Global solutions of functional fixed point equations via pseudo-spectral methods

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Research Training Group
Quantum and Gravitational Fields

Outline

- Motivation
- Basics of pseudo-spectral methods
- O(1) model in d=3 and d=2.4
- Gross-Neveu model in d=3
- Scalar-tensor model in d=3 (see also last talk by G. P. Vacca)
- Quo vadis f(R)-gravity?
- Summary and Outlook

Motivation

- generic situation in FRG-business: non-linear coupled ODEs/PDEs that capture critical physics near fixed point
- standard treatment: local expansions or shooting method
- downside: expensive, global aspects hard to calculate
- idea: expand in orthogonal polynomials that are defined on whole interval
  $\Rightarrow$ **(rational) Chebyshev polynomials**

Basics of pseudo-spectral methods I

- for simplicity, stick to $\mathbb{R}_+$
- reminder: Chebyshev polynomials (of first kind):

$$T_n(\cos(x)) := \cos(nx)$$

- rational Chebyshev polynomials:

$$R_n(x) := T_n\left(\frac{x-L}{x+L}\right), L > 0$$

- divide $\mathbb{R}_+$ into two regions:
  - $x \in [0, x_0]$: Chebyshev polynomials
  - $x \in [x_0, \infty]$: rational Chebyshev polynomials

Basics of pseudo-spectral methods II

- thus, expand any function f via

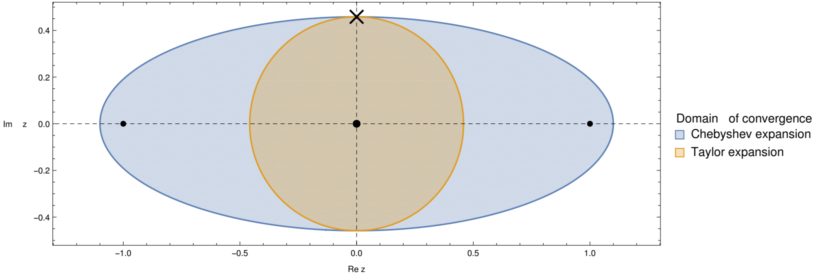
$$f(x) = \begin{cases} \sum_{i=0}^{N_c} c_i T_i\left(\frac{2x}{x_0} - 1\right), \; x \leq x_0\,, \\ f_\infty(x) \sum_{i=0}^{N_r} r_i R_i(x - x_0), \; x \geq x_0\,, \end{cases}$$

- two free numerical parameters:
  - x_0: matching point, should be large enough that essential physics "happens" for $x < x_0$
  - L: encodes the specific compactification
- both parameters can be used to optimize numerical convergence

Basics of pseudo-spectral methods III

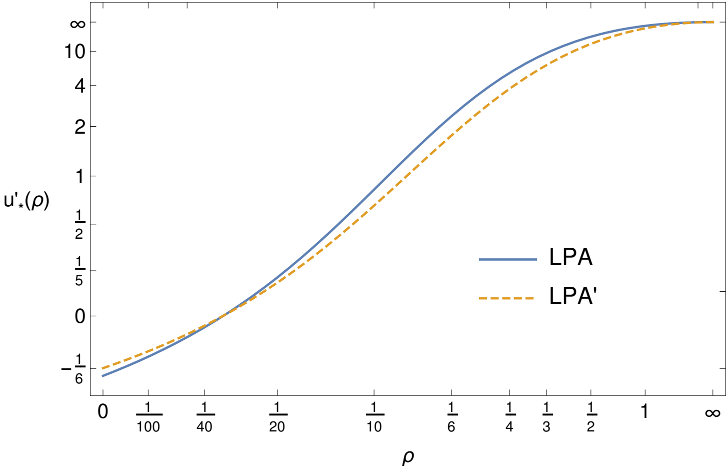
- evaluation of a Chebyshev series via recursive Clenshaw algorithm
- calculation of derivative of a Chebyshev series via recursive algorithm (again yields Chebyshev series)
- coefficients of the series encode the convergence properties and deliver estimate on (series) truncation error
- for "sufficiently nice" functions: **exponential convergence**, i.e. series coefficients decrease exponentially fast

Taylor expansion vs. Chebyshev expansion

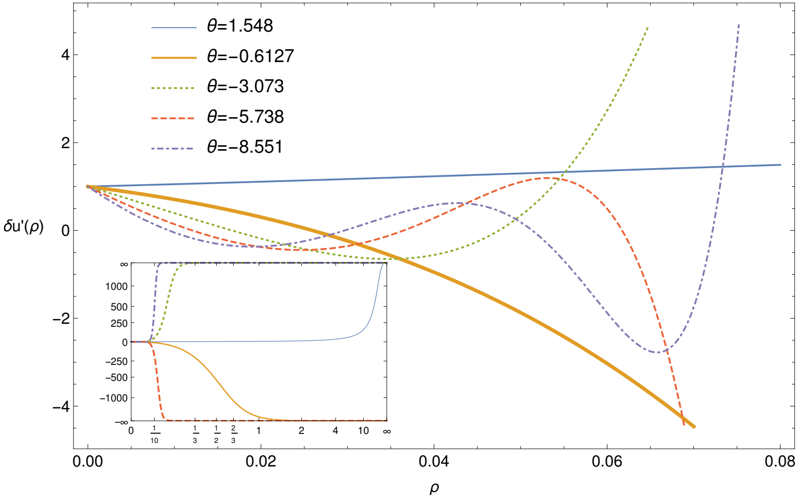


Questions?

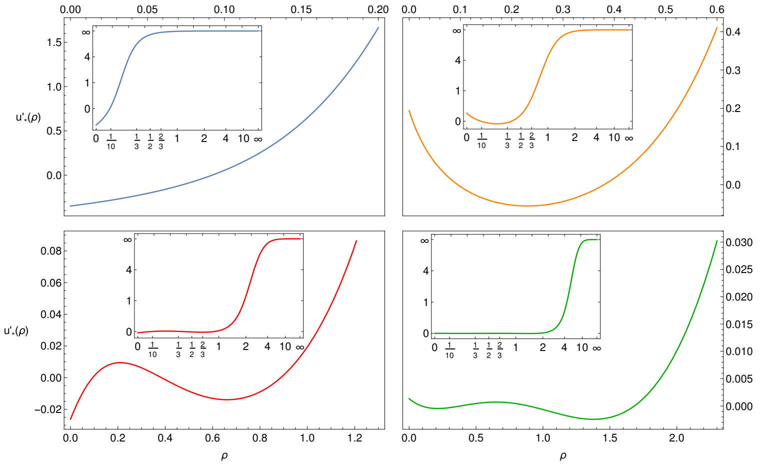
O(1) model in d=3, LPA and LPA'



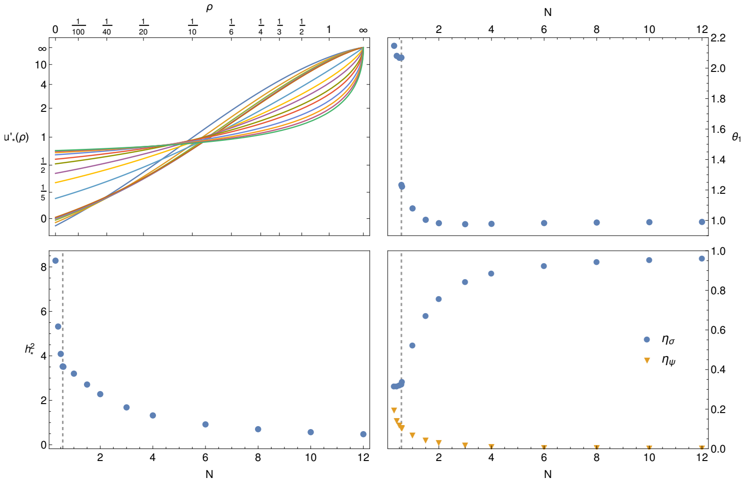
O(1) model in d=3, eigenperturbations in LPA'



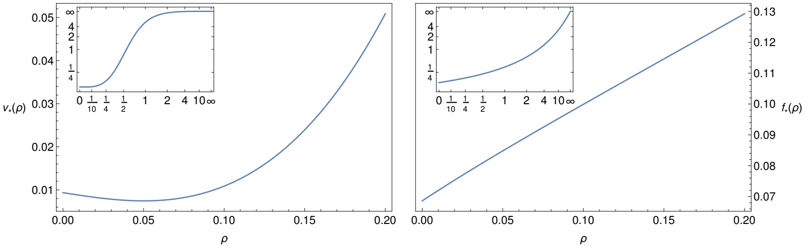
O(1) model in d=2.4, LPA'



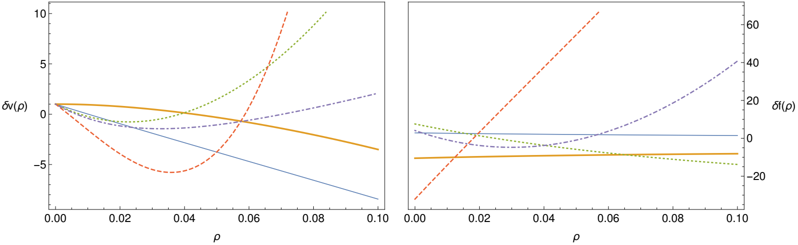
Gross-Neveu model in d=3, LPA'



Scalar-tensor model in d=3



Scalar-tensor model in d=3, eigenperturbations



$$\theta_1 = 3\,, \qquad \theta_2 = 1.913\,, \qquad \theta_3 = 1.180\,,$$

$$\theta_4 = 0.6679\,, \qquad \theta_5 = -0.2812\,, \qquad \theta_6 = -1.217$$

The problem with f(R)-gravity

- choice of regulator impedes globally well-defined solution(s)
- problem lies in “spectral adjusting”
- in case of f(R):

$$R_{\mathrm{TT}} \propto f'(R)$$

- local solutions give some evidence that for some R_0, $f'(R_0) = 0$
  $\rightarrow$ regulator changes sign, proper regularization questionable
- similar in scalar-tensor model

Summary

- pseudo-spectral methods are handy to represent solutions to fixed point equations
  - globally,
  - with high precision,
  - efficiently
- perturbations to fixed point equations can be resolved in that way as well $\rightarrow$ high-precision critical exponents
- application to flows: stay tuned

Controversial statement(s)

- local expansions are uncontrolled (anyway, we want to do non-perturbative physics!)
- shooting method gets expensive very fast (starting values for every operator, anomalous dimensions self-consistenly, . . .)
- (global) spectral methods solve both problems

Thank you for your attention!