Lambda} \ ,$$

$$A_1 = \frac{16\pi(d-3)}{(4\pi)^{d/2}\Gamma[d/2]}$$
$$A_2 = -\frac{16\pi(d-1)}{(4\pi)^{d/2}(d+2)\Gamma[d/2]}$$
$$B_1 = \frac{16\pi(d^5-4d^4-9d^3-48d^2+60d+24}{(4\pi)^{d/2}12d^2(d-1)\Gamma[d/2]}$$
$$B_2 = -\frac{16\pi(d^5-15d^3-58d^2+48}{(4\pi)^{d/2}12d^2(d-1)(d+2)\Gamma[d/2]}$$

At fixed point $\tilde{G}_*>0$
for $1<d\leq 6$

For type II cutoff (same results as K. Falls 2015)
$\tilde{G}_*>0$ for $0\leq d\leq 7$

A global flow from UV to IR exists. There is no singularity at $\tilde{\Lambda}=1/2$

E-H for $d \to 2 + \epsilon$

Comparison for pure gravity.
linear vs exponential parametrizations
Standard De Donder vs physical gauge fixing

See also: Nink, Codello-D'Odorico

$$S_{GF} = \frac{1}{2\alpha} \int d^d x \sqrt{\bar{g}}\, \bar{g}^{\mu\nu} \chi_\mu \chi_\nu \; ; \qquad \chi_\mu = \bar{\nabla}_\rho h^\rho{}_\mu - \frac{1+\beta}{d} \bar{\nabla}_\mu h$$

In York decomposition:

$$\chi_\mu = \left(\bar{\nabla}^2 + \frac{\bar{R}}{d} \right) \xi_\mu + \bar{\nabla}_\mu \left(\frac{d-1}{d} \left(\bar{\nabla}^2 + \frac{\bar{R}}{d-1} \right) \sigma - \frac{\beta}{d} h \right)$$

$-B_1$	St. $\beta = 0$	St. $\beta = \infty$	Phys. $\sigma' = 0$	Phys. $h = 0$
Linear	$\frac{38}{3}$	$\frac{26}{3}$	$\frac{38}{3}$	$\frac{26}{3}$
Exponential	$\frac{50}{3}$	$\frac{38}{3}$	$\frac{50}{3}$	$\frac{38}{3}$

In presence of Λ there is a dicontinuity and we find for h=0 $-B_1 = \frac{52}{3}$

Scalar-gravity system

Truncation with even potentials: $\Gamma_k[\phi, g] = \int d^dx \sqrt{g} \left(V(\phi) - F(\phi) R + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \right)$

Expanding also around a constant background $\phi = \bar{\phi} + \delta\phi$

Simple mixed gravity-scalar term: $\int dx \sqrt{g}\, \delta\phi \left[\left(V'(\bar{\phi}) - F'(\bar{\phi}) R \right) \frac{h}{2} - F'(\bar{\phi}) \frac{d-1}{d} \left(-\nabla^2 - \frac{R}{d-1} \right) s \right]$

The hessian, gauge fixed ($\xi'_\mu = 0$, $h = 0$) and for a shifted $\sigma'' = \sigma' + \cdots$ is diagonal:

$$\int dx \sqrt{\bar{g}} \left[F(\bar{\phi}) \frac{1}{4} h^{TT}{}_{\mu\nu} \left(-\bar{\nabla}^2 + \frac{2\bar{R}}{d(d-1)} \right) h^{TT\mu\nu} - \frac{(d-1)(d-2)}{4d^2} F(\bar{\phi}) \sigma'' (-\bar{\nabla}^2) \sigma'' \right.$$
$$\left. + \frac{1}{2} \delta\phi \left(-\bar{\nabla}^2 + V''(\bar{\phi}) - F''(\bar{\phi}) \bar{R} + 2 \frac{d-1}{d-2} \frac{F'(\bar{\phi})^2}{F(\bar{\phi})} \left(-\bar{\nabla}^2 - \frac{\bar{R}}{d-1} \right) \right) \delta\phi \right]$$

To plug this into the Wetterich equation we need to choose some appropriate coarse-graining cutoff operator: type I, type II, or (scalar-) pure cutoff.

We first consider a type I cutoff: $-\bar{\nabla}^2 \;\longrightarrow\; P_k(-\bar{\nabla}^2) = -\bar{\nabla}^2 + R_k(-\bar{\nabla}^2)$

This cutoff depends explicitely on $F_k(\bar{\phi})$

Going to dimensionless quantities $f(\varphi) = k^{2-d} F(\phi)$ $\quad v(\varphi) = k^{-d} V(\phi)$
we can obtain the flow equations.

Analysis of type I flow equations

To investigate fixed point solutions in this infinite dimensional space of "couplings" we consider in d dimensions the following cases:

A. The full equations

B. The ones in the "one loop" approximation, neglecting $\dot{F}_k(\bar{\phi})$ on the r.h.s of the flow equations. It can help to understand the cutoff dependence on $F_k(\bar{\phi})$

These equations have some analytic fixed point solutions of the kind :

$$v(\varphi) = v_0$$
$$f(\varphi) = f_0 + \frac{\xi}{2}\varphi^2$$

	A	B
FP1	$(v_{0A}, f_{0A}, \xi = 0)$	$(v_{0B}, f_{0B}, \xi = 0)$
FP2	-	$(v_{0B}, f_{02B}, \xi > 0)$
FP3	$(v_{03}, f_{03} = 0, \xi < 0)$	

We have analyzed the eigenperturbations of these solutions for d=3 and d=4 cases analytically or numerically.

For example for FP1 in d=4 of case A 4 relevant and 1 marginal directions:

$$v_0 = 0.00396 \qquad f_0 = 0.0069$$

Phenomenologically interesting

$$\theta_1 = 4, \qquad w_1^t = (\delta v, \delta f)_1 = (1, 0)$$
$$\theta_2 = 2.553, \qquad w_2^t = (\delta v, \delta f)_2 = (-1, 1.236)$$
$$\theta_3 = 2, \qquad w_3^t = (\delta v, \delta f)_3 = (c_{3v} + \varphi^2, c_{3f})$$
$$\theta_4 = 0.553, \qquad w_4^t = (\delta v, \delta f)_4 = (c_{4v0} - \varphi^2, c_{4f} + 1.236\varphi^2)$$
$$\theta_5 = 0, \qquad w_5^t = (\delta v, \delta f)_5 = (c_{5v0} + c_{5v0}\varphi^2 + \varphi^4, c_{5f0} + c_{5f2}\varphi^2)$$

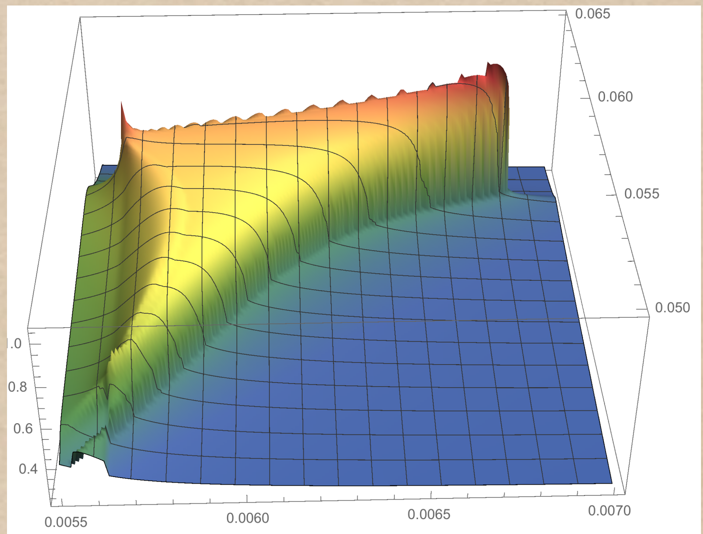
Further analysis of case B

In d=3 we expect to exists a deformation of the WF fixed point which in flat space belongs to the Ising universality class.

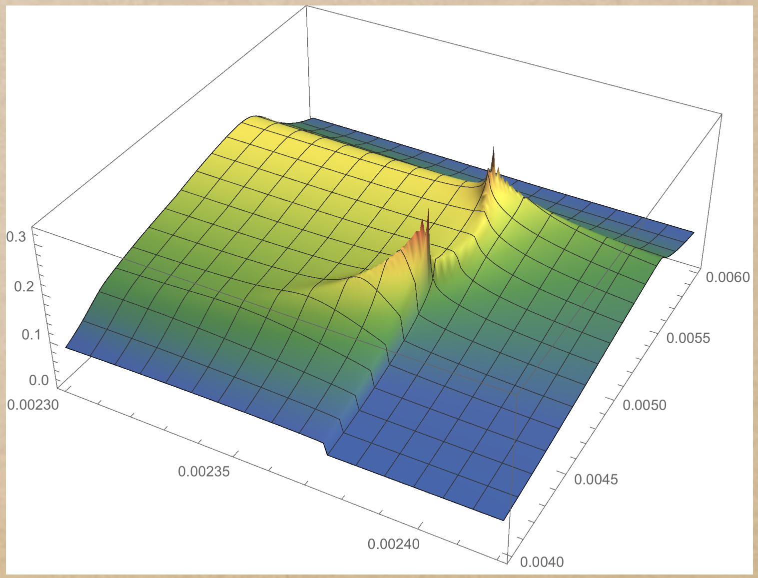
We have employed shooting methods (Morris), from the origin and the asyptotic region and various types of polynomial expansions as well.

Shooting method from the origin gives this picture:

$$0.0055 < v(0) < 0.0070 \text{ and } 0.050 < f(0) < 0.065.$$



Three spikes corresponds to FP1, FP2, and possibly a non trivial WF solution.
This solution, which we have investigated also with polynomial expansions, has the property to cross f=0 starting from f(0)>0, so that is defined as an analytic continuation.



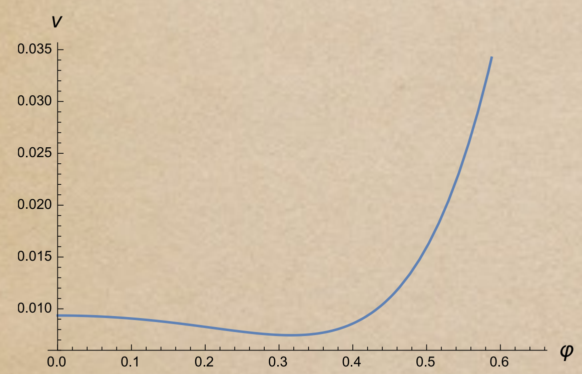
For d=4 from the shoting methos we have no indications that a WF type of fixed point do exist, similarly to the flat space case. In the region shown we see FP1 and FP2.

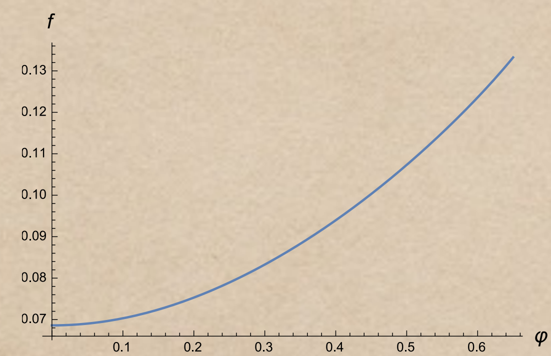
More on case A

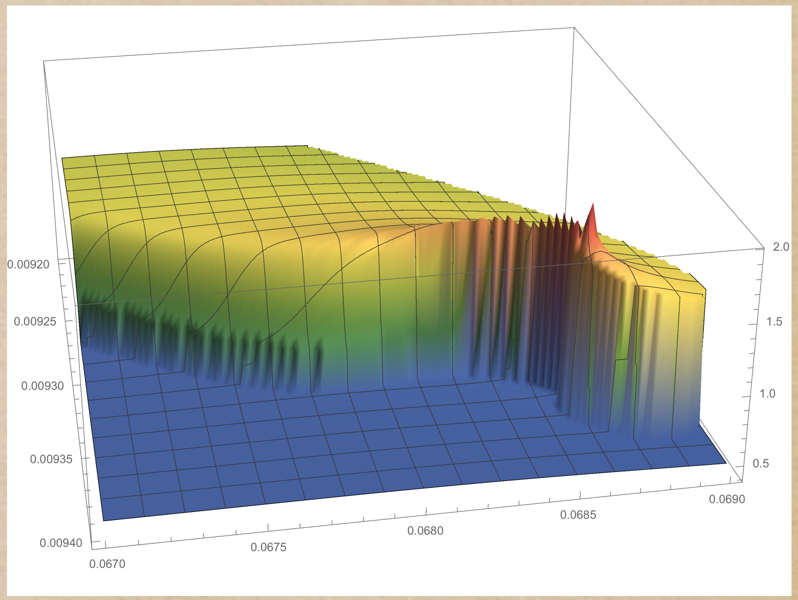
The search of a WF fixed point for these full equations was recently addressed (Borchardt-Knorr) using pseudospectral methods (based on Chebitchev polynomials).

For d=3 they show that there exist a WF-like solution, which is constructed with great precision. It has 4 relevant directions. f is always positive.

Indeed this solution can be found by shooting methods and standard polynomial expansion analysis







The full equations admit this solution, contrary to the "one loop" approximation.
These are schemes based on a spectrally adjusted cutoff so that both split symmetries are broken

$$\delta\phi \to \delta\phi + \delta\psi,\ \bar{\phi} \to \bar{\phi} - \delta\psi \qquad h_{\mu\nu} \to h_{\mu\nu} + \delta h_{\mu\nu},\ \omega \to \omega + \delta\omega,\ \bar{g}_{\mu\nu} \to \bar{g}_{\mu\nu} - \delta h_{\mu\nu} - 2\bar{g}_{\mu\nu}\delta\omega$$

Background-scalar independent cutoff

It is possible to explore the flow equations obtained in other cutoff schemes, in particular not spectrally adjusted. These so called pure cutoffs respect the scalar split symmetry. One cannot avoid instead the gravitational background dependence in the quadratic cutoff operator.

A linear cutoff $R_k(z) = \gamma k^a (k^2 - z)\theta(k^2 - z)$ leads to more complicated equations. The parameter γ could be used for optimizations in the minimal sensitivity sense.

Preliminary analysis: we can find easily the constant analytic solution (FP1) for any d.

E.g. in d=4 and $\gamma = 1$ we have $v_0 = 0.0299 \qquad f_0 = 0.01368$

The pattern for the critical exponents and the eigenperturbations is very similar to the FP1 of case A, with a slight change in some numbers.

$$\begin{aligned}
&\theta_1 = 4, && w_1^t = (\delta v, \delta f)_1 = (1, 0) \\
&\theta_2 = 2.307, && w_2^t = (\delta v, \delta f)_2 = (-1, 0.663) \\
&\theta_3 = 2, && w_3^t = (\delta v, \delta f)_3 = (c_{3v} + \varphi^2, c_{3f}) \\
&\theta_4 = 0.307, && w_4^t = (\delta v, \delta f)_4 = (c_{4v0} - \varphi^2, c_{4f} + 0.663\varphi^2) \\
&\theta_5 = 0, && w_5^t = (\delta v, \delta f)_5 = (c_{5v0} + c_{5v0}\varphi^2 + \varphi^4, c_{5f0} + c_{5f2}\varphi^2)
\end{aligned}$$

We have to complete the search for other less trivial global solutions, also in d=3.

Other interesting cutoff we want to investigate: power like type (Morris)

O(N) scalars coupled to gravity

Again for a local truncation il "LPA" one might consider the Lagrangian $F_k(\rho, R)$ as a generic function of R and $\rho = \varphi^a\varphi^a/2$, for a maximally symmtric background.
This is currently under investigation (R. Percacci, G.P.V)

It presents a grade of complexity very similar to the case of f(R) gravity which is included as a subset. Simpler models are obtained by expanding in power of R, at cosmological level being interesting essentially powers up to 2. On expanding one faces againg possible problems of far off-shellness.

Expansion up to a linear term.
Direct extension of the single scalar field case (P. Labus, R. Percacci, G.P.V to appear soon)

$u_k(\rho)$ $f_k(\rho)$ describe again our linear truncation of the EAA.

Flow equations are similar for the same cutoff scheme choice.
There are now two external physical parameters: d and N

Scalar O(N) coupled to gravity

We have written the equations for typeI-II cutoffs and for a pure cutoff similarly to the previous case.

Here I show some preliminary results for the type I cutoff for the full equations.

Analytic solutions for any (d,N): e.g. for d=4 we find			Physical for
FP1:	$u = \frac{1}{32\pi^2} + \frac{N}{128\pi^2}$	$f = \frac{169 - 12N}{2304\pi^2}$	$N < \frac{169}{12}$
FP3:	$u = \frac{1}{64\pi^2} + \frac{N}{128\pi^2}$	$f = \frac{41 - 6N - \sqrt{4(N-25)N + 1321}}{24(N-1)}\rho$	Never
FP4:	$u = \frac{1}{64\pi^2} + \frac{N}{128\pi^2}$	$f = \frac{41 - 6N + \sqrt{4(N-25)N + 1321}}{24(N-1)}\rho$	$1 < N < \frac{45}{2}$

FP1 is the usual fully constant solution with interesting physical implications.
FP3 is the branch with a finite limit at N=1.
FP4 is a new interesting possibly physical scaling solution with a non minimally coupling.
In the "one loop approximation there is also a FP2 solution similar to the single field case.

Example d=4, N=4:
FP1 has critical exponents: (4, 2.782, 2, 0.782, 0). Eigenperturbations similar to N=1 case.
FP4: not yet studied, this analysis is important for this fixed point to be considered physical or an artifact of the type I cutoff choice / or of the truncation.

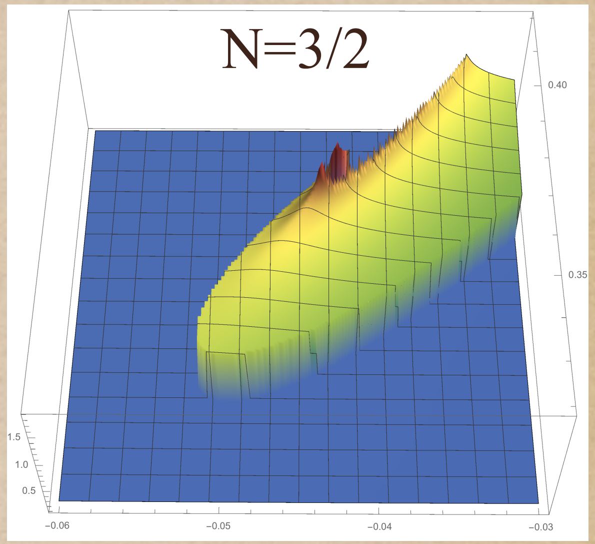
Scalar O(N) coupled to gravity

<u>Other cutoff schemes:</u>

Flow equations for the type II cutoff looks very similar, not yet investigated.

For a pure cutoff case we confirm the existence of FP1, with similar properties, the rest of the FP pattern is still to be explored.

For d=3 (again type I cutoff) we can find the existence of a non trivial WF fixed point, at least for N up to 2.
We have traced this with shooting methods and polynomial expansion.



Plane of first derivatives of u and f at the origin.

We are considering also the large N limit analysis for different cutoffs, since it should be possible to proceed analytically.

Conclusions

- We went back to the problem of scalar fields interacting with gravity. Depending on the truncations chosen as usual one may encounter difficulties to find fixed point solutions and in constructing global flows.
- The choice of how to parametrize the metric fluctuations can be important The exponential parametrization, being an interesting choice by itself, can help to bypass some bad features brought in by poor truncations.
- We also propose the use of a different kind of gauge fixing procedure related to the York decomposition.
- For a single scalar field case we obtain much simpler flow equations compared to the previous approach. We find some analytical solutions. In d=3 they admit a WF scaling solution.
- We have also used the same approach the analyze the linear O(N) scalar model coupled to gravity. It presents similar features but admit a new non minimally coupled scaling solution for N>1.

Outlook

- Type I and type II cutoff, being spectrally adjusted in this framework, may be dangerous, breaking the field splitting in the scalar sector.
  We have started to use alternative cutoffs. More work is needed.
  In the gravitational sector the well known issue of dependence on the backgroud metric has to be addressed. This is related to the double metric framework and the msWI.
- In this framework we expect no special difficulties to construct a global flow for the RG trajectories from the UV to the IR. These are needed also for any phenomenological application.
- At the level of larger truncations we have started to analyze the local truncation based on a lagrangian $F_k(\rho, R)$
  We expect to obtain much simpler equations that in a previous works.
- In this formalism it could be interesting to go beyond the maximally symmetric background.
- Anomalous dimensions? Fermions and vectors?

Many thanks for your attention!