Local Quantum Gravity

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Outline

1. Introduction
   - Motivation
   - Truncation

2. Momentum locality
   - Definition and examples
   - Proof for graviton two- and three-point function

3. Fixed points and phase diagram
Motivation

- Investigate the asymptotic safety scenario with a fully dynamical, minimally self-consistent ansatz
- Newton’s coupling from graviton three-point function
- No background-field approximation

\[ \Gamma_k[\bar{g}, \phi] \neq \Gamma_k[\bar{g} + \phi] \]  

- Vertex expansion to disentangle background and dynamical fields
Vertex expansion

\[ \Gamma_k[\bar{g}, \phi] = \sum_n \frac{1}{n!} \Gamma^{(\phi_1...\phi_n)}[\bar{g}, 0] \phi_1 ... \phi_n \quad \text{with} \quad \phi = (h, c, \bar{c}) \]

Graviton two-point function:

\[ \partial_t (\quad)^{-1} = -\frac{1}{2} \quad + \quad -2 \]

Graviton three-point function:

\[ \partial_t \quad = -\frac{1}{2} \quad + \quad 3 \quad - 3 \quad + \quad 6 \]
Vertex construction

RG-invariant ansatz for $n$-point function:

$$\Gamma(\phi_1...\phi_n) = \prod_{i=1}^{n} \sqrt{Z_{\phi_i}(p_i^2)G_n^{n-1}}\mathcal{T}^{(n)}(p_1, \ldots, p_n; \Lambda_n)$$  \hspace{1cm} (2)

$\mathcal{T}^{(n)}$ contains classical Einstein-Hilbert tensor structures

Graviton two-point function:

$$\Gamma^{(hh)} = Z_h(p^2)\mathcal{T}^{(2)} \sim Z_h(p^2)(p^2 - 2\Lambda_2)\Pi_{TT} + \text{other tensor structures}$$

Graviton three-point function:

$$\Gamma^{(hhh)} = \sqrt{Z_h(p_1^2)Z_h(p_2^2)Z_h(p_3^2)}G_3^{1/2}\mathcal{T}^{(3)}(p_1, p_2, p_3; \Lambda_3)$$
Vertex construction

RG-invariant ansatz for \( n \)-point function:

\[
\Gamma(\phi_1\ldots\phi_n) = \prod_{i=1}^{n} \sqrt{Z_{\phi_i}(p_i^2)} G_n^{2^{-1}} \mathcal{T}^{(n)}(p_1, \ldots, p_n; \Lambda_n)
\] (2)

\( \mathcal{T}^{(n)} \) contains classical Einstein-Hilbert tensor structures

Graviton two-point function:

\[
\Gamma^{(hh)} = Z_h(p^2) \mathcal{T}^{(2)} \sim Z_h(p^2)(p^2 + M^2)\Pi_{TT} + \text{other tensor structures}
\]

\( M^2 = -2\Lambda_2 = \text{graviton mass parameter} \)
We focus on the transverse-traceless part of the tensor structure:

- Split up into momentum dependent and momentum independent part

\[
\mathcal{T}^{(n)}(p_1, \ldots, p_n; \Lambda_n) = \Lambda_n \mathcal{T}^{(n)}(0; 1) + \mathcal{T}^{(n)}(p_1, \ldots, p_n; 0) \quad (3)
\]

- Contract with \( n \) transverse-traceless projectors

- Obtain two operators to project on flow of \( \Lambda_n \) and \( G_n \)

- RHS of flow equation

\[
\text{Flow}_{\Lambda}^{(3)} := \frac{\dot{\Gamma}^{(3)}}{Z_h^{3/2}(p^2)} \circ \Pi_{TT}^3 \circ \mathcal{T}^{(3)}(0; 1) \quad (4)
\]
Graviton three-point function depends on $p_1$, $p_2$ and $p_3$

- Momentum conservation: 3 degrees of freedom left ($|p_1|$, $|p_2|$ and $\vartheta_{12}$)
- Use maximally symmetric momentum configuration

$$p := |p_1| = |p_2| \quad \vartheta_{12} = 2\pi/3$$

- Only dependence on scalar $p$ is left
Momentum projection

- Bilocal projection for $G_n$

$$
\dot{g}_3 = (2 + 3\eta_h(k^2))g_3 - \frac{24}{19}(\eta_h(k^2) - \eta_h(0))\lambda_3 g_3 \\
+ 2\mathcal{N}_g \sqrt{g_3} k \left( \text{Flow}_G^{(3)}(k^2) - \text{Flow}_G^{(3)}(0) \right)
$$

(6)

- $p = 0$ projection for $\Lambda_n$

$$
\dot{\lambda}_3 = \left( \frac{3}{2} \eta_h(0) - 1 - \frac{\dot{g}_3}{2g_3} \right) \lambda_3 + \frac{\mathcal{N}_\lambda}{\sqrt{g_3} k} \text{Flow}_\Lambda^{(3)}(0)
$$

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- Full momentum dependence of $\eta_h(p^2)$ and $\eta_c(p^2)$ (Christiansen et al. 2014)
Truncation

- Flat Euclidean background

\[ \bar{g}_{\mu\nu} = \delta_{\mu\nu} + h_{\mu\nu} \]

- Closure of flow equations

\[ G_n(p_1, \ldots, p_n) \equiv G_3 =: G \]

\[ \Lambda_5 \equiv \Lambda_4 \equiv \Lambda_3 \]

- Parameters of the truncation

\[ G, \Lambda_3, M^2, \eta_h(p^2), \eta_c(p^2) \]
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Questions so far?
Investigation of momentum dependence

For graviton two-point function see Christiansen et al. 2012
Definition of momentum locality for an $n$-point vertex

$$\lim_{t_i/k^2 \to \infty} \frac{|\hat{\Gamma}_k^{(n)}(p_1, \ldots, p_n)|}{|\Gamma_k^{(n)}(p_1, \ldots, p_n)|} = 0$$

$t_i = \text{momentum channels}$

(each propagator must carry $p \to \infty$)

$|.| = \text{projection on tensor structure}$

- Implies the separation of scales
- An RG-step in the IR does not change physics in the UV
Examples

**Perturbatively renormalisable theories**

Example: \( S = \int \frac{1}{2} m^2 \phi^2 + \frac{1}{2} (\partial \phi)^2 + \lambda_1 \phi^4 \)

\[ \Gamma(\phi\phi) \sim p^2 \]
\[ \dot{\Gamma}(\phi\phi) = \sim p^0 \]
\[ \lim_{p/k \to \infty} \frac{\dot{\Gamma}(\phi\phi)}{\Gamma(\phi\phi)} = 0 \]

\[ \Gamma(4\phi) \sim p^0 \]
\[ \dot{\Gamma}(4\phi) = \sim p^{-2} \]
\[ \lim_{t_i/k^2 \to \infty} \frac{\dot{\Gamma}(4\phi)}{\Gamma(4\phi)} = 0 \]

**Momentum local!**

Applies to standard theories like gauge-theories in four dimensions
Examples

*Perturbatively non-renormalisable theories*

Example: \( S = \int x \left( \frac{1}{2} m^2 \phi^2 + \frac{1}{2} (\partial \phi)^2 + \lambda_1 \phi^4 + \lambda_2 \phi^2 (\partial \phi)^2 \right) \)

\[
\Gamma(\phi\phi) \sim p^2
\]
\[
\dot{\Gamma}(\phi\phi) = \bigcirc \quad \sim p^2
\]
\[
\lim_{p/k \to \infty} \frac{\dot{\Gamma}(\phi\phi)}{\Gamma(\phi\phi)} = \text{const.}
\]

\[
\Gamma(4\phi) \sim p^2
\]
\[
\dot{\Gamma}(4\phi) = \bigcirc \quad \sim p^2
\]
\[
\lim_{t_i/k^2 \to \infty} \frac{\dot{\Gamma}(4\phi)}{\Gamma(4\phi)} = \text{const.}
\]

*Not momentum local!*
Graviton two-point function

Truncation: $S \sim \int_x \sqrt{g} \ (2\Lambda - R) + \ldots$

$\Gamma^{(hh)} \sim p^2$  $\Gamma^{(hhh)} \sim p^2$  $\Gamma^{(4h)} \sim p^2$  $\Gamma^{(5h)} \sim p^2$

$\dot{\Gamma}^{(hh)} = -\frac{1}{2}$  $\sim p^2$  $+ \sim p^2$  $-2 \sim p^0$  $\sim p^0$

$\lim_{p/k \to \infty} \frac{\dot{\Gamma}^{(hh)}}{\Gamma^{(hh)}} = 0$

Non-trivially momentum local!
Graviton three-point function

Truncation: $S \sim \int \sqrt{g} \left( 2\Lambda - R \right) + \ldots$

$\Gamma^{(hh)} \sim p^2 \quad \Gamma^{(hhh)} \sim p^2 \quad \Gamma^{(4h)} \sim p^2 \quad \Gamma^{(5h)} \sim p^2$

$\dot{\Gamma}^{(hhh)} = -\frac{1}{2} \quad +3 \quad -3 \quad +6 \quad \sim p^0$

$\lim_{p_i/k \to \infty} \frac{\dot{\Gamma}^{(hhh)}}{\Gamma^{(hhh)}} = 0$

Non-trivially momentum local!
Holds for all momentum configurations with $p_i \to \infty$
Graviton three-point function

Link to diffeomorphism invariance!
Questions so far?
Phase diagram

$F_1$: UV fixed point  $F_2$: Gaußian fixed point  $F_3$: IR fixed point
Phase diagram

UV fixed point properties:

- \((g^*_{\text{UV}}, \mu^*_{\text{UV}}, \lambda^*_{3,\text{UV}}) = (0.66, -0.59, 0.11)\)
- \((\theta_{1/2}, \theta_3) = (-1.4 \pm 4.1 i, 14)\)
- Two attractive and one repulsive direction
- Two dimensional critical hypersurface
- Repulsive direction approximately along \(\lambda_3\)-axis
IR fixed point properties:

- \((g_{\text{IR}}^*, \mu_{\text{IR}}^*, \lambda_{3, \text{IR}}^*) = (0, \infty, -\infty)\)
- Classical scaling close to IR fixed point
- Non-trivial IR fixed point at \((0, -1, -\infty)\) not found \((\text{Christiansen et al. 2014})\)
- Expansion about non-flat background required
Higher order momentum dependence

Current approximation:

\[ G_n(p_1, \ldots, p_n) \equiv G_3 \]

*Is that a good approximation? (I.e. does \( Z_h(p^2) \) capture higher order momentum dependence?)*

Compare full momentum dependence of LHS with RHS (at the fixed point)

\[
\text{LHS} = \partial_t \left( Z_{h}^{3/2}(p^2) G^{1/2} \left( \frac{171}{32} p^2 + \frac{9}{4} \Lambda_3 \right) \right) \overset{?}{=} \text{Flow}_G^{(3)}(p^2) = \text{RHS}
\]

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Higher order momentum dependence

The momentum dependence fits!

<table>
<thead>
<tr>
<th>LHS</th>
<th>RHS</th>
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<tbody>
<tr>
<td>0.5</td>
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<tr>
<td>0.3</td>
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<tr>
<td>0.1</td>
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Three-point function

External momentum $p/k$
Summary

Setup:

- Genuine Newton’s coupling from graviton three-point function
- Minimally self-consistent system

Results:

- New notion of momentum locality
- Proof of momentum locality for graviton 2- and 3-point function
- UV fixed point, which supports the asymptotic safety scenario
- IR fixed point with classical scaling
Outlook

- Inclusion of matter degrees of freedom
  (see 'Asymptotic safety of gravity-matter systems'
    J. Meibohm, J. M. Pawlowski and MR
    arXiv:1510.07018)
- Graviton four-point function
- Expansion around non-flat background