

# Asymptotic safety of gravity-matter systems

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**PTFS**

## 1 Introduction

- Motivation
- Truncation

## 2 Anomalous dimensions

- Scheme for an approximated momentum dependence
- Bounds on the anomalous dimensions

## 3 Results

- Fixed points of dynamical couplings
- Background couplings

## 4 Summary and Outlook

## Why gravity-matter systems?

- Pure quantum gravity does not exist in nature
- Matter seems to matter Percacci, Perini (2003)  
Donà, Eichhorn, Percacci (2013)

Does the non-trivial UV fixed point persist within inclusion of matter?

Why a vertex expansion?

Features:

- Systematic approximation scheme
- Disentangling background and fluctuation fields
- Newton's coupling from graviton three-point function

Challenges:

- Long algebraic computations due to large tensor structures
- Physical choice of tensor projection
- Technical issues for non-flat backgrounds (so far)

# Vertex expansion

$$\begin{aligned}\Gamma_k[\bar{g}, \phi] &= \sum_{n=0}^{\infty} \frac{1}{n!} \Gamma_k^{(\phi_1 \dots \phi_n)}[\bar{g}, 0] \phi_1 \dots \phi_n \quad \text{with} \quad \phi = (h, c, \bar{c}, \psi, \bar{\psi}, \varphi) \\ &= \Gamma_k[\bar{g}, 0] + \Gamma_k^{(h)}[\bar{g}, 0] h + \frac{1}{2} \Gamma_k^{(2h)}[\bar{g}, 0] h^2 + \frac{1}{3!} \Gamma_k^{(3h)}[\bar{g}, 0] h^3 + \dots \\ &\quad + \frac{1}{2} \Gamma_k^{(\bar{c}c)}[\bar{g}, 0] \bar{c} c + \frac{1}{2} \Gamma_k^{(\bar{\psi}\psi)}[\bar{g}, 0] \bar{\psi} \psi + \frac{1}{2} \Gamma_k^{(\varphi\varphi)}[\bar{g}, 0] \varphi^2 + \dots\end{aligned}$$

- Computation of all  $n$ -point correlation functions up to third order
- Flow equation for each correlation function from Wetterich equation

# Examples for vertex flows

Graviton two-point function:

$$\begin{aligned}
 \partial_t (\text{double line})^{-1} = & -\frac{1}{2} \text{ring with top cross} + \text{ring with top cross and bottom dot} - 2 \text{dashed ring with top cross} \\
 & + N_s \left( -\frac{1}{2} \text{dashed ring with top cross and bottom dot} + \text{dashed ring with top cross and right dot} \right) \\
 & - 2N_f \left( -\frac{1}{2} \text{ring with top cross and bottom dot} + \text{ring with top cross and bottom dot with arrow} \right)
 \end{aligned}$$

Fermionic two-point function:

$$\partial_t (\text{arrow line})^{-1} = -\frac{1}{2} \text{ring with top cross and bottom dot} + \text{ring with top cross and bottom dot with arrow} + \text{ring with top cross and bottom dot with arrow and right dot}$$

# Generation of vertex dressing

Generating action:

$$S = S_{\text{EH}} + S_{\text{gh}} + S_{\text{gf}} + \int d^4x \sqrt{g} \bar{\psi}_i \not{\nabla} \psi_i + \frac{1}{2} \int d^4x \sqrt{g} g_{\mu\nu} \partial^\mu \varphi_j \partial^\nu \varphi_j$$

with  $i = 1, \dots, N_f$  and  $j = 1, \dots, N_s$

Tensor structures for vertices:

$$\mathcal{T}^{(\phi_1 \dots \phi_n)}(p_1, \dots, p_n; \Lambda_n) = \mathcal{S}^{(\phi_1 \dots \phi_n)}(p_1, \dots, p_n; \Lambda \rightarrow \Lambda_n, G_N \rightarrow 1) \quad (1)$$

Spin-connection via spin-base invariance formalism Weldon (2000); Gies, Lippoldt (2013)

RG-invariant ansatz for  $n$ -point function Christiansen, Knorr, Pawłowski, Rodigast (2014)

$$\Gamma^{(\phi_1 \dots \phi_n)} = \prod_{i=1}^n \sqrt{Z_{\phi_i}(p_i^2)} G_n^{\frac{n}{2}-1} \mathcal{T}^{(\phi_1 \dots \phi_n)}(p_1, \dots, p_n; \Lambda_n) \quad (2)$$

Graviton two-point function:

$$\Gamma^{(hh)} = Z_h(p^2) \mathcal{T}^{(hh)} \sim Z_h(p^2) (p^2 - 2\Lambda_2) \Pi_{\text{TT}} + \text{other tensor structures}$$

Fermionic two-point function:

$$\Gamma^{(\bar{\psi}\psi)} = Z_\psi(p^2) i\gamma_\mu p^\mu$$



RG-invariant ansatz for  $n$ -point function Christiansen, Knorr, Pawłowski, Rodigast (2014)

$$\Gamma^{(\phi_1 \dots \phi_n)} = \prod_{i=1}^n \sqrt{Z_{\phi_i}(p_i^2)} G_n^{\frac{n}{2}-1} \mathcal{T}^{(\phi_1 \dots \phi_n)}(p_1, \dots, p_n; \Lambda_n) \quad (2)$$

Graviton two-point function:

$$\Gamma^{(hh)} = Z_h(p^2) \mathcal{T}^{(hh)} \sim Z_h(p^2) (p^2 + M^2) \Pi_{TT} + \text{other tensor structures}$$

$$M^2 = -2\Lambda_2 = \text{graviton mass parameter}$$

- Transverse-traceless part of graviton  $n$ -point functions
- $\Gamma^{(3h)}$  at symmetric momentum configuration
- Bilocal momentum projection for  $\Gamma^{(3h)}$  Christiansen, Knorr, Meibohm, Pawłowski, MR
- Momentum dependence of  $\Gamma^{(\phi\phi)}$  encoded in anomalous dimensions

- Flow of  $n$ -point functions up to order three
- Flat Euclidean background  $g_{\mu\nu} = \delta_{\mu\nu} + h_{\mu\nu}$
- Closure of flow equations (gravity sector)

$$G_n(p_1, \dots, p_n) \equiv G_3 =: G$$

$$\Lambda_5 \equiv \Lambda_4 \equiv \Lambda_3$$

- Minimally coupled scalars and fermions
- No matter-matter-interactions

## Parameters of the truncation

$$G, \Lambda_3, M^2, \eta_h(p^2), \eta_c(p^2), \eta_\psi(p^2), \eta_\varphi(p^2), N_s, N_f$$

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# Fully momentum dependent anomalous dimensions

The anomalous dimensions obey coupled Fredholm integral equations, e.g.

$$\eta_h(p^2) \sim \int d\Omega dq q^3 (r(q^2) - \eta_i(q^2)r(q^2)) f_i(p, q, G, \Lambda_3, M^2)$$

In our setup: Four coupled Fredholm integral equations

→ Requires large numerical effort for solution (Liouville-Neumann series)

## Approximation

Evaluate anomalous dimensions at the peak of the integral  $q = k$

Full momentum dependence Christiansen, Knorr, Meibohm, Pawłowski, MR (2015)

$$\begin{aligned}(g^*, \mu^*, \lambda_3^*) &= (0.66, -0.59, 0.11) \\ (\theta_{1/2}, \theta_3) &= (-1.4 \pm 4.1 i, 14)\end{aligned}\tag{3}$$

Approximated momentum dependence

$$\begin{aligned}(g^*, \mu^*, \lambda_3^*) &= (0.62, -0.57, 0.095) \\ (\theta_{1/2}, \theta_3) &= (-1.3 \pm 4.1 i, 12)\end{aligned}\tag{4}$$

Error of 6% (15%) on fixed point values (critical exponents)

## Key message

Momentum dependence of the anomalous dimensions is important

# Bounds on the anomalous dimensions - from the regulator

The regulator is proportional to  $Z_\phi \sim k^{-\eta_\phi}$

$$R_k^\phi(p^2) = \Gamma_k^{(\phi\phi)}(p^2) r_k^\phi(p^2) \Big|_{M^2=0} \quad (5)$$

But the regulator has to satisfy (with  $i = 1$  for fermions and  $i = 2$  else)

$$\lim_{k \rightarrow \infty} R_k^\phi(p) \sim \lim_{k \rightarrow \infty} Z_\phi k^i \sim \lim_{k \rightarrow \infty} k^{i-\eta_\phi} \xrightarrow{!} \infty \quad (6)$$

## Bounds on the anomalous dimensions - from the regulator

$$\eta_h < 2, \quad \eta_c < 2, \quad \eta_\varphi < 2, \quad \eta_\psi < 1$$

## Bounds on the anomalous dimensions - from the flow

Contributions to the flow have the typical shape

$$\text{diagram} \sim (\# - \eta_\phi)$$

with  $\# = \{4, 5, 6, \dots\}$  for analytic approximation

The sign of the diagram is regulator dependent for  $\eta_\phi > \#$

### Bounds on the anomalous dimensions - from the flow

$$\eta_h \lesssim 4, \quad \eta_c \lesssim 10, \quad \eta_\varphi \lesssim 8, \quad \eta_\psi \lesssim 5$$

We take the most conservative bound!



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## Questions so far?

# Flow equations in analytic approximation

Flow of dynamical couplings (matter contributions)

$$\begin{aligned}\partial_t g &\sim -\frac{43}{570\pi} g^2 N_s - \frac{3599}{11400\pi} g^2 N_f, \\ \partial_t \lambda_2 &\sim -\frac{1}{24\pi} g N_s + \frac{4}{9\pi} g N_f,\end{aligned}\tag{7}$$

compared to background couplings in background-field approximation

$$\begin{aligned}\partial_t \bar{g} &\sim +\frac{1}{6\pi} \bar{g}^2 N_s + \frac{1}{3\pi} \bar{g}^2 N_f, \\ \partial_t \bar{\lambda} &\sim +\frac{1}{12\pi} (3 + 2\bar{\lambda}) \bar{g} N_s - \frac{1}{3\pi} (3 - \bar{\lambda}) \bar{g} N_f.\end{aligned}\tag{8}$$

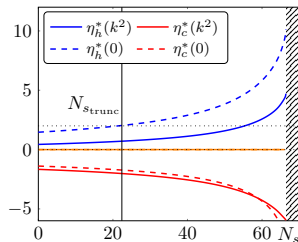
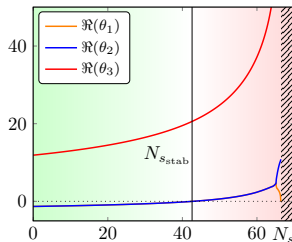
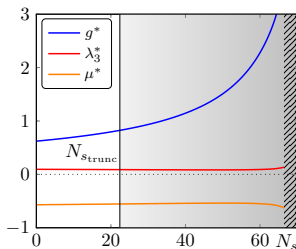
Do the different signs indicate an error?

**No, it's just a computation of different couplings!**

# Reminder: Truncation

- $g$  from  $(\Gamma^{(3h)}(p = k) - \Gamma^{(3h)}(p = 0))$ ; numerical integration necessary  
 $Z_h(p^2)$  models remaining  $p$ -dependence Christiansen, Knorr, Meibohm, Pawłowski, MR
- $\lambda_3$  from  $\Gamma^{(3h)}(p = 0)$ ; analytic equation
- $\mu$  from  $\Gamma^{(2h)}(p = 0)$ ; analytic equation
- $\eta_\phi(p^2)$  from  $\Gamma^{(\phi\phi)}(p^2)$  with approximated  $p$ -dependence
- Remark:  $\eta_\varphi(p^2) = 0$  due to choice of graviton gauge

# Inclusion of scalars



Reliability bound on truncation ( $\eta_h^* > 2$ )

$$N_{s_{trunc}} \approx 21.5$$

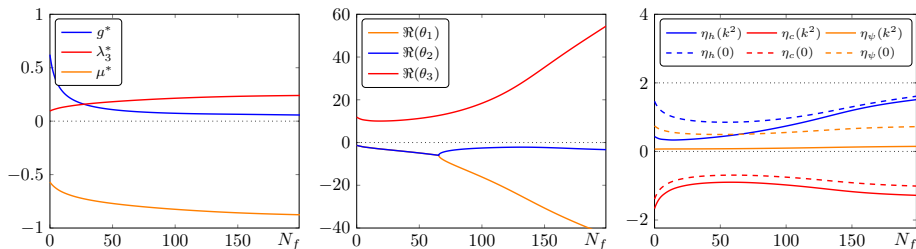
Loss of an attractive UV fixed point

$$N_{s_{stab}} \approx 42.6$$

Complete loss of the UV fixed point

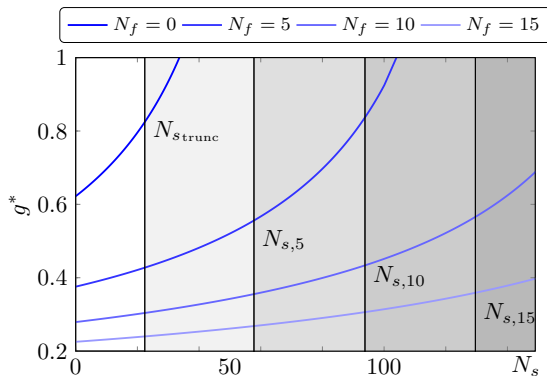
$$N_{s_{max}} \approx 66.4$$

# Inclusion of fermions



Reliable, attractive UV fixed point for all  $N_f$

# Mixed scalar-fermion systems



One fermion compensates for  $\approx 7.1$  scalars

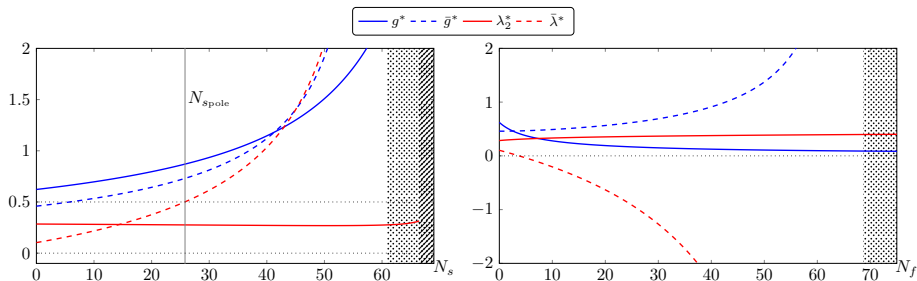
- The background couplings  $\bar{g}$  and  $\bar{\lambda}$  are observables for  $k \rightarrow 0$
- Their flow depends only on dynamical quantities

$$\begin{aligned}\partial_t \left( \frac{k^2}{\bar{g}} \right) &= F_{R^1}(\mathbf{g}, \lambda; N_s, N_f) \\ \partial_t \left( \frac{\bar{\lambda} k^4}{\bar{g}} \right) &= F_{R^0}(\mathbf{g}, \lambda; N_s, N_f)\end{aligned}\tag{9}$$

- Same equations as earlier computations if  $(\mathbf{g}, \lambda) \rightarrow (\bar{g}, \bar{\lambda})$

Codello, Percacci, Rahmede (2008); Donà, Eichhorn, Percacci (2013)

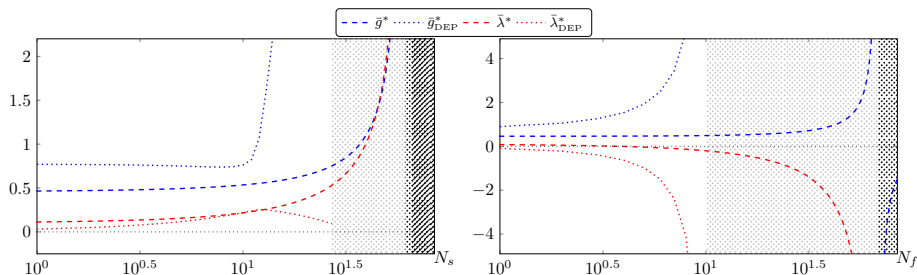
# Background couplings vs dynamical couplings



Qualitative agreement only for small  $N_s$  and  $N_f$



# Comparison with previous results



DEP: Donà, Eichhorn, Percacci (2013)

Good qualitative agreement for small  $N_s$  and  $N_f$

## Setup:

- Vertex expansion up to third order
- Genuine Newton's coupling from graviton three-point function
- Minimally coupled scalars and fermions

## Results:

- All numbers of fermions are compatible with asymptotic safety
- Need to improve truncation for intermediate and large  $N_s$
- Background and dynamical couplings deviate for large  $N_s$  and  $N_f$

## Improvement in the matter sector (Matter-matter-interactions)

- Yukawa-coupling Held, Eichhorn, Pawłowski; in preparation
- $\varphi^4$ -coupling Fischer, Pawłowski; in preparation

## Improvement in the gravity/mixed sector

- Graviton four-point function Denz, Pawłowski, MR; in preparation
- $h\varphi\varphi$ -coupling Donà, Eichhorn, Labus, Percacci; arXiv:1512.01589  
Eichhorn, Labus, MR; in preparation
- Expansion about non-flat backgrounds