Background Independence in a Background Dependent Renormalization Group

Zoë Slade

University of Southampton

Based on: Peter Labus, Tim R. Morris and Zoë H. Slade, arXiv:1603.04772

Contents

Background independence

- Background independence is a fundamental requirement for a theory of quantum gravity.

Background field method

$$\tilde{g}_{\mu\nu} = \bar{g}_{\mu\nu} + \tilde{h}_{\mu\nu}$$

- Use background metric $\bar{g}_{\mu\nu}$ to build Laplacian operator $\bar{\nabla}^2$.
- Cutoff R_k is function of background metric:

$$R_k(-\bar{\nabla}^2)$$

- Background independence lost (at intermediate k).
- Imposing modified split Ward identity restores background independence in limit $k \to 0$.

Background independence in conformally reduced gravity

Conformally reduced gravity

$$\tilde{g}_{\mu\nu} = f(\tilde{\phi})\,\hat{g}_{\mu\nu}$$

Findings of J. A. Dietz and T. R. Morris (arXiv:1502.07396):

- Background independence can in general be in conflict with the existence of fixed points, unless careful choices are made.
- Background independent description of flow can be uncovered if cutoff is power law

$$R_k \propto p^{-2n}$$

Conformally reduced gravity

$$\tilde{g}_{\mu\nu} = f(\tilde{\phi})\,\delta_{\mu\nu} = f(\chi + \tilde{\varphi})\,\delta_{\mu\nu} \quad , \quad \bar{g}_{\mu\nu} = f(\chi)\,\delta_{\mu\nu}$$

- Background field χ, fluctuation field $\tilde{\varphi}$.
- Parameterisation f is chosen to be independent of k.

The flow equation

$$\partial_t \Gamma_k[\varphi, \chi] = \frac{1}{2}\mathrm{Tr}\left[\frac{1}{\sqrt{\bar{g}}\sqrt{\bar{g}}}\frac{\delta^2\Gamma_k}{\delta\varphi\delta\varphi} + R_k[\chi]\right]^{-1}\partial_t R_k[\chi]$$

where $\varphi = \langle\tilde{\varphi}\rangle$

In the conformal truncation, cutoff depends on background field χ:

$$R_k[-\bar{\nabla}^2] \to R_k[\chi]$$

The modified split Ward identity

The Ward identity

$$\frac{1}{\sqrt{\bar{g}}}\left(\frac{\delta\Gamma_k}{\delta\chi}-\frac{\delta\Gamma_k}{\delta\varphi}\right)=\frac{1}{2}\mathrm{Tr}\left[\frac{1}{\sqrt{\bar{g}}\sqrt{\bar{g}}}\frac{\delta^2\Gamma_k}{\delta\varphi\delta\varphi}+R_k[\chi]\right]^{-1}\frac{1}{\sqrt{\bar{g}}}\left\{\frac{\delta R_k[\chi]}{\delta\chi}+\frac{d}{2}\partial_\chi \ln f\, R_k[\chi]\right\}$$

- Keeps track of background dependence.
- Background independence realised if RHS vanishes i.e. if Γ_k is function of total field $\phi = \chi + \varphi$.
- Background independence automatically restored in limit $k \to 0$ (since R_k vanishes).
- Need Ward identity to control arbitrary enlargement of theory space.
- Not an optional extra!
- Also used to uncover a background independent description.

The derivative expansion

$$\Gamma_k[\varphi,\chi] = \int d^d x \sqrt{\bar{g}} \left(-\frac{1}{2} K(\varphi,\chi) \bar{g}^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi + V(\varphi,\chi) \right)$$

- Slowly varying background field χ.

Flow equation for potential V:

$$\partial_t V(\varphi,\chi) = f(\chi)^{-\frac{d}{2}} \int dp\, p^{d-1} Q_p \dot{R}_p$$

Ward identity for V:

$$\partial_\chi V - \partial_\varphi V + \frac{d}{2} \partial_\chi \ln f\, V = f(\chi)^{-\frac{d}{2}} \int dp\, p^{d-1} Q_p \left(\partial_\chi R_p + \frac{d}{2} \partial_\chi \ln f\, R_p \right)$$

where

$$Q_p = \left(\partial_\varphi^2 V - p^2 \frac{K}{f} + R_p \right)^{-1}$$

The derivative expansion

Flow equation for K:

$$f^{-1}\partial_t K(\varphi,\chi) = 2f^{-\frac{d}{2}}\int dp\, p^{d-1} P_p(\varphi,\chi)\dot{R}_p$$

Ward identity for K:

$$f^{-1}\left(\partial_\chi K - \partial_\varphi K + \frac{d-2}{2}\partial_\chi \ln f\, K\right) = 2f^{-\frac{d}{2}}\int dp\, p^{d-1} P_p(\varphi,\chi)\left(\partial_\chi R_p + \frac{d}{2}\partial_\chi \ln f\, R_p\right)$$

where

$$P_p = P_p(V, K/f, R_p)$$

Compatibility

Compatibility: Flow equation and Ward identity satisfied simultaneously for all values of k.

Test for compatibility:

1. Write Ward identity as $\mathcal{W} = 0$
2. Take RG time derivative $\implies \dot{\mathcal{W}}$
3. If $\dot{\mathcal{W}} = 0$ then follows without further constraints, we have compatibility.

Compatibility at the exact level

Ward identity: (arXiv:hep-th/9802064 & 9809020)

$$\mathcal{W}_\omega \equiv \frac{\delta \Gamma_k}{\delta \chi_\omega} - \frac{\delta \Gamma_k}{\delta \varphi_\omega} - \frac{1}{2}\Delta_{xy}\frac{\delta r_{yx}}{\delta \chi_\omega} = 0 \tag{1}$$

where

$$\Delta_{xy} \equiv \left(\frac{\delta^2 \Gamma_k}{\delta \varphi_x \delta \varphi_y} + r_{xy}\right)^{-1} \quad \text{and} \quad r_{xy} \equiv \sqrt{\bar{g}(x)}\sqrt{\bar{g}(y)}R_k(x,y)$$

Taking the RG time derivative gives

$$\dot{\mathcal{W}}_\omega = \frac{\delta \dot{\Gamma}_k}{\delta \chi_\omega} - \frac{\delta \dot{\Gamma}_k}{\delta \varphi_\omega} + \frac{1}{2}\left[\Delta\left(\frac{\delta^2 \dot{\Gamma}_k}{\delta \varphi \delta \varphi} + \dot{r}\right)\Delta\right]_{xy}\frac{\delta r_{yx}}{\delta \chi_\omega} - \frac{1}{2}\Delta_{xy}\frac{\delta \dot{r}_{yx}}{\delta \chi_\omega}$$

Compatibility at the exact level

Substitute in flow equation $\dot{\Gamma}_k$:

$$\dot{\mathcal{W}}_\omega = -\frac{1}{2}(\Delta \dot{r} \Delta)_{zz'} \frac{\delta^2}{\delta\varphi_{z'}\delta\varphi_z}\left(\frac{\delta\Gamma}{\delta\chi_\omega} - \frac{\delta\Gamma}{\delta\varphi_\omega}\right) + \frac{1}{4}\left(\frac{\delta^2}{\delta\varphi_z\delta\varphi_{z'}}\Delta_{uu'}\right)\dot{r}_{u'u}\,\Delta_{z'y}\frac{\delta r_{yx}}{\delta\chi_\omega}\Delta_{xz}$$

Expanding out second term:

$$\left(\frac{\delta^2}{\delta\varphi_z\delta\varphi_{z'}}\Delta_{uu'}\right)\dot{r}_{u'u}\,\Delta_{z'y}\frac{\delta r_{yx}}{\delta\chi_\omega}\Delta_{xz} \tag{2}$$

$$= \Delta_{xz}\Bigg(\Delta_{uv}\Gamma_{zvs}\Delta_{sv'}\Gamma_{z'v's'}\Delta_{s'u'} + \Delta_{uv'}\Gamma_{v's'z'}\Delta_{s'v}\Gamma_{zvs}\Delta_{su'} \tag{3}$$

$$- \Delta_{uv'}\Gamma_{v's'zz'}\Delta_{s'u'}\Bigg)\dot{r}_{u'u}\Delta_{z'y}\frac{\delta r_{yx}}{\delta\chi_\omega}$$

$$= (\Delta\dot{r}\Delta)_{s'v'}\frac{\delta^2}{\delta\varphi_{v'}\delta\varphi_{s'}}\Delta_{xy}\frac{\delta r_{yx}}{\delta\chi_\omega} \qquad \text{where} \quad \Gamma_{x_1\ldots x_n} \equiv \frac{\delta^n\Gamma_k}{\delta\varphi_{x_1}\ldots\delta\varphi_{x_n}}$$

Compatibility at the exact level

$$\implies \dot{\mathcal{W}}_\omega = -\frac{1}{2}(\Delta \dot{r} \Delta)_{zz'} \frac{\delta^2}{\delta\varphi_{z'}\delta\varphi_z}\left(\frac{\delta\Gamma}{\delta\chi_\omega} - \frac{\delta\Gamma}{\delta\varphi_\omega}\right) + (\Delta \dot{r} \Delta)_{s'v'} \frac{\delta^2}{\delta\varphi_{v'}\delta\varphi_{s'}} \Delta_{xy} \frac{\delta r_{yx}}{\delta\chi_\omega}$$

$$\therefore \quad \dot{\mathcal{W}}_\omega = -\frac{1}{2}\mathrm{tr}\left(\Delta \dot{r} \Delta \frac{\delta^2}{\delta\varphi\delta\varphi}\right)\mathcal{W}_\omega$$

The flow equation and Ward identity are compatible.

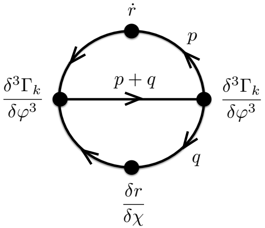
Compatibility at the exact level

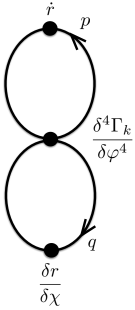
$$\left(\frac{\delta^2}{\delta\varphi_z\delta\varphi_{z'}}\Delta_{uu'}\right)\dot{r}_{u'u}\,\Delta_{z'y}\frac{\delta r_{yx}}{\delta\chi_\omega}\Delta_{xz} \tag{4}$$

$$= \Delta_{xz}\Big(\Delta_{uv}\Gamma_{zvs}\Delta_{sv'}\Gamma_{z'v's'}\Delta_{s'u'} + \Delta_{uv'}\Gamma_{v's'z'}\Delta_{s'v}\Gamma_{zvs}\Delta_{su'}$$

$$- \Delta_{uv'}\Gamma_{v's'zz'}\Delta_{s'u'}\Big)\dot{r}_{u'u}\Delta_{z'y}\frac{\delta r_{yx}}{\delta\chi_\omega} \tag{5}$$

$$= (\Delta\dot{r}\Delta)_{s'v'}\frac{\delta^2}{\delta\varphi_{v'}\delta\varphi_{s'}}\Delta_{xy}\frac{\delta r_{yx}}{\delta\chi_\omega}$$





Compatibility in the derivative expansion

Flow of Ward identity for V:

$$\dot{\mathcal{W}}^{(V)} = -\int_p Q_p^2 \dot{R}_p \left(\partial_\varphi^2 \mathcal{W}^{(V)} - p^2 \mathcal{W}^{(K)}\right) - \int_{p,q} Q_p^2 \left(\partial_\varphi^2 Q_q - 2p^2 P_q\right) [\dot{R}, \partial_\chi R + \gamma R]_{qp}$$

where $\int_p \equiv f(\chi)^{-d/2} \int dp p^{d-1}$ and $\gamma \equiv \frac{d}{2} \partial_\chi \ln f$.

Compatibility realised if

$$\left[\dot{R}, \partial_\chi R + \gamma R\right]_{qp} \equiv \dot{R}_q(\partial_\chi R_p + \gamma R_p) - \dot{R}_p(\partial_\chi R_q + \gamma R_q) = 0 \qquad (6)$$

$$\implies \frac{(\partial_\chi R_p + \gamma R_p)}{\dot{R}_p} = \frac{(\partial_\chi R_q + \gamma R_q)}{\dot{R}_q} \quad \text{(ratio independent of momentum)}$$

$$\implies \boxed{\partial_\chi R_p + \gamma R_p = F(\chi, t)\, \dot{R}_p} \qquad (7)$$

Compatibility in the derivative expansion

Flow of Ward identity for K:

- More complicated
- Get additional commutator-like terms:

$$\left(\partial_\chi R_p + \gamma R_p\right)\partial^k_{p^2}\dot{R}_p - \dot{R}_p\partial^k_{p^2}\left(\partial_\chi R_p + \gamma R_p\right) \tag{8}$$

These vanish if, again,

$$\boxed{\partial_\chi R_p + \gamma R_p = F(\chi, t)\,\dot{R}_p} \tag{9}$$

- This provides a necessary and sufficient condition to ensure compatibility in the derivative expansion.

Required form of cutoff R_k

Denote dimensionless quantities with a $\overline{\text{bar}}$:

$$\varphi = k^{\eta/2}\bar{\varphi}, \qquad \chi = k^{\eta/2}\bar{\chi}, \qquad f(\chi) = k^{d_f}\bar{f}(\chi),$$

$$V(\varphi,\chi) = k^{d_V}\bar{V}(\bar{\varphi},\bar{\chi}), \qquad K(\varphi,\chi) = k^{d_R-2+d_f}\bar{K}(\bar{\varphi},\bar{\chi})$$

where $d_V = d(1-d_f/2)$ and $d_R = d_V - \eta$.

By dimensions, we have

$$R(p^2/f) = -k^{d_R}\, r\left(\frac{p^2}{k^{2-d_f}f}\right) = -k^{d_R}\, r(\hat{p}^2) \tag{10}$$

$$\implies \gamma\dot{R}_p = d_V\left[\partial_\chi R_p + \gamma R_p\right] - \eta\gamma R_p \tag{11}$$

Required form of cutoff R_k

$$\gamma \dot{R}_p = d_V \left[\partial_\chi R_p + \gamma R_p\right] - \eta\gamma R_p \tag{12}$$

- If $\eta = 0$ compatibility condition $\boxed{\partial_\chi R_p + \gamma R_p = F(\chi, t)\, \dot{R}_p}$ automatically satisfied ($F(\chi, t) = \gamma/d_v$).
- If $\eta \neq 0$ the compatibility condition implies

$$\hat{p}\frac{d}{d\hat{p}} r(\hat{p}^2) = -2n\, r(\hat{p}^2) \tag{13}$$

for some constant $n = d/2(\eta F/(d_v F - \gamma) - 1)$ and

$$\therefore \quad r(\hat{p}^2) \propto \hat{p}^{-2n} \tag{14}$$

Local potential approximation (LPA)

$$\partial_t \bar{V} + d_V \bar{V} - \frac{\eta}{2}\,\bar{\varphi}\,\frac{\partial \bar{V}}{\partial \bar{\varphi}} - \frac{\eta}{2}\,\bar{\chi}\,\frac{\partial \bar{V}}{\partial \bar{\chi}} = \int_0^\infty d\hat{p}\,\hat{p}^{d-1}\,\frac{d_R\,r - \frac{d_V}{d}\,\hat{p}\,r'}{\hat{p}^2 + r - \partial^2_{\bar{\varphi}}\bar{V}}$$

$$\frac{\partial \bar{V}}{\partial \bar{\chi}} - \frac{\partial \bar{V}}{\partial \bar{\varphi}} + \bar{\gamma}\,\bar{V} = \bar{\gamma}\int_0^\infty d\hat{p}\,\hat{p}^{d-1}\,\frac{r - \frac{1}{d}\,\hat{p}\,r'}{\hat{p}^2 + r - \partial^2_{\bar{\varphi}}\bar{V}}$$

where $r' = dr(\hat{p}^2)/d\hat{p}$ and from the change to dimensionless variables:

$$\bar{\gamma} = \frac{d}{2}\frac{\partial}{\partial \bar{\chi}}\ln \bar{f}\left(\mathrm{e}^{\eta t/2}\mu^{\eta/2}\bar{\chi}\right) \tag{15}$$

Ward identity forces $\bar{V}$ to depend on t through $\bar{\gamma} \implies$ **fixed points forbidden in general**, unless...

1. $\eta = 0$ or
2. set f to be power law: $f \propto \chi^\rho \implies \bar{\gamma} = \frac{d}{2}\frac{\rho}{\bar{\chi}} \quad , \quad \rho$ const.

Local potential approximation (LPA) - Example 1

Choose:

- $f(\chi)$ power law
- $d = 4$
- Optimised cutoff $r(\hat{p}^2) = (1-\hat{p}^2)\theta(1-\hat{p}^2)$

$$\partial_t \bar{V} + d_V \bar{V} - \frac{\eta}{2}\bar{\varphi}\,\partial_{\bar{\varphi}}\bar{V} - \frac{\eta}{2}\bar{\chi}\,\partial_{\bar{\chi}}\bar{V} = \left(\frac{d_R}{6} + \frac{\eta}{12}\right)\frac{1}{1-\partial^2_{\bar{\varphi}}\bar{V}} \tag{16}$$

$$\partial_{\bar{\chi}}\bar{V} - \partial_{\bar{\varphi}}\bar{V} + \bar{\gamma}\bar{V} = \frac{\bar{\gamma}}{6}\frac{1}{1-\partial^2_{\bar{\varphi}}\bar{V}} \tag{17}$$

Combine equations:

$$2\partial_t \bar{V} + \eta\bar{V} - (\eta\bar{\varphi} - \alpha\bar{\chi})\,\partial_{\bar{\varphi}}\bar{V} - (\eta+\alpha)\bar{\chi}\partial_{\bar{\chi}}\bar{V} = 0 \tag{18}$$

Local potential approximation (LPA) - Example 1

Combined equation:

$$2\partial_t \bar{V} + \eta \bar{V} - (\eta\bar{\varphi} - \alpha\bar{\chi})\,\partial_{\bar{\varphi}} \bar{V} - (\eta+\alpha)\bar{\chi}\partial_{\bar{\chi}} \bar{V} = 0 \tag{19}$$

Solve by method of characteristics:

$$\frac{d\bar{V}}{dt} = -\frac{\eta}{2}\bar{V}\,, \quad \frac{d\bar{\chi}}{dt} = -\frac{\alpha+\eta}{2}\bar{\chi}\,, \quad \frac{d\bar{\varphi}}{dt} = \frac{\alpha\bar{\chi} - \eta\bar{\varphi}}{2} \tag{20}$$

$$\Longrightarrow \bar{V} = \mathrm{e}^{-\eta t/2}\,\hat{V}(\hat{\phi},\hat{\chi}) = \mathrm{e}^{-\eta t/2}\,\hat{V}\left(\,\mathrm{e}^{\eta t/2}[\bar{\varphi}+\bar{\chi}],\mathrm{e}^{(\eta+\alpha)t/2}\bar{\chi}\right) \tag{21}$$

where hatted variables $\hat{V},\hat{\phi},\hat{\chi}$ are initial data. Substitute $\bar{V}$ back into flow equation:

$$\hat{\chi}\partial_{\hat{\chi}}\hat{V} + 2\rho\hat{V} = \frac{\rho}{3}\frac{1}{\mathrm{e}^{-\frac{\eta}{2}t} - \partial^2_{\hat{\phi}}\hat{V}} \tag{22}$$

$\Longrightarrow$ No solutions unless $\eta = 0$.

Local potential approximation (LPA) - Example 2

Choose:

- $f(\chi)$ general and instead $\eta = 0$
- $d = 4$
- Optimised cutoff $r(\hat{p}^2) = (1 - \hat{p}^2)\theta(1 - \hat{p}^2)$

Combined equation:

$$\partial_t \bar{V} + \frac{2 - d_f}{\partial_\chi \ln \bar{\bar{f}}} \left(\partial_\varphi \bar{V} - \partial_\chi \bar{V}\right) = 0 \tag{23}$$

whose characteristic curves satisfy:

$$\frac{d\chi}{dt} = \frac{d_f - 2}{\partial_\chi \ln \bar{\bar{f}}}, \quad \frac{d\varphi}{dt} = \frac{2 - d_f}{\partial_\chi \ln \bar{\bar{f}}}, \quad \frac{d\bar{V}}{dt} = 0 \tag{24}$$

Local potential approximation (LPA) - Example 2

$$\frac{d\chi}{dt} = \frac{d_f - 2}{\partial_\chi \ln \bar{f}}, \quad \frac{d\varphi}{dt} = \frac{2 - d_f}{\partial_\chi \ln \bar{f}}, \quad \frac{d\bar{V}}{dt} = 0 \tag{25}$$

From first equation:

$$\hat{t} = t + \frac{\ln \bar{f}}{2 - d_f} \tag{26}$$

From first two equations: $\phi = \varphi + \chi$ is an integration constant for the characteristics.

From last equation: $\bar{V}$ is also a constant for the characteristics.

Thus solution is given by:

$$\bar{V} = \hat{V}(\phi, \hat{t}) \tag{27}$$

Local potential approximation (LPA) - Example 2

Substituting $\bar{V}$ into flow:

$$\partial_{\hat{t}}\hat{V} + d_V\hat{V} = \frac{d_V}{6}\frac{1}{1-\partial_\phi^2\hat{V}} \tag{28}$$

- Background independent i.e. independent of χ (and of f).
- Fixed points in t coincide with fixed points in $\hat{t}$:

$$\partial_t\bar{V} = \partial_{\hat{t}}\hat{V}$$

- When flow and Ward identity compatible can uncover a background independent (and f independent) description.

Polynomial truncations

Expand potential and equations in a double power series:

$$\bar{V}(\bar{\varphi}, \bar{\chi}) = \sum_{n,m=0}^{\infty} a_{nm} \bar{\varphi}^n \bar{\chi}^m \tag{29}$$

Fixed point equations:

$$d_V \bar{V} - \frac{\eta}{2} \bar{\varphi} \frac{\partial \bar{V}}{\partial \bar{\varphi}} - \frac{\eta}{2} \bar{\chi} \frac{\partial \bar{V}}{\partial \bar{\chi}} = \int_0^\infty d\hat{p}\, \hat{p}^{d-1} \frac{d_R\, r - \frac{d_V}{d}\, \hat{p}\, r'}{\hat{p}^2 + r - \partial_{\bar{\varphi}}^2 \bar{V}} \tag{30}$$

$$\frac{\partial \bar{V}}{\partial \bar{\chi}} - \frac{\partial \bar{V}}{\partial \bar{\varphi}} + \bar{\gamma}\, \bar{V} = \bar{\gamma} \int_0^\infty d\hat{p}\, \hat{p}^{d-1} \frac{r - \frac{1}{d}\, \hat{p}\, r'}{\hat{p}^2 + r - \partial_{\bar{\varphi}}^2 \bar{V}} \tag{31}$$

$$\bar{\gamma} = \frac{d}{2} \frac{\partial}{\partial \bar{\chi}} \ln \bar{f} \left(\mathrm{e}^{\eta t/2} \mu^{\eta/2} \bar{\chi} \right) \tag{32}$$

Polynomial truncations

Choose:

- $f(\chi)$ power law
- d=4
- Optimised cutoff $r(\hat{p}^2) = (1 - \hat{p}^2)\theta(1 - \hat{p}^2)$

At zeroth order:

$$d_V a_{00} = \left(\frac{d_R}{6} + \frac{\eta}{12}\right)\frac{1}{1 - 2a_{20}}, \qquad 2\rho\, a_{00} = \frac{\rho}{3}\frac{1}{1 - 2a_{20}} \tag{33}$$

$$\implies d_V = d_R + \eta/2 \tag{34}$$

but previously had $d_V = d_R + \eta$

$\implies$ Fixed points excluded unless $\eta = 0$.

Polynomial truncations

$$d_V\,\bar{V} - \frac{\eta}{2}\,\bar{\varphi}\,\frac{\partial \bar{V}}{\partial \bar{\varphi}} - \frac{\eta}{2}\,\bar{\chi}\,\frac{\partial \bar{V}}{\partial \bar{\chi}} = \int_0^\infty d\hat{p}\,\hat{p}^{d-1}\,\frac{d_R\,r - \frac{d_V}{d}\,\hat{p}\,r'}{\hat{p}^2 + r - \partial^2_{\bar{\varphi}}\bar{V}} \tag{35}$$

$$\frac{\partial \bar{V}}{\partial \bar{\chi}} - \frac{\partial \bar{V}}{\partial \bar{\varphi}} + \bar{\gamma}\,\bar{V} = \bar{\gamma}\int_0^\infty d\hat{p}\,\hat{p}^{d-1}\,\frac{r - \frac{1}{d}\,\hat{p}\,r'}{\hat{p}^2 + r - \partial^2_{\bar{\varphi}}\bar{V}} \tag{36}$$

Count number of equations:

- Plug in $\bar{V}(\bar{\varphi}, \bar{\chi}) = \sum_{n,m=0}^{\infty} a_{nm}\bar{\varphi}^n\bar{\chi}^m$.
- Act with $\frac{\partial^{i+j}}{\partial\bar{\varphi}^i\,\partial\bar{\chi}^j}$ then set $\bar{\varphi} = \bar{\chi} = 0$ to obtain system at order $r = i + j$.
- For a particular r_* have $2(r_* + 1)$ equations.

Up to order r:

$$n_{\text{eqn}}(r) = \sum_{i=0}^{r} 2\,(i+1) = r^2 + 3r + 2 \tag{37}$$

Polynomial truncations

$$d_V \bar{V} - \frac{\eta}{2} \bar{\varphi} \frac{\partial \bar{V}}{\partial \bar{\varphi}} - \frac{\eta}{2} \bar{\chi} \frac{\partial \bar{V}}{\partial \bar{\chi}} = \int_0^\infty d\hat{p}\, \hat{p}^{d-1} \frac{d_R\, r - \frac{d_V}{d}\, \hat{p}\, r'}{\hat{p}^2 + r - \partial^2_{\bar{\varphi}} \bar{V}} \tag{38}$$

$$\frac{\partial \bar{V}}{\partial \bar{\chi}} - \frac{\partial \bar{V}}{\partial \bar{\varphi}} + \bar{\gamma}\, \bar{V} = \bar{\gamma} \int_0^\infty d\hat{p}\, \hat{p}^{d-1} \frac{r - \frac{1}{d}\, \hat{p}\, r'}{\hat{p}^2 + r - \partial^2_{\bar{\varphi}} \bar{V}} \tag{39}$$

Count number of coefficients on LHS:

- Plug in $\bar{V}(\bar{\varphi}, \bar{\chi}) = \sum_{n,m=0}^{\infty} a_{nm} \bar{\varphi}^n \bar{\chi}^m$.
- For any fixed pair (i, j)
  - Ward identity $\ni a_{ij}, a_{i+1,j}, a_{i,j+1}$ whilst flow equation $\ni a_{ij}$

Up to order r from LHS:

$$\left\{ a_{00}, a_{01}, \ldots, a_{0,r+1}, a_{10}, \ldots, a_{1,r}, \ldots, a_{2,r-1}, \ldots, a_{r+1,0} \right\} \tag{40}$$

Polynomial truncations

Count number of coefficients on RHS:

- Propagator $\left(\frac{1}{1-\partial^2 \bar{V}}\right)$ gives $r+1$ coefficients.
- $\bar{\gamma}$ gives $r+2$ coefficients.
- Not forgetting η and d_f.

Up to order r from both LHS and RHS:

$$\begin{aligned} n_{\text{coeff}}(r) &= n_{\text{lhs}} + (r+1) + (r+2) + 2 \\ &= \frac{1}{2} r^2 + \frac{9}{2} r + 8 \end{aligned} \tag{41}$$

Polynomial truncations

Number of equations:

$$n_{\text{eqn}}(r) = r^2 + 3r + 2 \tag{42}$$

Number of coefficients:

$$n_{\text{coeff}}(r) = \frac{1}{2}\,r^2 + \frac{9}{2}\,r + 8 \tag{43}$$

- Asymptotically twice as many equations as coefficients.
- Number of equations and coefficients equal when $r = 5.3$
- Equations either over-constrained or highly redundant beyond six-point level.

Summary and conclusions

- Investigated the potential conflict between fixed points and background independence.
- Compatibility guaranteed at exact level.
- In the derivative expansion, compatibility only guaranteed if $\eta = 0$ or cutoff R_k is power law.
- If incompatible then no solutions - confirmed with LPA example.
- If compatible, fixed points can still be forbidden:

$$\frac{\partial \bar{V}}{\partial \bar{\chi}} - \frac{\partial \bar{V}}{\partial \bar{\varphi}} + \bar{\gamma}\, \bar{V} = \bar{\gamma} \int_0^\infty d\hat{p}\, \hat{p}^{d-1}\, \frac{r - \frac{1}{d}\, \hat{p}\, r'}{\hat{p}^2 + r - \partial_{\bar{\varphi}}^2 \bar{V}} \tag{44}$$

$$\bar{\gamma} = \frac{d}{2} \frac{\partial}{\partial \bar{\chi}} \ln \bar{f} \left(\mathrm{e}^{\eta t/2} \mu^{\eta/2} \bar{\chi} \right) \tag{45}$$

Summary and conclusions

- Expanding in vertices, the flow and Ward identity shown to become either over-constrained or highly redundant beyond the six-point level.
- If compatible, can combine flow and Ward identity to uncover a background independent description.
- What about full gravity?

	parametrisation f			cutoff profile r	
η	type	d_f	runs	power-law	not power-law
$\neq 0$	not power-law	any	yes	~~FP~~ $\widehat{\mathrm{FP}}$	incompatible
	power law $f = \chi^\rho$	$\neq \rho\eta/2$	yes	$\mathrm{FP} \neq \widehat{\mathrm{FP}}$	
		$= \rho\eta/2$	no	$\mathrm{FP} = \widehat{\mathrm{FP}}$	
$= 0$	any	$\neq 0$	yes	$\mathrm{FP} = \widehat{\mathrm{FP}}$	
		$= 0$	no		