# Generalized Parametrization Dependence in Quantum Gravity

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Research Training Group
Quantum and Gravitational Fields

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# Motivation

- physical observables should not depend on computational details $\Rightarrow$ Why study parametrization dependence?
- for example perturbation theory of finite order defers these dependencies to higher orders
- nonperturbative truncations depend on the "details"
- use certain criteria for "good parametrizations"

- a-priori criteria:
  - parametrizations supporting the identification of physical degrees of freedom
  - symmetry preserving formulations
  - hard gauge fixing ($\alpha \to 0$)
- a-posteriori criteria:
  - principle of minimum sensitivity
    $\Rightarrow$ stability serves as measure for quality of approximation

# Setup

- assume the metric as the fundamental degree of freedom
- single metric Einstein-Hilbert truncation

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu} + \frac{\tau}{2} h_\mu{}^\alpha h_{\alpha\nu} + \mathcal{O}(h^3),$$

$$\Gamma_{\mathrm{grav}}[g] = -\mathcal{Z}_R \int \mathrm{d}^d x \sqrt{g}\,(R - 2\Lambda), \qquad \mathcal{Z}_R = \frac{1}{16\pi G_{\mathrm{N}}}$$

- gauge-fixing condition and action

$$F_\mu[\bar{g}, g] = \left(\delta_\mu^\alpha \bar{D}^\beta - \frac{1+\beta}{d} \bar{g}^{\alpha\beta} \bar{D}_\mu\right) g_{\alpha\beta},$$

$$\Gamma_{\mathrm{gf}}[\bar{g}, g] = \frac{\mathcal{Z}_R}{2\alpha} \int \mathrm{d}^d x \sqrt{g}\; \bar{g}^{\alpha\beta} F_\alpha[\bar{g}, g] F_\beta[\bar{g}, g]$$

- ghost action

$$\Gamma_{\rm gh}[\bar{g}, g, \bar{C}, C] = -\int \mathrm{d}^d x \sqrt{g}\ \bar{C}_\mu \mathcal{M}^\mu{}_\nu[\bar{g}, g] C^\nu, \quad \mathcal{M}^\mu{}_\nu[\bar{g}, g] = \frac{\delta F^\mu[\bar{g}, g]}{\delta v^\nu}$$

- in transverse decomposition $\delta v^\mu = \delta v^{\mathrm{T}\mu} + \bar{D}^\mu \delta\chi$

$$\delta F^\mu[\bar{g}, g] \simeq (\delta^\mu_\nu \bar{D}^2 + \bar{R}^\mu_\nu)\delta v^{\mathrm{T}\nu} + \frac{1}{2}\left((d-1-\beta)\bar{D}^\mu \bar{D}_\nu + 4\bar{R}^\mu_\nu\right)\bar{D}^\nu \delta\chi$$

$\Rightarrow$ incomplete gauge fixing for $\beta = d - 1$

- employ York decomposition for the metric fluctuation

$$h_{\mu\nu} = h^{\mathrm{T}}_{\mu\nu} + 2\bar{D}_{(\mu}\xi^{\mathrm{T}}_{\nu)} + \left(2\bar{D}_\mu\bar{D}_\nu - \frac{2}{d}\bar{g}_{\mu\nu}\bar{D}^2\right)\sigma + \frac{1}{d}\bar{g}_{\mu\nu}h$$

$$\bar{D}^\mu h^{\mathrm{T}}_{\mu\nu} = 0, \qquad \bar{g}^{\mu\nu}h^{\mathrm{T}}_{\mu\nu} = 0, \qquad \bar{D}^\mu\xi^{\mathrm{T}}_\mu = 0$$

and the ghost field $C^\mu = C^{\mathrm{T}\mu} + \bar{D}^\mu\eta$

- test influence of the field redefinition

$$\left(\sqrt{\Delta - Ric}\,\xi^{\mathrm{T}}\right)^\mu \to \xi^{\mathrm{T}\mu}, \quad \sqrt{\Delta^2 + \frac{4}{3}D_\mu R^{\mu\nu}D_\nu}\;\sigma \to \sigma,$$

$$\sqrt{\Delta}\,\eta \to \eta, \qquad \Delta = -D^2$$

# Calculation

- using FRG to calculate $\beta$-functions for dimensionless Newtons constant $g = \frac{k^{d-2}}{16\pi \mathcal{Z}_R}$ and cosmological constant $\lambda = \frac{\Lambda}{k^2}$
- choose universal RG machine motivated regulator with Litim-cutoff: $\Delta \rightarrow \Delta + R_k(\Delta)$ [Benedetti et al. '11]
- split inverse regularized propagator into kinetic part $\mathcal{P}$ and curvature part $\mathcal{F}$

$$\Gamma^{(2)} + \mathcal{R}_k = \mathcal{P} + \mathcal{F} \quad \Rightarrow \quad (\Gamma^{(2)} + \mathcal{R}_k)^{-1} = \sum_{n=0}^{\infty} (-\mathcal{P}^{-1}\mathcal{F})^n \mathcal{P}^{-1}$$

- degeneracy in the scalar sector for hard gauge fixing ($\alpha \to 0$)

$$\mathcal{P}_{(\sigma h)} \xrightarrow{\alpha \to 0} \frac{1}{\alpha} \begin{pmatrix} \mathcal{O}(1) & \mathcal{O}(\beta) \\ \mathcal{O}(\beta) & \mathcal{O}(\beta^2) \end{pmatrix}, \qquad \mathcal{P}^{-1}_{(\sigma h)} \xrightarrow{\alpha \to 0} \begin{pmatrix} \mathcal{O}(\beta^2) & \mathcal{O}(\beta) \\ \mathcal{O}(\beta) & \mathcal{O}(1) \end{pmatrix}$$

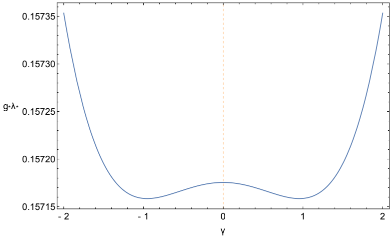
- spectral adjustment of the regulator $\mathcal{R}_k \sim \mathcal{P}$ can lead to a divergence for $\alpha \to 0$

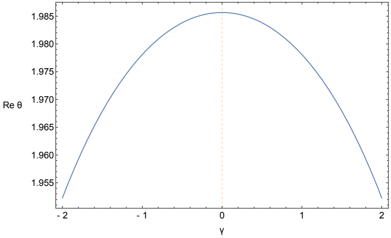
$$\mathrm{Tr}\left[(\Gamma^{(2)} + \mathcal{R}_k)^{-1} \partial_t \mathcal{R}_k\right] \sim \frac{1}{\alpha} \mathcal{O}(\beta^2)$$

$\Rightarrow$ can be dealt with by choosing $\beta = \gamma \cdot \sqrt{\alpha}$

# Results

- linear split ($\tau = 0$) without field redefinition

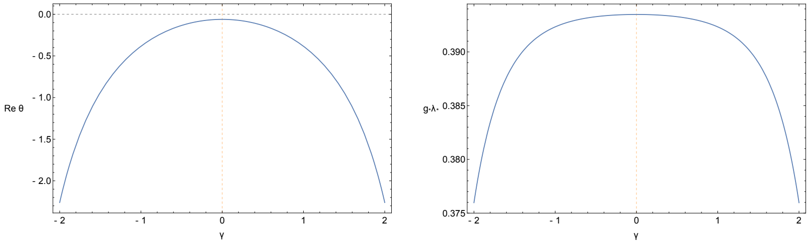



for $\alpha = \gamma = 0$ we find:

$g_* = 0.879, \quad \lambda_* = 0.179, \quad g_*\lambda_* = 0.157, \quad \theta = 1.986 \pm i3.064$

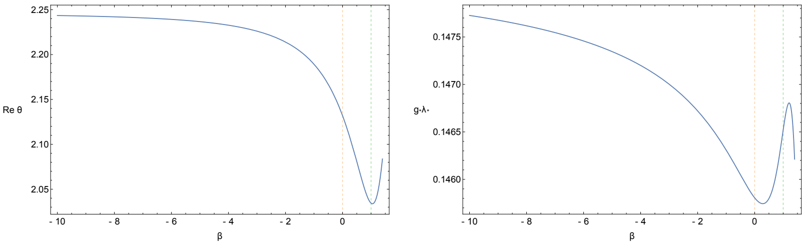
- exponential split ($\tau = 1$) without field redefinition




high parametrization dependence

$\Rightarrow$ bad truncation/parametrization

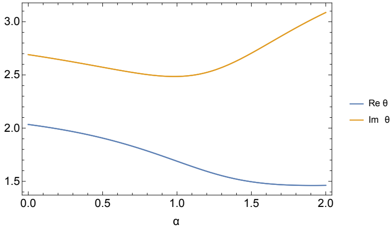
- linear split ($\tau = 0$) with field redefinition

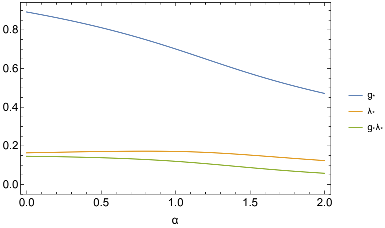



for $\alpha = 0$, $\beta = 1$ we find:
$g_* = 0.893, \quad \lambda_* = 0.164, \quad g_*\lambda_* = 0.147, \quad \theta = 2.034 \pm i2.691$

for $\alpha = 0$, $\beta = \infty$ we find:
$g_* = 0.983, \quad \lambda_* = 0.151, \quad g_*\lambda_* = 0.148, \quad \theta = 2.245 \pm i2.794$

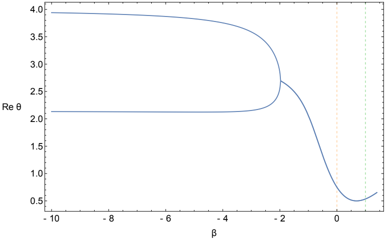
- linear split ($\tau = 0$) with field redefinition

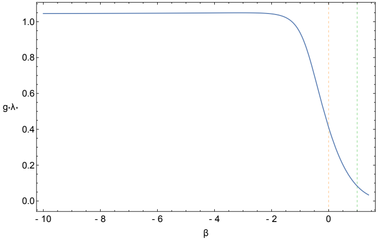



for $\alpha = 1$, $\beta = 1$ we find:
$g_* = 0.701, \quad \lambda_* = 0.172, \quad g_*\lambda_* = 0.120, \quad \theta = 1.689 \pm i2.486$

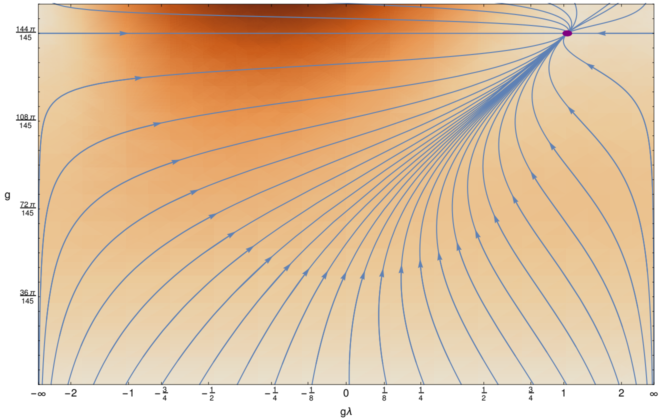
- exponential split ($\tau = 1$) with field redefinition



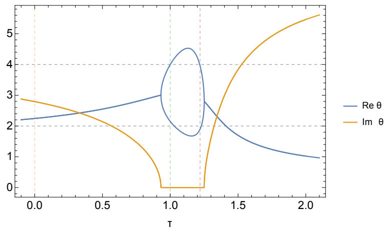

for $\beta = \infty$ we find ($\alpha$ dependence drops out):
$g_* = 3.120, \;\; \lambda_* = 0.331, \;\; g_*\lambda_* = 1.033, \;\; \theta_1 = 4, \;\; \theta_2 = 2.148$

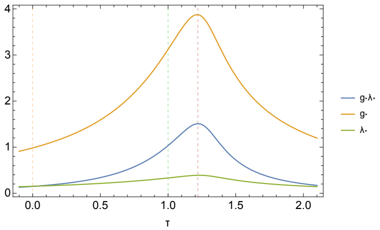
- exponential split ($\tau = 1$) with field redefinition




analytical solution for the phase diagram

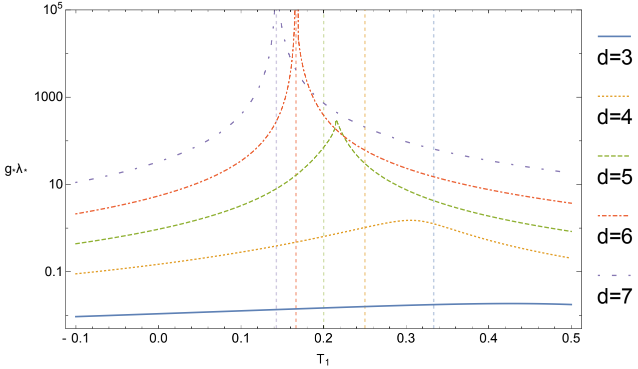
- generalized parametrization (arbitrary $\tau$) with field redefinition




for $\tau \simeq 1.22$, $\alpha = 0$, $\beta = \infty$ we find:
$g_* = 3.873, \quad \lambda_* = 0.389, \quad g_*\lambda_* = 1.508, \quad \theta_1 = 3.957, \quad \theta_2 = 1.898$

- generalized parametrization with field redefinition
  $g_{\mu\nu} \simeq \bar{g}_{\mu\nu} + h_{\mu\nu} + \frac{1}{2}(\tau h_{\mu\rho} h^{\rho}{}_{\nu} + \tau_2 h h_{\mu\nu} + \tau_3 \bar{g}_{\mu\nu} h_{\rho\sigma} h^{\rho\sigma} + \tau_4 \bar{g}_{\mu\nu} h^2)$
- arbitrary dimensions ($\alpha = 0$, $\beta = \infty$)




flow equations only depend on $T_1 = \frac{\tau}{d} + \tau_3$
(dependence on $\tau_2$ and $\tau_4$ drops out)

## summary

- gauge fixing can be a subtle issue
- interplay between parametrization and gauge fixing
- study of parametrization dependence can reveal bad truncations/parametrizations
- existence of UV fixed point for wide range of parameters
- there are regions of minimal sensitivity pointing to favored parametrizations