Local Quantum Gravity

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IMPRS

PTFS

Outline

1. Introduction
   - Motivation
   - Truncation
2. Momentum locality
   - Definition and examples
   - Proof for graviton two- and three-point function
3. Fixed points and phase diagram

Motivation

- Investigate the asymptotic safety scenario with a fully dynamical, minimally self-consistent ansatz
- Newton's coupling from graviton three-point function
- No background-field approximation

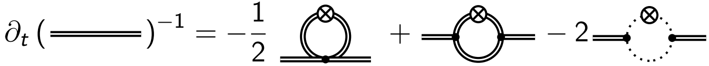
$$\Gamma_k[\bar{g}, \phi] \neq \Gamma_k[\bar{g} + \phi] \tag{1}$$

- Vertex expansion to disentangle background and dynamical fields

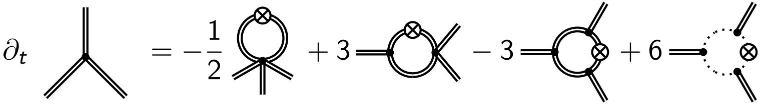
Vertex expansion

$$\Gamma_k[\bar{g},\phi] = \sum_n \frac{1}{n!}\Gamma^{(\phi_1 \dots \phi_n)}[\bar{g},0]\,\phi_1 \dots \phi_n \qquad \text{with} \qquad \phi = (h, c, \bar{c})$$

Graviton two-point function:



Graviton three-point function:



Vertex construction

RG-invariant ansatz for n-point function:

$$\Gamma^{(\phi_1 \ldots \phi_n)} = \prod_{i=1}^{n} \sqrt{Z_{\phi_i}(p_i^2)} G_n^{\frac{n}{2}-1} \mathcal{T}^{(n)}(p_1, \ldots, p_n; \Lambda_n) \tag{2}$$

$\mathcal{T}^{(n)}$ contains classical Einstein-Hilbert tensor structures

Graviton two-point function:

$$\Gamma^{(hh)} = Z_h(p^2)\mathcal{T}^{(2)} \sim Z_h(p^2)(p^2 - 2\Lambda_2)\Pi_{\text{TT}} + \text{other tensor structures}$$

Graviton three-point function:

$$\Gamma^{(hhh)} = \sqrt{Z_h(p_1^2) Z_h(p_2^2) Z_h(p_3^2)} G_3^{1/2} \mathcal{T}^{(3)}(p_1, p_2, p_3; \Lambda_3)$$

Vertex construction

RG-invariant ansatz for n-point function:

$$\Gamma^{(\phi_1\ldots\phi_n)} = \prod_{i=1}^{n} \sqrt{Z_{\phi_i}(p_i^2)}\, G_n^{\frac{n}{2}-1} \mathcal{T}^{(n)}(p_1,\ldots,p_n;\Lambda_n) \qquad (2)$$

$\mathcal{T}^{(n)}$ contains classical Einstein-Hilbert tensor structures

Graviton two-point function:

$$\Gamma^{(hh)} = Z_h(p^2)\mathcal{T}^{(2)} \sim Z_h(p^2)(p^2 + M^2)\Pi_{\mathrm{TT}} + \text{other tensor structures}$$

$$M^2 = -2\Lambda_2 = \text{graviton mass parameter}$$

Tensor projection

We focus on the transverse-traceless part of the tensor structure:

- Split up into momentum dependent and momentum independent part

$$\mathcal{T}^{(n)}(p_1, \dots, p_n; \Lambda_n) = \Lambda_n \mathcal{T}^{(n)}(0; 1) + \mathcal{T}^{(n)}(p_1, \dots, p_n; 0) \qquad (3)$$

- Contract with n transverse-traceless projectors
- Obtain two operators to project on flow of Λ_n and G_n
- RHS of flow equation

$$\text{Flow}^{(3)}_{\Lambda} := \frac{\dot{\Gamma}^{(3)}}{Z_h^{3/2}(p^2)} \circ \Pi^3_{\text{TT}} \circ \mathcal{T}^{(3)}(0; 1) \qquad (4)$$

Momentum projection

Graviton three-point function depends on p_1, p_2 and p_3

- Momentum conservation: 3 degrees of freedom left ($|p_1|$,$|p_2|$ and ϑ_{12})
- Use maximally symmetric momentum configuration

$$p := |p_1| = |p_2| \qquad\qquad \vartheta_{12} = 2\pi/3 \tag{5}$$

- Only dependence on scalar p is left

Momentum projection

- Bilocal projection for G_n

$$\dot{g}_3 = (2 + 3\eta_h(k^2))g_3 - \frac{24}{19}(\eta_h(k^2) - \eta_h(0))\lambda_3 g_3 + 2\,\mathcal{N}_g\,\sqrt{g_3}\,k\left(\text{Flow}_G^{(3)}(k^2) - \text{Flow}_G^{(3)}(0)\right) \tag{6}$$

- $p = 0$ projection for Λ_n

$$\dot{\lambda}_3 = \left(\frac{3}{2}\eta_h(0) - 1 - \frac{\dot{g}_3}{2g_3}\right)\lambda_3 + \frac{\mathcal{N}_\lambda}{\sqrt{g_3}k}\text{Flow}_\Lambda^{(3)}(0) \tag{7}$$

- Full momentum dependence of $\eta_h(p^2)$ and $\eta_c(p^2)$ *(Christiansen et al. 2014)*

Truncation

- Flat Euclidean background

$$\bar{g}_{\mu\nu} = \delta_{\mu\nu} + h_{\mu\nu}$$

- Closure of flow equations

$$G_n(p_1, \ldots, p_n) \equiv G_3 =: G$$

$$\Lambda_5 \equiv \Lambda_4 \equiv \Lambda_3$$

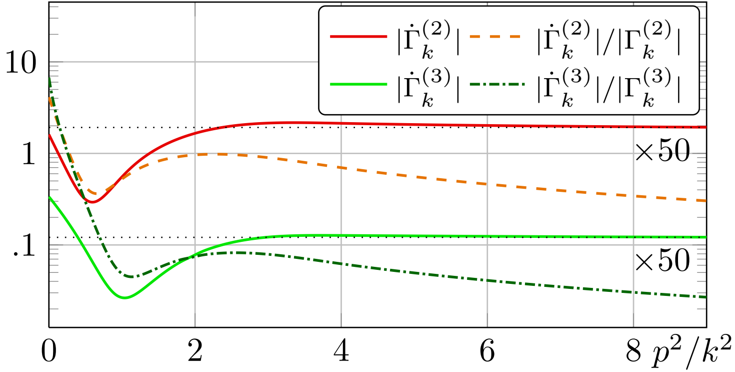
- Parameters of the truncation

$$G, \Lambda_3, M^2, \eta_h(p^2), \eta_c(p^2)$$

Outline

Questions so far?

Investigation of momentum dependence



Unexpected momentum dependence

For graviton two-point function see *Christiansen et al. 2012*

Definition of momentum locality for an n-point vertex

$$\lim_{t_i/k^2 \to \infty} \frac{|\dot{\Gamma}_k^{(n)}(p_1,\ldots,p_n)|}{|\Gamma_k^{(n)}(p_1,\ldots,p_n)|} = 0$$

$t_i =$ momentum channels

(each propagator must carry $p \to \infty$)

$|\,.\,| =$ projection on tensor structure

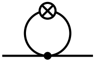
- Implements the separation of scales
- An RG-step in the IR does not change physics in the UV

Examples

Perturbatively renormalisable theories

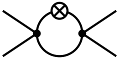
Example: $S = \int_x \frac{1}{2} m^2 \phi^2 + \frac{1}{2}(\partial\phi)^2 + \lambda_1 \phi^4$

$\Gamma^{(\phi\phi)} \sim p^2$

$\dot{\Gamma}^{(\phi\phi)} =$  $\sim p^0$

$\lim_{p/k\to\infty} \frac{\dot{\Gamma}^{(\phi\phi)}}{\Gamma^{(\phi\phi)}} = 0$

$\Gamma^{(4\phi)} \sim p^0$

$\dot{\Gamma}^{(4\phi)} =$  $\sim p^{-2}$

$\lim_{t_i/k^2\to\infty} \frac{\dot{\Gamma}^{(4\phi)}}{\Gamma^{(4\phi)}} = 0$

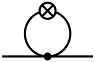
Momentum local!

Applies to standard theories like gauge-theories in four dimensions

Perturbatively non-renormalisable theories

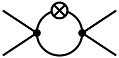
Example: $S = \int_x \frac{1}{2} m^2 \phi^2 + \frac{1}{2} (\partial\phi)^2 + \lambda_1 \phi^4 + \lambda_2 \phi^2 (\partial\phi)^2$

$\Gamma^{(\phi\phi)} \sim p^2$

$\dot{\Gamma}^{(\phi\phi)} =$  $\sim p^2$

$\lim_{p/k \to \infty} \frac{\dot{\Gamma}^{(\phi\phi)}}{\Gamma^{(\phi\phi)}} = \text{const.}$

$\Gamma^{(4\phi)} \sim p^2$

$\dot{\Gamma}^{(4\phi)} =$  $\sim p^2$

$\lim_{t_i/k^2 \to \infty} \frac{\dot{\Gamma}^{(4\phi)}}{\Gamma^{(4\phi)}} = \text{const.}$

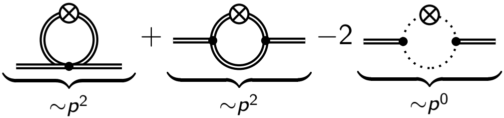
Not momentum local!

Graviton two-point function

Truncation: $S \sim \int_x \sqrt{g}\,(2\Lambda - R) + \dots$

$\Gamma^{(hh)} \sim p^2$ $\Gamma^{(hhh)} \sim p^2$ $\Gamma^{(4h)} \sim p^2$ $\Gamma^{(5h)} \sim p^2$

$\dot{\Gamma}^{(hh)} = -\frac{1}{2}$



$\sim p^0$

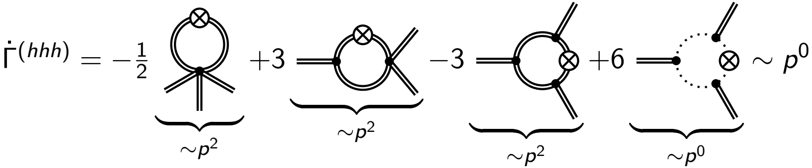
$$\lim_{p/k\to\infty} \frac{\dot{\Gamma}^{(hh)}}{\Gamma^{(hh)}} = 0$$

Non-trivially momentum local!

Graviton three-point function

Truncation: $S \sim \int_x \sqrt{g}\,(2\Lambda - R) + \dots$

$\Gamma^{(hh)} \sim p^2$ $\quad\Gamma^{(hhh)} \sim p^2$ $\quad\Gamma^{(4h)} \sim p^2$ $\quad\Gamma^{(5h)} \sim p^2$

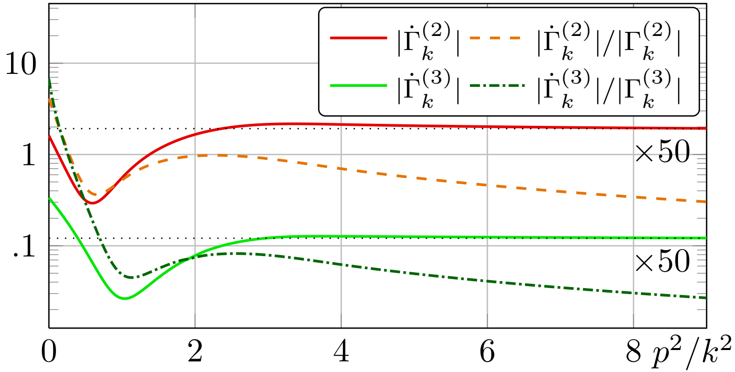


$$\lim_{p_i/k\to\infty} \frac{\dot{\Gamma}^{(hhh)}}{\Gamma^{(hhh)}} = 0$$

Non-trivially momentum local!

Holds for all momentum configurations with $p_i \to \infty$

Graviton three-point function

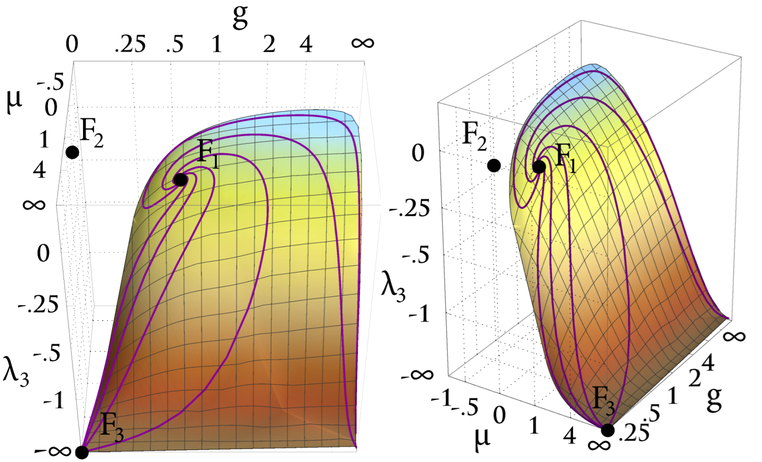


Link to diffeomorphism invariance!

Outline

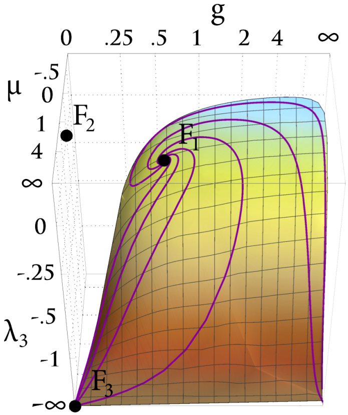
Questions so far?

Phase diagram



F_1: UV fixed point F_2: Gaußian fixed point F_3: IR fixed point

Phase diagram

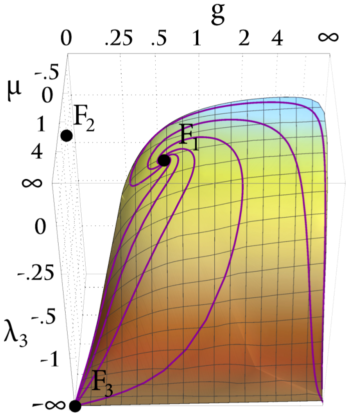


(Analytic approximation)

UV fixed point properties:

- $(g^*_{\mathrm{UV}}, \mu^*_{\mathrm{UV}}, \lambda^*_{3,\mathrm{UV}}) = (0.66, -0.59, 0.11)$
- $(\theta_{1/2}, \theta_3) = (-1.4 \pm 4.1\, i, 14)$
- Two attractive and one repulsive direction
- Two dimensional critical hypersurface
- Repulsive direction approximately along λ_3-axis

Phase diagram



(Analytic approximation)

IR fixed point properties:

- $(g^*_{\mathrm{IR}}, \mu^*_{\mathrm{IR}}, \lambda^*_{3,\mathrm{IR}}) = (0, \infty, -\infty)$
- Classical scaling close to IR fixed point
- Non-trivial IR fixed point at $(0, -1, -\infty)$ not found *(Christiansen et al. 2014)*
- Expansion about non-flat background required

Higher order momentum dependence

Current approximation:

$$G_n(p_1, \ldots, p_n) \equiv G_3$$

Is that a good approximation?

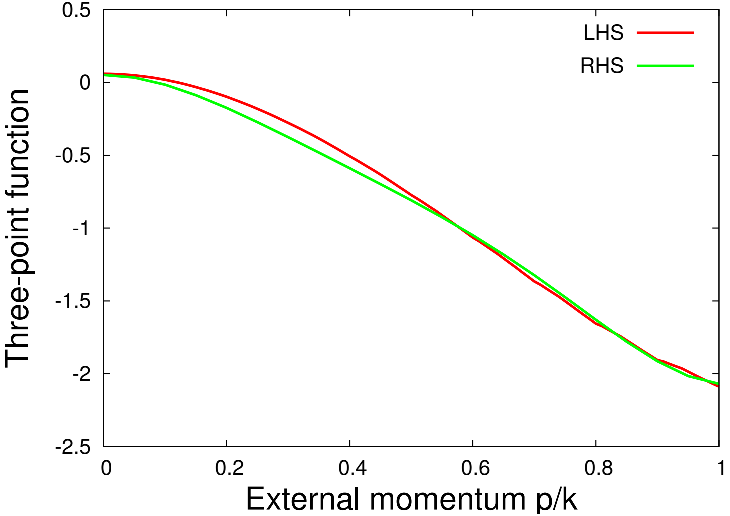
(I.e. does $Z_h(p^2)$ capture higher order momentum dependence?)

Compare full momentum dependence of LHS with RHS (at the fixed point)

$$\text{LHS} = \partial_t \left({Z_h}^{3/2}(p^2) G^{1/2} \left(\frac{171}{32} p^2 + \frac{9}{4} \Lambda_3 \right) \right) \stackrel{?}{=} \text{Flow}_G^{(3)}(p^2) = \text{RHS} \quad (8)$$

Higher order momentum dependence

The momentum dependence fits!



Summary

Setup:

- Genuine Newton's coupling from graviton three-point function
- Minimally self-consistent system

Results:

- New notion of momentum locality
- Proof of momentum locality for graviton 2- and 3-point function
- UV fixed point, which supports the asymptotic safety scenario
- IR fixed point with classical scaling

Outlook

- Inclusion of matter degrees of freedom

  (see 'Asymptotic safety of gravity-matter systems'

  J. Meibohm, J. M. Pawlowski and MR

  arXiv:1510.07018)

- Graviton four-point function
- Expansion around non-flat background