Non-minimal coupling and scalar mass in Higgs-Yukawa model with asymptotic safe gravity

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Plan

1. Introduction
   i. Model
   ii. Set-up

2. Results

3. Higgs inflation and fine-tuning problems

Introduction

- Asymptotic Safety
  - Suggested by S. Weinberg (1979)
  - Existence of non-trivial fixed point (NTFP) is essential.
  - UV critical surface is defined.
  - If its dimension is finite, Non-perturbatively renormalizable.

- For quantum gravity,
  - Functional Renormalization Group approach
  - Many studies have shown
    - NTFP exists
    - Dimension of UV critical surface is stably three.
    - Then, the quantum gravity can be renormalizable.

Introduction

- Matter fields coupled to quantum gravity
  - Scalar-gravity model
    - The quartic coupling λ becomes irrelevant.
    - The **mass $\boldsymbol{m}$** and the **non-minimal coupling ξ** become relevant.

[R. Percacci, D. Perini' 03]
[G. Narain, R. Percacci '09]

  - Higgs-Yukawa model without non-minimal coupling
    - The **Yukawa coupling $\boldsymbol{y}$** becomes irrelevant.

[O. Zanusso, L. Zambelli, G. P. Vacca, R. Percacci '10]

  - How is the combined case?

Consider **Higgs-Yukawa model non-minimally coupled to gravity.**

- Toy model of Higgs inflation

[F. Bezrukov, M. Shaposhnikov, '08]

  - Non-minimal coupling $\xi\phi^2 R$ plays crucial role.
  - To realize Higgs inflation, needs large ξ
    - At least $\xi \sim 10$ … Is it possible?

[Y. Hamada, H. Kawai, K-y. Oda, S. C. Park, '14]
[J.L. Cook, L. M. Krauss, A. J. Long, S. Sabharwa;, 14]

Model: Higgs-Yukawa model ($d = 4$)

- Non-minimally coupled to asymptotically safe gravity

$$\Gamma_\Lambda = \int d^4x \sqrt{g} \Big[V_\Lambda(\phi^2) - F_\Lambda(\phi^2) R + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + \bar{\psi} \nabla\!\!\!/ \psi + y_\Lambda \phi \bar{\psi} \psi \Big] + S_{\rm gh} + S_{\rm gf}$$

- Potentials

$$V(\phi^2) = \Lambda_{\rm cc} + m^2 \phi^2 + \lambda \phi^4 + \cdots$$

Cosmological Const.

$$F(\phi^2) = M_{\rm pl}^2 + \xi \phi^2 + \cdots$$

Planck mass (Newton const.)

Non-minimal coupling

Set-up

$$\Phi = (\phi, \psi, \bar{\psi}, \bar{g}_{\mu\nu})$$

$$\Upsilon = (\varphi, \chi, \bar{\chi}, h_{\mu\nu})$$

- Use the background method $\Phi = \bar{\Phi} + \Upsilon$
  - de-Sitter metric is used.
- Gauge and ghost action

$$\Sigma^\mu = \bar{\partial}_\nu h^{\nu\mu} - \frac{\beta + 1}{4} \bar{\partial}^\mu h$$

$$S_{\mathrm{gf}} = \frac{1}{2\alpha} \int d^4x \sqrt{\bar{g}} F(\phi^2) \bar{g}^{\mu\nu} \Sigma_\mu \Sigma_\nu \qquad (1)$$

$$S_{\mathrm{gh}} = \int d^4x \sqrt{\bar{g}} \bar{C}_\mu \left[-\delta^\rho_\mu \bar{\partial}^2 - \left(1 - \frac{1+\beta}{2}\right) \bar{\partial}_\mu \bar{\partial}_\rho + \bar{R}^\rho_\mu \right] C_\rho \qquad (2)$$

  - de-Donder gauge (Landau gauge) $\alpha = 0, \quad \beta = 1$
- York decomposition

$$h_{\mu\nu} = h^\perp_{\mu\nu} + \partial_\mu \xi_\nu + \partial_\nu \xi_\mu + \left(\partial_\mu \partial_\nu - \frac{1}{4} \bar{g}_{\mu\nu} \partial^2 \right) \sigma + \frac{1}{4} \bar{g}_{\mu\nu} h$$

$$C_\mu = C^\perp_\mu + \partial_\mu C$$

$$\bar{C}_\mu = \bar{C}^\perp_\mu + \partial_\mu \bar{C}$$

- Cutoff function
  - Optimized cutoff

[P. Dona, R. Percacci, '13]

For scalar and gravity

$$R_\Lambda(z) = (\Lambda^2 - z)\theta(\Lambda^2 - z)$$

For fermion

$$R_\Lambda\left(z - \frac{R}{4}\right) = \left(\Lambda^2 - \left(z - \frac{R}{4}\right)\right) \theta\left(\Lambda^2 - \left(z - \frac{R}{4}\right)\right)$$

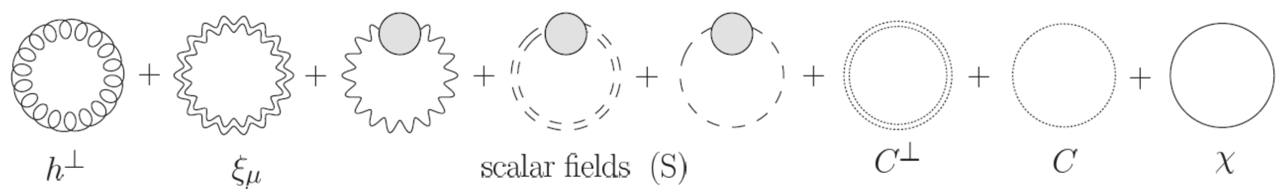
Type II

Wetterich equation

- Wetterich equation for the system

$$\partial_t\Gamma_\Lambda = \frac{1}{2}\mathrm{Tr}\,\frac{\partial_t\mathcal{R}_\Lambda}{\Gamma_\Lambda^{(1,1)}+\mathcal{R}_\Lambda}\bigg|_{h^\perp h^\perp} + \frac{1}{2}\mathrm{Tr}'\,\frac{\partial_t\mathcal{R}_\Lambda}{\Gamma_\Lambda^{(1,1)}+\mathcal{R}_\Lambda}\bigg|_{\xi\xi} + \frac{1}{2}\mathrm{Tr}''\,\frac{\partial_t\mathcal{R}_\Lambda}{\Gamma_\Lambda^{(1,1)}+\mathcal{R}_\Lambda}\bigg|_{\mathrm{SS}}$$
$$- \mathrm{Tr}\,\frac{\partial_t\mathcal{R}_\Lambda}{\Gamma_\Lambda^{(1,1)}+\mathcal{R}_\Lambda}\bigg|_{\overline{\chi}\chi} - \mathrm{Tr}\,\frac{\partial_t\mathcal{R}_\Lambda}{\Gamma_\Lambda^{(1,1)}+\mathcal{R}_\Lambda}\bigg|_{\bar{C}^\perp C} - \mathrm{Tr}\,\frac{\partial_t\mathcal{R}_\Lambda}{\Gamma_\Lambda^{(1,1)}+\mathcal{R}_\Lambda}\bigg|_{\bar{C}C}$$

$$\partial_t\Gamma_\Lambda =$$



Fermionic fluctuation

- Dimensionless scale

$$t = \log\frac{\Lambda_0}{\Lambda}, \qquad \Lambda = \Lambda_0 e^{-t}$$

$$\partial_t = -\Lambda\frac{\partial}{\partial\Lambda}$$

Critical exponent θ_j

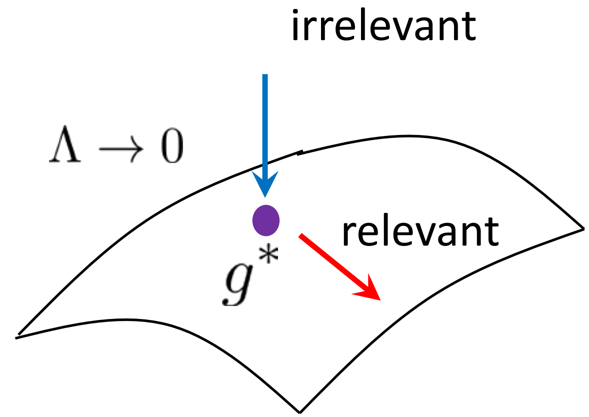
- linearized beta function around g^*

$$\partial_t g_i = \beta_i(g)$$

$$\simeq \underbrace{\beta(g^*)}_{=0} + \sum_{j=1}^{N} \left.\frac{\partial \beta_i}{\partial g_j}\right|_{g=g^*} (g_j - g_j^*)$$

Eigenvalue

$$g_i(\Lambda) = g_i^* + \sum_j^N \zeta_j^i \left(\frac{\Lambda_0}{\Lambda}\right)^{\theta_j}$$



$\theta_j > 0$

Go away from g^*

$\theta_j < 0$ come close to g^*

The flow with a positive θ: relevant

negative θ: irrelevant

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Without fermion

[R. Percacci, D. Perini' 03]
[G. Narain, R. Percacci '09]

- Scalar-gravity system
  - 5 dimensional theory space: $\{M_{\text{Pl}}^2,\ \Lambda_{\text{cc}},\ m^2,\ \xi,\ \lambda\}$
  - Gaussian-matter FP exists.
  - Critical exponents:

$$M_{\text{pl}}^{2\,*} = 2.38 \times 10^{-2}$$
$$\Lambda_{\text{cc}}^{*} = 8.82 \times 10^{-3}$$

	$M_{\text{pl}}^2,\ \Lambda_{\text{cc}}$	$m^2,\ \xi$	λ
$\theta_i =$	$2.143 \pm 2.879i$	$0.143 \pm 2.879i$	-2.627

- Non-minimal coupling $\xi\phi^2 R$ found to be **relevant**.
- ξ is a free parameter.
- Then, in principle, ξ can have large value.

With fermion

[K-y Oda, **M. Y.**, '15]

- 6 dimensional theory space: $\{M_{\rm Pl}^2,\ \Lambda_{\rm cc},\ m^2,\ \xi,\ \lambda,\ y\}$
- Gaussian-matter FP exists.
- Critical exponents:

$M_{\rm pl}^{2\,*} = 1.63 \times 10^{-2}$

$\Lambda_{\rm cc}^{*} = 3.72 \times 10^{-3}$

	$M_{\rm pl}^2,\ \Lambda_{\rm cc}$	$m^2,\ \xi$	λ	y
$\theta_i =$	$1.509 \pm 2.4615i$	$-0.4909 \pm 2.461i$	-2.6069	-1.464

[O. Zanusso, L. Zambelli,
G. P. Vacca, R. Percacci '10]

- Fermion fluctuation makes non-minimal coupling $\xi\phi^2 R$ **irrelevant**.
- m^2 and ξ cannot be free parameters!

Why m^2 and ξ become irrelevant?

- Effect of fermionic fluctuation

Without fermion → With fermion

$$M_{\rm pl}^{2\,*} = 2.38 \times 10^{-2} \quad\rightarrow\quad M_{\rm pl}^{2\,*} = 1.63 \times 10^{-2}$$

$$\Lambda_{\rm cc}^{*} = 8.82 \times 10^{-3} \quad\rightarrow\quad \Lambda_{\rm cc}^{*} = 3.72 \times 10^{-3}$$

- The matrix $$\frac{\partial \beta_i}{\partial g_j} \simeq \begin{pmatrix} \frac{\partial \beta_\xi}{\partial \xi} & \frac{\partial \beta_\xi}{\partial m^2} \\ \frac{\partial \beta_{m^2}}{\partial \xi} & \frac{\partial \beta_{m^2}}{\partial m^2} \end{pmatrix}$$

Its eigenvalues = critical exponents

Around the Gaussian-matter FP → $$\theta_i \simeq \frac{1}{2}\left(\frac{\partial \beta_\xi}{\partial \xi} + \frac{\partial \beta_{m^2}}{\partial m^2}\right)$$

$$\begin{pmatrix} 2.85544 & -6.51993 \\ 2.40051 & -2.57031 \end{pmatrix} \quad\rightarrow\quad \begin{pmatrix} 1.6814 & -5.39674 \\ 1.99718 & -2.66334 \end{pmatrix}$$

$$\theta_i \simeq \frac{2.85544 - 2.57031}{2} > 0 \qquad\qquad \theta_i \simeq \frac{1.6814 - 2.66334}{2} < 0$$

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Higgs inflation

[F. Bezrukov, M. Shaposhnikov, ’08]

- The action (Jordan frame)

$$S_J = \int d^4x\sqrt{-g}\left\{ \left(1+\xi\frac{h^2}{M_{\rm pl}^2}+\cdots\right)\frac{M_{\rm pl}^2}{2}R + \frac{1}{2}\partial_\mu h\partial^\mu h - V(h^2)\right\}$$

- Conformal transformation (Jordan frame ⇒ Einstein frame)

$$\left(1+\xi\frac{h^2}{M_{\rm pl}^2}+\cdots\right)g_{\mu\nu} \to g^E_{\mu\nu} \quad \Rightarrow \quad V(h^2) \to \frac{V(h^2)}{\left(1+\xi\frac{h^2}{M_{\rm pl}^2}+\cdots\right)^2}$$

- Obtain the plat potential for $h \gg M_{\rm pl}$.
- To realize Higgs inflation, needs large ξ
  - At least $\xi \sim 10$ … Is it possible?

[Y. Hamada, H. Kawai, K-y. Oda, S. C. Park, ’14]
[J.L. Cook, L. M. Krauss, A. J. Long, S. Sabharwa;, 14]

Can ξ become large?

- ξ is irrelevant in our model.
- ξ should be generated by relevant couplings in low energy.
- The canonical dimension of ξ is zero
- The quantum fluctuation is small.

$$
\begin{aligned}
\partial_t \xi_2 = &-\frac{1}{576\pi^2}\Bigg[\frac{1+2\lambda_2}{\xi_0-\lambda_0}\left(9+\frac{39\xi_0}{\xi_0-\lambda_0}+\frac{60\xi_0^2}{(\xi_0-\lambda_0)^2}\right)+\frac{3(3+32\xi_2)}{\xi_0-\lambda_0}-\frac{6\xi_0(11+2\xi_2)}{(\xi_0-\lambda_0)^2}\\
&\quad -\frac{60\xi_0^2(1+2\xi_2)}{(\xi_0-\lambda_0)^3}+\frac{216\xi_2(1+2\xi_2)^2}{(1+2\lambda_2)^3(\xi_0-\lambda_0)}+\frac{9[\lambda_0(5-2\xi_2)-2\xi_0(1+2\xi_2)](1+2\xi_2)}{(1+2\lambda_2)(\xi_0-\lambda_0)^2}\\
&\quad +\frac{27(1+2\xi_2)\left(1-10\xi_2-16\xi_2^2\right)}{(1+2\lambda_2)^2(\xi_0-\lambda_0)}+\frac{108\xi_0\xi_2(1+2\xi_2)^2}{(1+2\lambda_2)^2(\xi_0-\lambda_0)^2}+\frac{72\lambda_4}{(1+2\lambda_2)^2}\frac{1+12\xi_2+2\lambda_2}{1+2\lambda_2}\Bigg]\\
&+\frac{\partial_t\xi_0-2\xi_0}{1152\pi^2\xi_0}\Bigg[\frac{1+2\lambda_2}{\xi_0-\lambda_0}\left(3+\frac{18\xi_0}{\xi_0-\lambda_0}+\frac{20\xi_0^2}{(\xi_0-\lambda_0)^2}\right)+\frac{15\xi_2}{\xi_0}-\frac{6(1+\xi_2)}{\xi_0-\lambda_0}-\frac{10\xi_0(3+4\xi_2)}{(\xi_0-\lambda_0)^2}\\
&\quad -\frac{20\xi_0^2(1+2\xi_2)}{(\xi_0-\lambda_0)^3}-\frac{3[\lambda_0-\xi_0(5-4\xi_2)](1+2\xi_2)}{(1+2\lambda_2)(\xi_0-\lambda_0)^2}+\frac{36\xi_0\xi_2(1+2\xi_2)^2}{(1+2\lambda_2)^2(\xi_0-\lambda_0)^2}\Bigg]\\
&+\frac{\partial_t\xi_2}{1152\pi^2\xi_0}\Bigg[-15+\frac{54\xi_0}{\xi_0-\lambda_0}+\frac{20\xi_0^2}{(\xi_0-\lambda_0)^2}-\frac{6\xi_0(7+2\xi_2)}{(1+2\lambda_2)(\xi_0-\lambda_0)}-\frac{144\xi_0\xi_2(1+2\xi_2)}{(1+2\lambda_2)(\xi_0-\lambda_0)}\Bigg]\\
&-\frac{N_{\rm f}y^2}{48\pi^2},
\end{aligned}
$$

$$\xi_0 = M_{\rm pl}^2/\Lambda^2$$

$$\lambda_0 = \Lambda_{\rm cc}/\Lambda^4$$

$$\xi_2 = \xi$$

Fine-tuning problem in Higgs

- Higgs mass

$$m_R^2 = m_0^2 + \left(\frac{\lambda}{16\pi^2} + \cdots \right) \Lambda_{\rm pl}^2$$

$$(10^2\ \mathrm{GeV})^2 \ = \ -(10^{19}\ \mathrm{GeV})^2 \ + \ (10^{19}\ \mathrm{GeV})^2$$

- The symmetries protect a mass of fermion and gauge field.
  - Chiral symmetry

$$\cancel{m_0 \bar{\psi}\psi}$$

  - Gauge symmetry

$$\cancel{m_0^2 A_\mu A^\mu}$$

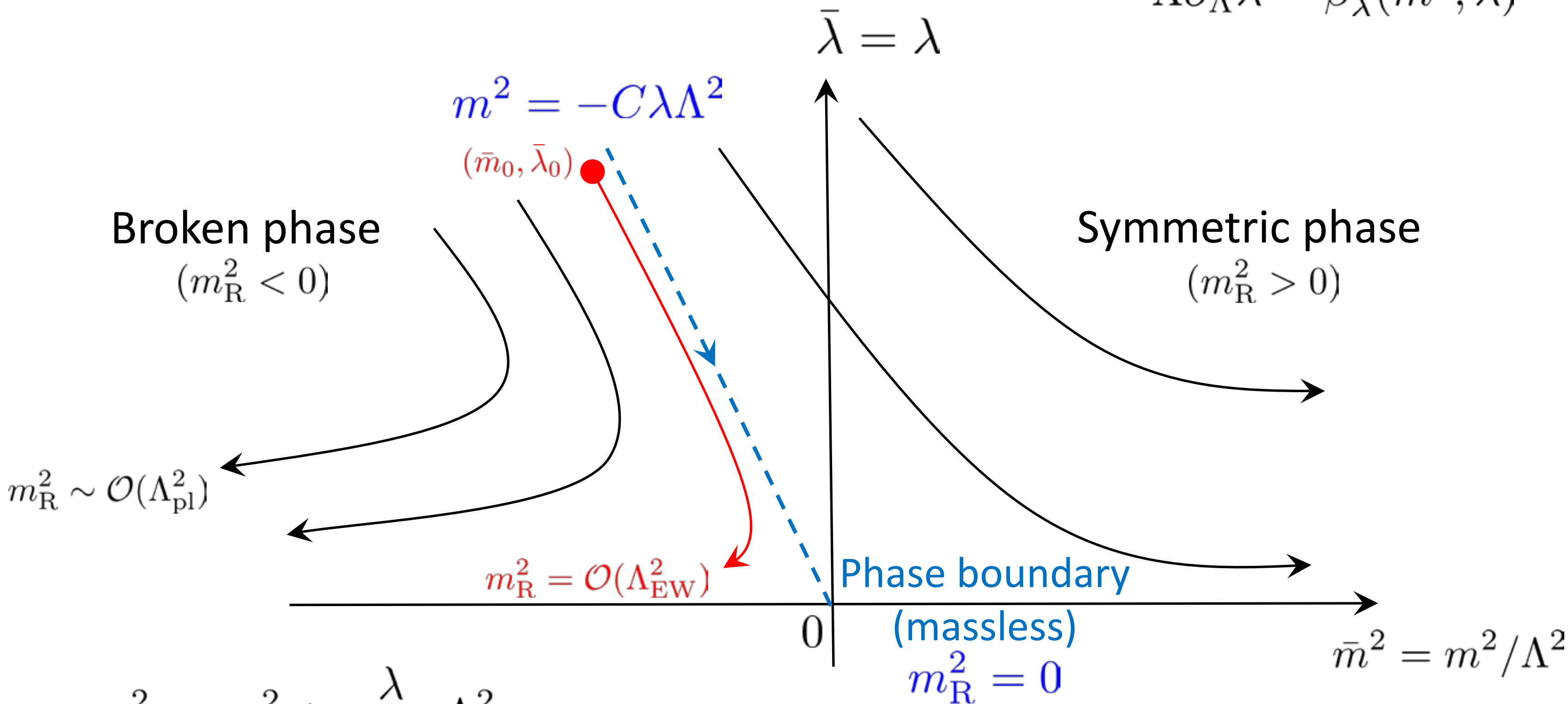
Fine-tuning problem in viewpoint of Wilson's RG

[Cf. H. Aoki, S. Iso, '14]

$$\mathcal{L}_{\rm bare}|_{\Lambda=\Lambda_{\rm pl}} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{m_0^2}{2}\phi^2 - \frac{\lambda_0}{4}\phi^4$$

$$-\Lambda\partial_\Lambda \bar{m}^2 = \beta_{\bar{m}^2}(\bar{m}^2, \bar{\lambda})$$

$$-\Lambda\partial_\Lambda \bar{\lambda} = \beta_{\bar{\lambda}}(\bar{m}^2, \bar{\lambda})$$



$$m_{\rm R}^2 = m_0^2 + \frac{\lambda}{16\pi^2}\Lambda_{\rm pl}^2$$

Fine-tune problem = Why is the Higgs close to critical?

Fine-tuning problems

- The scalar mass is irrelevant, thus not free parameter.
- The scalar mass should be generated by relevant couplings, $M_{\rm pl}, \Lambda_{\rm cc}$.

The criticality of the universe ⇔ The criticality of the Higgs

- Once the scalar mass is generated, the mass grows up in low energy scale due to the canonical scaling .

$$\partial_t \lambda_2 = 2\lambda_2 - \frac{1}{48\pi^2}\left[\frac{9\lambda_0(1+2\xi_2)}{2(\xi_0-\lambda_0)^2} - \frac{9(2\lambda_0-\xi_0)(1+2\xi_2)^2}{2(1+2\lambda_2)(\xi_0-\lambda_0)^2} - \frac{9(1+2\xi_2)^2}{2(1+2\lambda_2)^2(\xi_0-\lambda_0)} - \frac{18\lambda_4}{(1+2\lambda_2)^2}\right]$$
$$+ \frac{\partial_t\xi_0 - 2\xi_0}{96\pi^2\xi_0}\left[-\frac{2\xi_2}{\xi_0} + \frac{3\xi_0(1+2\xi_2)}{2(\xi_0-\lambda_0)^2} - \frac{3\xi_0(1+2\xi_2)^2}{2(1+2\lambda_2)(\xi_0-\lambda_0)^2}\right]$$
$$+ \frac{1}{96\pi^2}\frac{\partial_t\xi_2}{\xi_0}\left[2 - \frac{3\xi_0}{\xi_0-\lambda_0} + \frac{6\xi_0(1+2\xi_2)}{(1+2\lambda_2)(\xi_0-\lambda_0)}\right] - \frac{N_{\rm f}y^2}{8\pi^2},$$

$\xi_0 = M_{\rm pl}^2/\Lambda^2$

$\lambda_0 = \Lambda_{\rm cc}/\Lambda^4$

$\xi_2 = \xi$

$\lambda_2 = m^2$

Fine-tuning problems

- To realize the criticality: $\Lambda_{\mathrm{cc}} \sim 0,\ m^2 \sim 0$.
- The relevant couplings, $M_{\mathrm{pl}}, \Lambda_{\mathrm{cc}}$, must be fine-tuned.
- The fine-tuning problem still remains.
- Our result indicates that both the fine-tuning problems of Λ_{cc} and m^2 are related.

- Is there a trajectory both the universe and the Higgs are critical?
- How to guarantee to choose the trajectory?
  - Symmetry?

- cf. The classical scale symmetry
  - It makes the Higgs critical.

[W. A. Bardeen, '95]
[H. Aoki, S. Iso, '14]

Summary

- Higgs-Yukawa model non-minimally coupled to quantum gravity
  - Toy model of Higgs inflation
- The fermionic fluctuation makes ξ and m^2 irrelevant.
- ξ cannot become large in low energy.
- Fine-tune problem still remains.
  - Fine-tuning for relevant couplings is required.
  - Cosmological constant ⇔ Higgs mass

- Gauge and cutoff scheme dependence
- Extension of theory space.
  - Gauge fields