Asymptotically safe inflation from quadratic gravity

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Quantum generation of initial spectrum

Quantum fluctuations

$$(a\,\delta\phi_k)'' + \left(k^2 + \frac{z''}{z}\right)(a\,\delta\phi_k) = 0 \qquad\qquad z = \frac{a\,(\partial_t \phi)}{H}$$

Two-point correlations function:

$$\langle 0|\mathcal{R}_k \mathcal{R}_p|0\rangle = \frac{(2\pi)^3}{2\,k^3}\,\mathcal{P}_{\mathcal{R}}(k)\,\delta^3(k+p) \qquad\qquad \mathcal{R} = -\,H\,\frac{\delta\phi}{\partial_t \phi}$$

Spectral index and tensor-to-scalar-ratio

$$\mathcal{P}_{\mathcal{R}}(k) = A_s \left(\frac{k}{k_0}\right)^{n_s - 1 + \frac{\log(k/k_0)}{2}\frac{\mathrm{d}n_s}{\mathrm{d}\log(k)}}$$

$$\mathcal{P}_t(k) = A_t \left(\frac{k}{k_0}\right)^{n_t} \qquad\qquad r \equiv \frac{A_t}{A_s} \qquad\qquad k_0 \sim 0.05\,\mathrm{Mpc}^{-1}$$ **pivot scale**

Slow-roll inflation and power spectrum

Scalar field - **inflaton** - us a perfect fluid with:

$$\rho_\phi \sim \frac{1}{2}(\partial_t \phi)^2 + V(\phi) \qquad p_\phi \sim \frac{1}{2}(\partial_t \phi)^2 - V(\phi)$$

Definitions of **slow-roll parameters**:

$$\epsilon(\phi) = \frac{1}{2\,\kappa^2}\left(\frac{V'(\phi)}{V(\phi)}\right)^2 \qquad \eta(\phi) = \frac{1}{\kappa^2}\left(\frac{V''(\phi)}{V(\phi)}\right)$$

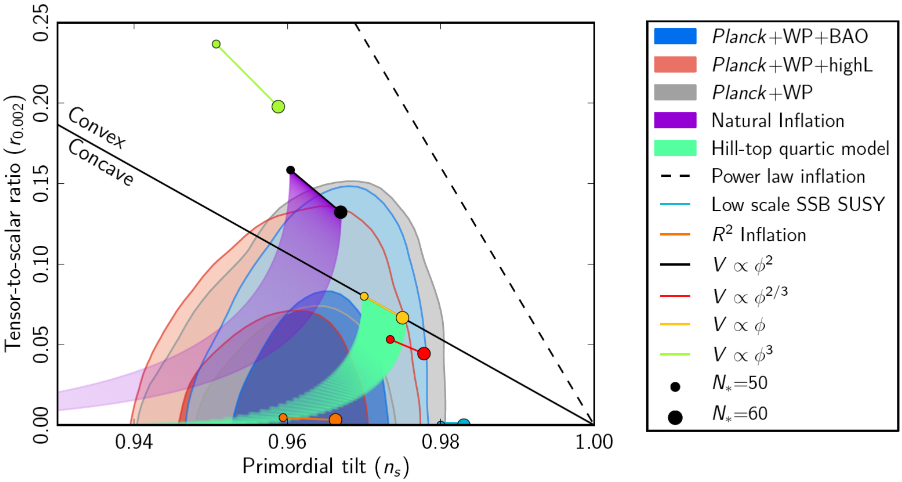
$$\text{Slow-roll inflation} \quad \Leftrightarrow \quad \epsilon(\phi) \ll 1 \quad \wedge \quad \eta(\phi) \ll 1$$

The values of the slow-roll parameters identify:

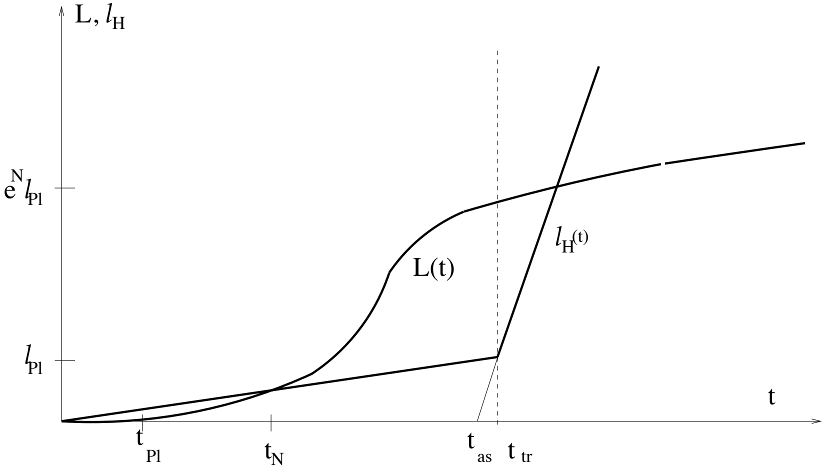
Spectral index n_s, tensor-to-scalar-ratio r

$$n_s = 1 + 2\,\eta(\phi_i) - 6\,\epsilon(\phi_i) \qquad r = 16\,\epsilon(\phi_i)$$

Inflationary models and cosmic data



Problem: Super-Planckian initial conditions



$H(t) \sim a/t \quad a > 1$ A. Bonanno, M. Reuter, JCAP 08 (2007) 024

Asymptotically safe inflation from quadratic gravity

AS inflation Weinberg Phys. Rev. D 81, 083535 (2010)

Consider a general truncation to obtain a de Sitter solution which is unstable but lasts $N > 60$ e-folds.

Let us consider the **UV Lagrangian**:

$$\mathcal{L}_k = \frac{k^2}{16\pi g(k)}[R - 2\,k^2\lambda(k)] - \beta(k)R^2 \qquad (1)$$

Where g_k, λ_k, β_k are dimensionless running coupling constants, such that:

$$\lim_{k\to\infty}\{g_k, \lambda_k, \beta_k\} = \{g_*, \lambda_*, \beta_*\}$$

By solving the linearized flow equations for $g(k)$, $\lambda(k)$ and $\beta(k)$, we obtain:

$$g_k = \frac{6\pi c_1 k^2}{6\pi\mu^2 + 23c_1(k^2 - \mu^2)} \tag{2}$$

$$\beta_k = \beta_* + b_0 \left(\frac{k^2}{\mu^2}\right)^{-\frac{\theta_3}{2}} \qquad \lambda_k \sim c_0\, k^{-2} \tag{3}$$

Where:

- μ is an infrared renormalization scale
- θ_3 is the critical exponent relative to $\beta(k)$
- b_0, c_0 and c_1 are free parameters

$b_0,\ c_0,\ c_1 \Longleftrightarrow$ relevant directions of the UV critical surface

The effective action can be obtained by substituting into the UV action:

$$\lambda_{\rm uv} \longrightarrow \lambda(k) \qquad g_{\rm uv} \longrightarrow g(k) \qquad \beta_{\rm uv} \longrightarrow \beta(k)$$

Cutoff scale identification?

k as function of the cosmological time t:

$$k(t) = \xi/t, \qquad \xi > 0 \tag{4}$$

However this identification breaks diffeomorphism invariance.

- By noting that: $a(t) \sim t^\alpha \quad \Rightarrow \quad R \sim 1/t^2 \sim k^2$
- We can identify:

$$k^2 \to \xi R \tag{5}$$

A. Bonanno, Phys. Rev. D 85, 081503 (2012)

E. J. Copeland, C. Rahmede, I. D. Saltas, Phys. Rev. D 91, 103530 (2015)

We thus obtain the **effective action at the inflationary era**:

$$S = \frac{1}{2\kappa^2} \int \sqrt{-g} \left[R + \alpha R^{2-\frac{\theta_3}{2}} + \frac{R^2}{6m^2} - \Lambda \right] d^4 x \tag{6}$$

Where:

- $\kappa^2 = \frac{48\,\pi^2 c_1}{6\pi\mu^2 - 23(\mu^2 + 2\,\xi\, c_0)c_1} \equiv 8\pi G_N$
- $\Lambda = \frac{\mu^2 c_0\,(6\pi - 23 c_1)}{6\pi\mu^2 - 23(\mu^2 + 2\xi c_0)c_1}$
- $\alpha = -2\,\mu^{\theta_3} b_0\, M_{\rm pl}^{-2}$

Inflation in f(R) gravity model

Let us consider the following general action:

$$S[g_{\mu\nu}] = \frac{1}{2\kappa^2} \int \sqrt{-g}\, \{R + F(R)\}\, \mathrm{d}^4 x$$

If $F''(R) \neq 0$, we can do a **conformal transformation**:

$$g_{\mu\nu} \quad \longrightarrow \quad g^{\mathrm{E}}_{\mu\nu} = \varphi g_{\mu\nu}$$

So that:

$$S[g^{\mathrm{E}}_{\mu\nu}] \equiv \int \sqrt{-g_{\mathrm{E}}} \left\{ \frac{\varphi R_{\mathrm{E}}}{2\kappa^2} - \frac{1}{2} g^{\mu\nu}_{\mathrm{E}} \partial_\mu \phi\, \partial_\nu \phi - V(\phi) \right\} \mathrm{d}^4 x$$

$$V(\phi) = \frac{1}{2\kappa^2 \varphi^2} \{(\varphi - 1) \cdot \chi(\varphi) - F(\chi(\varphi))\} \qquad \varphi = \mathrm{e}^{\sqrt{\frac{2}{3}}\, \kappa\phi}$$

In our case:

$$\varphi(\chi) \equiv 1 + \alpha \left(2 - \frac{\theta_3}{2}\right) \chi^{1-\frac{\theta_3}{2}} + \frac{\chi}{3m^2} \quad (7)$$

This relation, due to its non-linearity, cannot be inverted.

In our case, as θ_3 is rather close to unity, and in order to have an *analytical expression for* $V(\phi)$, we set $\theta_3 = 1$.

In this way we explicitly obtain the two branches:

$$\chi_\pm = \frac{3}{8}\left(27\alpha^2 m^4 + 8m^2\varphi - 8m^2 \pm 3\sqrt{3}\sqrt{27\alpha^4 m^8 + 16\alpha^2 m^6(\varphi - 1)}\right)$$

With the reality condition $\chi \geq 1 - 27m^2\alpha^2/16$.

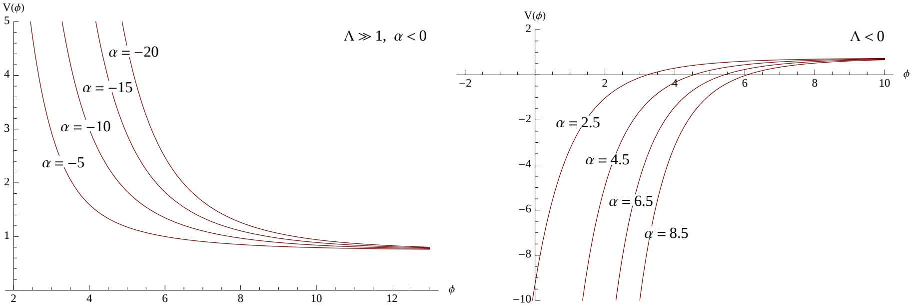
The **scalar inflationary potential** is the following:

$$V_{\pm}(\phi) = -\frac{m^2\, e^{-2\sqrt{\frac{2}{3}}\kappa\phi}}{256\,\kappa^2}\Bigg\{-192\,\left(e^{\sqrt{\frac{2}{3}}\kappa\phi}-1\right)^2+3\,\alpha^4-128\,\Lambda+$$
$$+3\,\alpha^2\left(\alpha^2+16\,e^{\sqrt{\frac{2}{3}}\kappa\phi}-16\right)\pm 6\,\alpha^2\sqrt{\alpha^2\left(\alpha^2+16\,e^{\sqrt{\frac{2}{3}}\kappa\phi}-16\right)}+$$
$$+\sqrt{32}\alpha\left(\left(\alpha^2+8\,e^{\sqrt{\frac{2}{3}}\kappa\phi}-8\right)\mp\sqrt{\alpha^2\left(\alpha^2+16\,e^{\sqrt{\frac{2}{3}}\kappa\phi}-16\right)}\right)^{\frac{3}{2}}\Bigg\}$$

We can study the **inflationary scenario** coming from the potential $V(\phi)$.

The shape of the potential depends on the values (α, Λ).

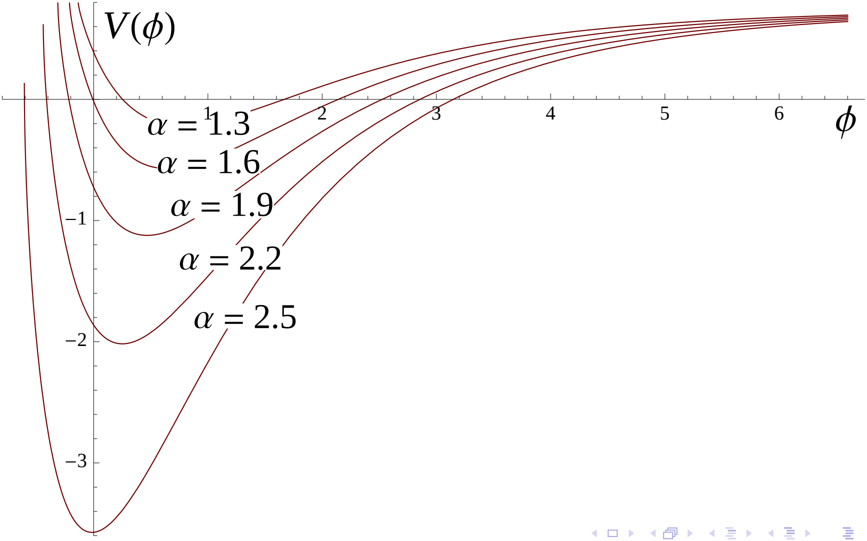
Examples:



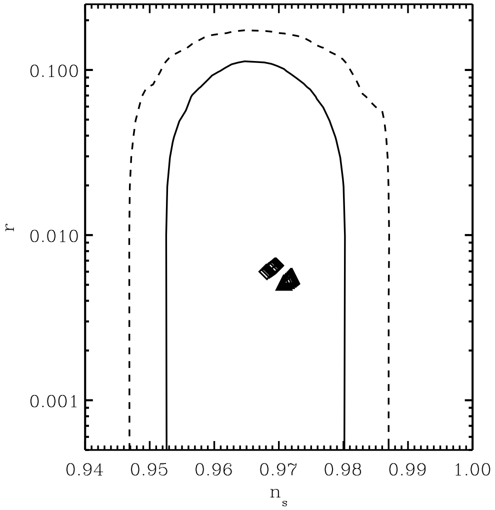
We choose ranges for α and Λ such that:

- $V(\phi)$ has a minimum (**oscillatory** phase);
- We can have a "graceful" **exit** from inflation $\Leftrightarrow V(\phi_{\mathrm{min}}) \leq 0$.

These features are verified for $V(\phi) = V_+(\phi)$ if $\alpha \in [1, 3]$ and $\Lambda \in [0, 1.5]$



- **Planck 2015**: $n_s = 0.968 \pm 0.006$ $\quad r < 0.11$
- **AS inflation**: $n_s \in [0.968, 0.970]$ $\quad r \in [0.005, 0.006]$



Oscillatory phase after inflation

After the end of inflation, the inflaton field ϕ begins to oscillate around the minimum ϕ_{min} of $V(\phi)$.

To study this phase, we can approximate:

$$V(\phi) \sim \frac{a}{2}\left[(\phi - \phi_{\text{min}})^2 - b\right]$$

Where:

- $\phi_{\text{min}} = \phi_{\text{min}}(\alpha, \Lambda)$
- $a(\alpha, \Lambda) = V''(\phi_{\text{min}})$
- $b(\alpha, \Lambda) = -2\,\frac{V(\phi_{\text{min}})}{V''(\phi_{\text{min}})}$

In particular:

$$\phi_{\min}(\alpha,\Lambda)=\frac{\sqrt{\frac{3}{2}}\left(3\alpha^3\left(\alpha^2-4\right)-32\alpha\Lambda+4\left(\alpha^2-6\right)|\alpha|^3\right)}{6\alpha\left(\alpha^2-8\right)\left(\alpha^2+2\right)-64\alpha\Lambda+8\left(\alpha^2-9\right)|\alpha|^3}$$

$$a(\alpha,\Lambda)=\frac{48+18\,\alpha^2-3\,\alpha^4+32\,\Lambda-4\,\alpha^3\,|\alpha|+36\,\alpha\,|\alpha|}{24}$$

$$\begin{aligned}b(\alpha,\Lambda)\ &=\frac{8\alpha\left(15\alpha^4-3\alpha^6-96\Lambda+8\alpha^2(15+4\Lambda)\right)|\alpha|}{\frac{8}{3}\left(48+18\alpha^2-3\alpha^4+32\Lambda-4\alpha\left(\alpha^2-9\right)|\alpha|\right)^2}\\&+\frac{-25\alpha^8+132\alpha^6-384\alpha^2\Lambda}{\frac{8}{3}\left(48+18\alpha^2-3\alpha^4+32\Lambda-4\alpha\left(\alpha^2-9\right)|\alpha|\right)^2}\\&+\frac{48\alpha^4(21+4\Lambda)-1024\Lambda(3+\Lambda)}{\frac{8}{3}\left(48+18\alpha^2-3\alpha^4+32\Lambda-4\alpha\left(\alpha^2-9\right)|\alpha|\right)^2}\end{aligned}$$

The time evolution of the field $\phi(t)$ is given by

$$\ddot{\phi}(t) + 3\,H(t)\,\dot{\phi}(t) + V'(\phi(t)) = 0 \tag{8}$$

Where:

$$H(t) \;=\; \left[\frac{1}{3}\left(\frac{1}{2}\dot{\phi}(t)^2 + V(\phi(t))\right)\right]^{1/2}$$

Putting:

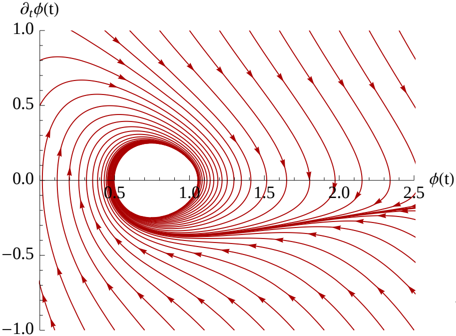
- $x(t) = \sqrt{a}\,(\phi(t) - \phi_{\mathrm{min}})$
- $y(t) = \dot{\phi}(t)$

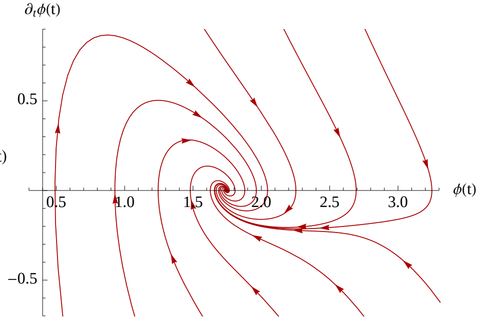
The original equation is equivalent to the following **dynamical system**:

$$\begin{cases} \dot{y} = -\left[\frac{3}{2}\left(y^2 + x^2 - \mathrm{ab}\right)\right]^{\frac{1}{2}} y - \sqrt{a}\,x \\ \dot{x} = \sqrt{a}\,y \end{cases} \tag{9}$$

The **long time behavior** is determined by the sign of $ab = -2\,V(\phi_{\min})$

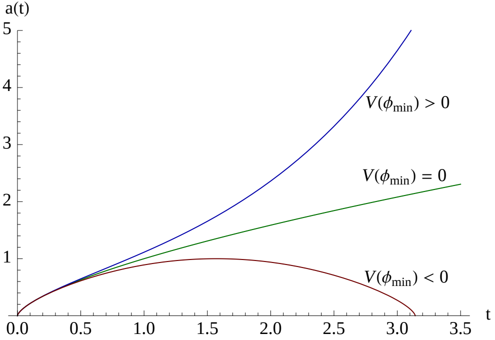
- $V(\phi_{\min}) < 0 \Rightarrow$ **Limit cycle behavior** (our case)
- $V(\phi_{\min}) > 0 \Rightarrow (\phi_{\min}, V(\phi_{\min}))$ is an **attractive node**
- $V(\phi_{\min}) = 0$ is an **Hopf bifurcation point**





This analysis can be useful to determine the **scale factor** $a(t)$. We obtain:

$$a(t) = \begin{cases} \left[\sin\left(\sqrt{\frac{3}{4}\,|V(\phi_{\rm min})|}\; t\right)\right]^{2/3} & V(\phi_{\rm min}) < 0 \\ t^{2/3} & V(\phi_{\rm min}) = 0 \\ \left[\sinh\left(\sqrt{\frac{3}{4}\,|V(\phi_{\rm min})|}\; t\right)\right]^{2/3} & V(\phi_{\rm min}) > 0 \end{cases} \qquad (10)$$



Conclusions

- Our model is significatly different from the Starobinsky model because it predicts a tensor-to-scalar ratio which is significantly higher, and a dynamics characterized by a limit-cycle behavior at the inflation exit;
- It is in agreement with Planck 2015 data;
- Present CMB data can put important constraints on the structure of the effective Lagrangian at the Planck scale;
- Limitation: simple tensorial structure of the effective Lagrangian which assumes a functional dependence of the $f(R)$ type.