On the two-loop counterterm in Quantum Gravity

Benjamin Knorr

in collaboration with H. Gies, S. Lippoldt and F. Saueressig

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Theoretisch-Physikalisches Institut
Friedrich-Schiller-Universität Jena

Asymptotic Safety Seminar

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Quantum and Gravitational Fields

Outline

- Short recap: why perturbative QG is thought to fail
- AS and the Goroff-Sagnotti term: setup and analytical results
- AS and the Goroff-Sagnotti term: explicit calculations
- Summary

Perturbative QG

- Einstein-Hilbert action:

$$S_{\text{EH}} = \frac{1}{16\pi G_N} \int \sqrt{-\det g}\,(R - 2\Lambda)$$

- naive power-counting suggests that gravity is perturbatively nonrenormalisable: $[G_N] = -[M_{\text{Pl}}^2] = -2$
- possible way around: (on-shell) cancellations

A basis of tensor structures

- introduce a basis of curvature invariants (in 4 dimensions):
  - order 0: "1"
  - order 1: R
  - order 2: $R^2, R_{\mu\nu}R^{\mu\nu}, C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma}$
  - order 3: $R\Box R, R_{\mu\nu}\Box R^{\mu\nu}, R^3, R\,R_{\mu\nu}R^{\mu\nu}, R_\mu{}^\nu R_\nu{}^\rho R_\rho{}^\mu,$ $R_{\mu\nu}R_{\rho\sigma}C^{\mu\rho\nu\sigma}, R\,C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma}, C_{\mu\nu}{}^{\rho\sigma}C_{\rho\sigma}{}^{\tau\omega}C_{\tau\omega}{}^{\mu\nu}$
  - ...

1-loop renormalisation I

- due to negative mass dimension of G_N, order 2 invariants are generated by renormalisation (see, e.g., t'Hooft and Veltman)
- schematically,

$$\Delta\Gamma_{\text{div}} \sim \frac{1}{\epsilon} \int \sqrt{-\det g} \left(\alpha R^2 + \beta R_{\mu\nu} R^{\mu\nu} + \gamma C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} \right)$$

- in 4 dimensions, the Euler characteristic

$$\chi = \frac{1}{32\pi^2} \int \sqrt{-\det g} \left(\frac{2}{3} R^2 - 2 R_{\mu\nu} R^{\mu\nu} + C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} \right)$$

is a topological invariant $\Rightarrow$ eliminate C^2-term

1-loop renormalisation II

- remaining divergent part of effective action:

$$\Delta\Gamma_{\text{div}} \sim \frac{1}{\epsilon} \int \sqrt{-\det g} \left(\alpha' R^2 + \beta' R_{\mu\nu} R^{\mu\nu} \right)$$

- go on-shell (with $\Lambda = 0$): $R = 0, R_{\mu\nu} = 0$
  $\Rightarrow$ gravity (with vanishing cosmological constant) is one-loop finite!
- does this hold to all orders?

2-loop renormalisation - Goroff and Sagnotti

- at the 2-loop level, only one invariant remains on-shell, the "Goroff-Sagnotti term" $C_{\mu\nu}{}^{\rho\sigma} C_{\rho\sigma}{}^{\tau\omega} C_{\tau\omega}{}^{\mu\nu}$
- Goroff and Sagnotti (1985), van de Ven (1991): gravity is 2-loop divergent,

$$\Delta\Gamma_{\text{div}} = (32\pi G_N)\,\frac{-209}{2880(4\pi)^4}\,\frac{1}{\epsilon}\int\sqrt{-\det g}\; C_{\mu\nu}{}^{\rho\sigma} C_{\rho\sigma}{}^{\tau\omega} C_{\tau\omega}{}^{\mu\nu}$$

- there is no topological invariant that could rescue us here
- it is believed that this problem reoccurs to all higher orders $(C^4 \ldots)$

AS and the Goroff-Sagnotti term - some expectations

Will the Asymptotic Safety Program fail once a “proper perturbative counterterm” is included?

- answer from outside: yes, because nature is obviously made up of (strings/loops/. . .)[1]
- answer from inside: see below!

[1] depending on whom you ask. . .

Questions?

AS and the Goroff-Sagnotti term - setup

Will the Asymptotic Safety Program fail once a "proper perturbative counterterm" is included?

- investigate minimal setup: Einstein and Hilbert meet Goroff and Sagnotti on Weinbergs BBQ:

$$\Gamma_k = \int \sqrt{\det g} \left[\frac{1}{16\pi G_N} (-R + 2\Lambda) + \sigma \, C_{\mu\nu}{}^{\rho\sigma} C_{\rho\sigma}{}^{\tau\omega} C_{\tau\omega}{}^{\mu\nu} \right]$$

- curvature expansion along the lines of the URGM
- single metric

AS&GS: analytical insights I

- What can we conclude without explicit calculation?

$$\Gamma^{(2)} \sim \frac{1}{G_N} \cdot (1, R) + \sigma \cdot \left(C, C^2, C^3 \right)$$

- C is traceless $\rightarrow \beta_g$ and β_λ are independent of σ!

Conclusion 1: The AS fixed point of the EH truncation is not influenced by the GS term.

AS&GS: analytical insights II

- we can even write down the form of the beta function of σ:

$$\dot{\sigma} = c_0 + (2 + c_1)\sigma + c_2\sigma^2 + c_3\sigma^3$$

- straightforward consequence of the curvature expansion of the flow equation and the schematic form of $\Gamma^{(2)}$
- independent of the truncation: inclusion of $R^2, C^2, \ldots$ does not change this form (but c_i are changed)
- central question: $c_3 \neq 0$?

AS&GS: analytical insights III

- this can be answered by inspection of the explicit form of $\Gamma^{(2)}$
- only the term $\sim \sigma C$ (as a vertex insertion) contributes to c_3
- it projects the propagator onto its identity component[2], thus c_3 is nonzero

Conclusion 2: The coupling σ always has a real fixed point.

Conclusion 3=1+2: Asymptotic Safety withstands the inclusion of the two-loop counterterm.

[2] details still to be figured out

AS&GS: analytical insights IV

- explicit calculation shows that schematically,

$$c_3 = \gamma \int \mathrm{d}p \, \left(G^{TT}(p^2) \right)^4 \dot{R}_k(p^2) > 0$$

which is even gauge-independent, and positivity is independent of the truncation

- G^{TT}: scalar part of the identity component of propagator
- EH: $G^{TT} = 32\pi G_N \, (p^2 + R_k(p^2) - 2\Lambda)^{-1}$

Conclusion 4: There is a real fixed point where σ is UV-repulsive.

Questions?

AS&GS: explicit calculation

Several ingredients to actually calculate the beta function of σ:

- heavy use of tensor algebra package xAct
- terms including R and $R_{\mu\nu}$ can be dropped, as they cannot combine to C^3 (formally, project onto K3 background)[3]
- optimised regulator and harmonic gauge fixing ($\alpha = \beta = 1$) simplify calculation tremendously
- lots of patience and coffee

[3]covariantly constant doesn't do the trick: $C \Delta C = 3C^3 + \dots$

AS&GS: fixed points

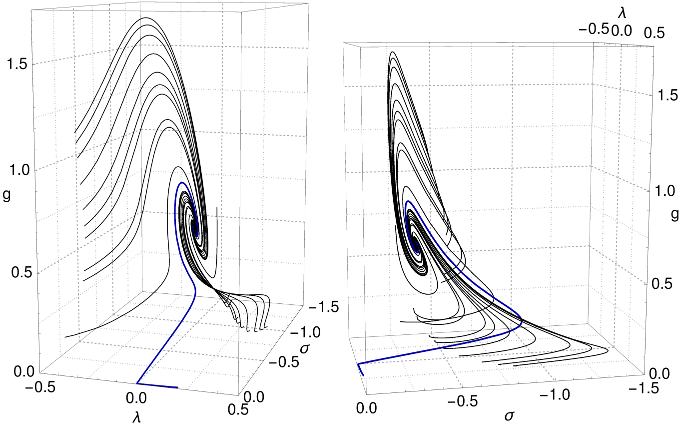
- scenario A: $\Lambda = 0$

	g_*	σ_*	θ_g	θ_σ
FP 1	1.64	-0.226	2.09	-77.4
FP 2	1.64	-0.0186	2.09	5.62
FP 3	1.64	-0.00227	2.09	-6.06

- scenario B: “full” flow

g_*	λ_*	σ_*	$\theta_{1,2}$	θ_σ
0.707	0.193	-0.305	$1.48 \pm 3.04\,\mathbf{i}$	-79.4

AS&GS: phase diagram



Summary and Outlook

- Asymptotic Safety withstands the Goroff-Sagnotti term
- it seems that the coupling is completely irrelevant, in accord with its mass dimension
- partly answered: what happens when one includes more invariants?
- unanswered: what happens beyond the background approximation?

Thank you for your attention!

Questions? Comments?