# Quantum-gravity effects on a Higgs-Yukawa model

Aaron Held

held@thphys.uni-heidelberg.de

Heidelberg University

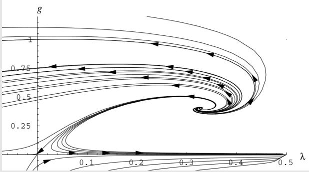
based on work with A. Eichhorn and J. M. Pawlowski [1604.02041]

May 23, 2016

# Motivation & Setup

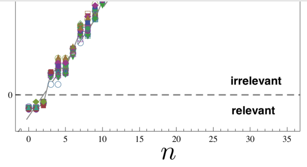
# Asymptotic Safety Programme

## Existence of fixed point




[1, 0110054]

## Finite # relevant directions




[2, 1301.4191]

## Persistence under SM matter

- effects of matter on gravity
- conjectured FP persists when including SM matter

## Effects on SM phenomenology

- effects of gravity on matter
- agreement with infra-red SM phenomenology?
- can AS explain free parameters?
- Higgs-mass? Yukawa-couplings?

# Gravitating Higgs-Yukawa: the model

## Toy model for the Higgs-Yukawa sector of the SM

$$\frac{1}{2}\int d^4x\sqrt{g}\left(g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + m_\phi^2\phi^2\right) + i\int d^4x\sqrt{g}\bar{\psi}\nabla\!\!\!/\,\psi$$

- scalar $\mathbb{Z}_2$-symmetry: $\phi \to -\phi$
- fermion chiral symmetry: $\psi \to e^{i\gamma_5\vartheta}\psi$

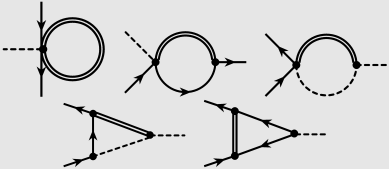
## Yukawa-term reducing the separate symmetries

$$i\,y\int d^4x\sqrt{g}\,\phi\,\bar{\psi}\psi$$

- combined discrete chiral symmetry with $\vartheta = \frac{\pi}{2}$

# Gravitating Higgs-Yukawa: key challenges

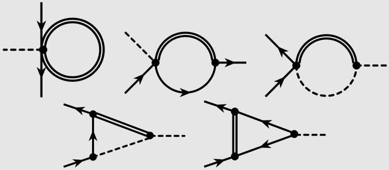
**Part I:**
Fixed points for the Yukawa-coupling



- irrelevance of a Gaussian Yukawa coupling, as in previous studies?
- dependence on graviton gauge?

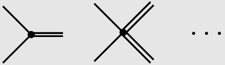
# Gravitating Higgs-Yukawa: key challenges

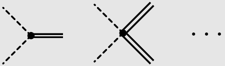
**Part I:**
Fixed points for the Yukawa-coupling



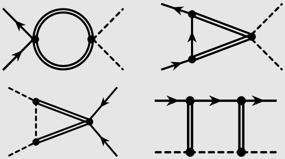
- irrelevance of a Gaussian Yukawa coupling, as in previous studies?
- dependence on graviton gauge?

**Part II:**
Gravity-induced mixed matter couplings

$S_{\psi,\text{kin}} \Longrightarrow$ 

$S_{\phi,\text{kin}} \Longrightarrow$ 

- **non-vanishing** mixed quartic diagrams?



# Embedding in gravity truncation

**In general:**
flat space linear split

- flat space linear split: $g_{\mu\nu} = \delta_{\mu\nu} + h_{\mu\nu}$
- De-Donder gauge fixing: $F_\mu = \left(\delta_\mu^\rho \bar{D}^\sigma - \frac{1-\beta}{4}\bar{g}^{\rho\sigma}\bar{D}_\mu\right) h_{\rho\sigma}$
- flat graviton propagator: $1/\Gamma^{(hh)}_{k,\mathrm{grav}}$
- fixed point values for $g^*$, $\mu_h^*$, $\eta_i^*$

**Specifically:**
flat vertex expansion

- fluctuation field expansion of minimally coupled matter [3, 1510.07018] $\Gamma_k[\bar{g},\Phi] = \sum_n \frac{1}{n!}\Gamma^{(n)}[\bar{g},0]\Phi^n$
- canonical (relevance) ordering
- consider lowest induced order

# Part I: Fixed points for the Yukawa-coupling

# Fixed gauge ($\alpha = 0,\ \beta = 1$) results

## Benchmarking

- recover pure-matter contribution from FRG results in [4]
- for $\eta_h \to 0,\ \eta_{\phi/\psi} \to 0$ agreement with [5, 1510.03734]:

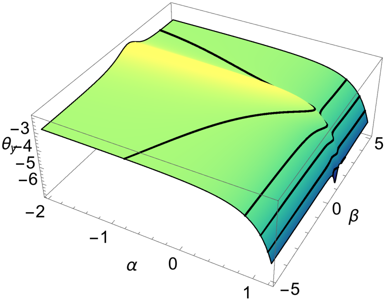
$$\beta_y = \underbrace{\frac{y^3(2+\mu_\phi)}{16\pi^2(1+\mu_\phi)^2}}_{\text{pure-matter}} + \underbrace{g\,y\,\frac{(29-2\mu_\phi+5\mu_\phi^2)}{20\pi(1+\mu_\phi)^2}}_{\text{gravity-matter}} \quad \text{with: } \mu_\phi = \frac{m_\phi^2}{k^2}$$

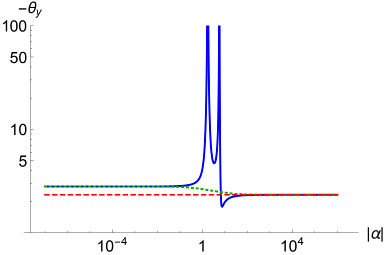
## Truncations in gravity sector

- will trivially (Gaussian) extend any truncation (due symmetry), i.e. for [3, 1510.07018]:

$$\theta_y = -\frac{\partial \beta_y}{\partial y}\Big|_{y=0, g=g_* \dots} \approx -2.33$$

# Gauge-dependence

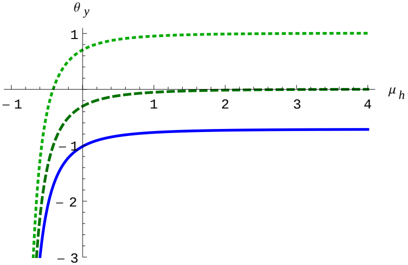



## Poles due to graviton $P\dot{R}P$-combination

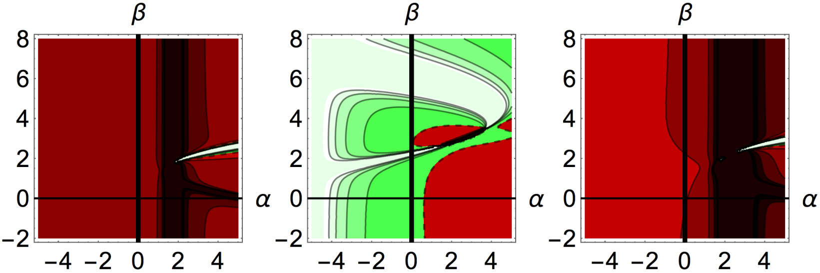
$$P\dot{R}P = \frac{1}{\left((\beta-3)^2 + 2\mu_h\left(2\alpha - \beta^2 + 2\alpha\mu_h + 3\right)\right)^2} \times$$
$$\times \frac{1}{(\mu_h+1)^2}\frac{1}{(\alpha\mu_h+1)^2} \times \ \mathit{analytic}$$

# Dependence of $\theta_y$ on graviton mass-parameter




- $\theta_y > 0$ only possible for $\mu_h$ large and positive
- positive $\eta_{\phi/\psi}$ protect from $\theta_y > 0$ (blue line); negative $\eta_{\phi/\psi}$ support for $\theta_y > 0$ (light green line);
- in the regime of $\mu_h \geq 0$ we observe strong gauge-dependence

# Dependence of gauge-stability on graviton mass-parameter




(a) fluctuating $g^* = 0.6$, $\mu_h^* = -0.6$ from [3], 1510.07018

(b) background $g^* = 0.9$, $\mu_h^* = 0.1$ from [6], 1311.2898

(c) dynamical $g^* = 0.7$, $\mu_h^* = -0.4$ from bimetric calculation [7], 1404.4537

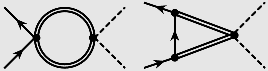
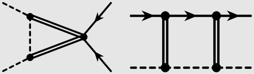
# Summary Part I

- apart from gauge-poles in $P\dot{R}P$, $\theta_y$ **is negative**: especially around $\alpha = 0$
- "gauge-stability" crucially depends on a **fluctuating** $\mu_h^*$
- in this toy model agreement with a SM top-Yukawa coupling ($y^* \approx 0.4/\sqrt{2}$) is only possible for either $\mu_h \gg 0$ or strong negative $\eta_{\phi/\psi} <\approx -1$
- $\Rightarrow$ **irrelevant** ($\theta_y < 0$) **Yukawa coupling** at $y^* = 0$

# Part II: Gravity-induced mixed matter couplings

# 2-fermion–2-scalar couplings at lowest momentum order

**gravity induced couplings are at least $\mathcal{O}(p^3)$:**

 $\propto p^3 + \mathrm{ho}$,  $\propto p^5 + \mathrm{ho}$

**complete basis of $p^3$-tensor structures:**

$$
\begin{aligned}
&\mathcal{X}_1 \int d^4x\sqrt{g}\ \left[(\partial_\mu\phi D^\nu\partial_\nu\phi)\,(\bar{\psi}\gamma^\mu\psi)\right]\\
+&\mathcal{X}_2 \int d^4x\sqrt{g}\ \left[(\partial_\nu\phi D^\nu\partial_\mu\phi)\,(\bar{\psi}\gamma^\mu\psi)\right]\\
+&\mathcal{X}_3 \int d^4x\sqrt{g}\ \left[(\partial_\nu\phi\partial_\mu\phi)\,(\bar{\psi}\gamma^\mu\nabla^\nu\psi)\right]\\
+&\mathcal{X}_4 \int d^4x\sqrt{g}\ \left[(\partial_\nu\phi\partial^\nu\phi)\,(\bar{\psi}\gamma^\mu\nabla_\mu\psi)\right]
\end{aligned}
$$

# Projections

## Simplifying $\mathcal{X}_{1+2}$-$\mathcal{X}_{3+4}$-projection

$$\mathrm{Tr}\left(\frac{\delta}{\delta\phi(p_1)}\frac{\delta}{\delta\phi(p_2)}\frac{\delta}{\delta\psi(p_3)}\frac{\delta}{\delta\bar{\psi}(-p_1-p_2-p_3)}\Gamma_k\gamma_\rho p_{\mathrm{ext}}^\rho\right)\Big|_{\mathcal{O}(p^4)}$$

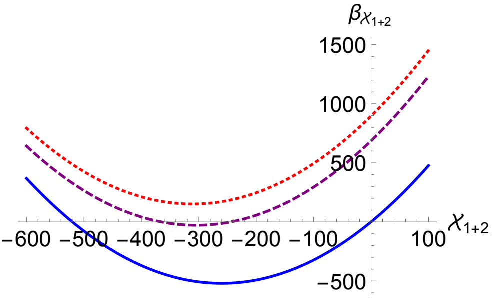
- choosing $p_1 = p_2$ and $p_3 = 0$ projects on $\mathcal{X}_1 + \mathcal{X}_2 = \mathcal{X}_{1+2}$
- choosing $p_1 = -p_2 = -p_3$ projects on $\mathcal{X}_3 + \mathcal{X}_4 = \mathcal{X}_{3+4}$

## $\mathcal{X}_i$-projection in fully symmetric momentum configuration

$$p := |p_1| = |p_2| = |p_3| \quad \& \quad \vartheta_{12} = \vartheta_{13} = \vartheta_{23} = -\frac{1}{3}\,.$$

- projection on $\mathcal{X}_i$ by choice of derivatives and $p_{\mathrm{ext}}$ (e.g., $p_{\mathrm{ext}} \cdot p_1 = 0$)

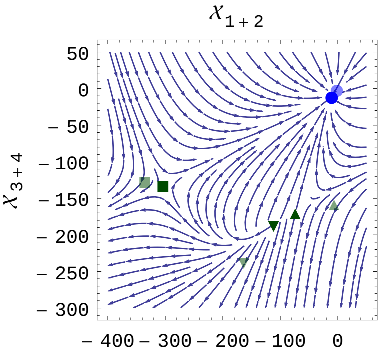
# Mechanism for annihilation of the GFP in $\mathcal{X}_{1+2}$

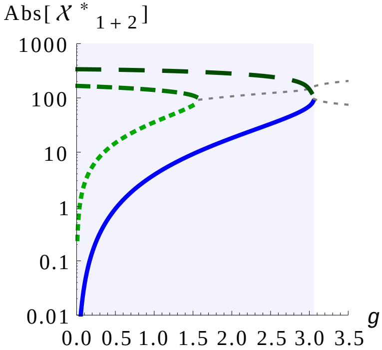



$$\beta_{\mathcal{X}_i} = 4\mathcal{X}_{1+2} + \#_a\, g^2 + \sum_j \#_{b_j} g\, \mathcal{X}_j + \sum_{i,j} \#_{c_{ij}} \mathcal{X}_i \mathcal{X}_j$$

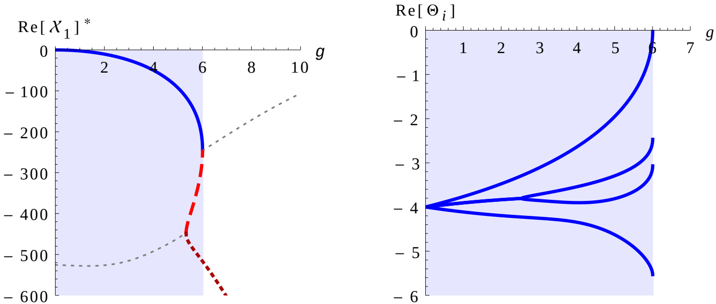
- for $g \neq 0$ non-Gaussian FPs are unavoidable
- depending on signs of $\#_{a,b_j,c_{ij}}$ FP annihilations occur

# Annihilation in simplified truncation ($\mathcal{X}_{1+2}$–$\mathcal{X}_{3+4}$)



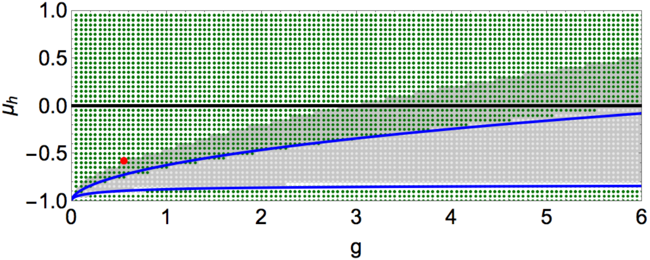



# Fixed point structure of disentangled $\mathcal{X}_i$-truncation




- without gravity 16 complex fixed points: GFP and 5 further real ones
- complex pair pulled into the real plane
- shift of critical exponents $\theta_i$
- FP collision with $\theta_i = 0$

# Existence of the shifted GFP




- all three truncations qualitatively agree in excluding a shifted GFP with all $\mathcal{X}_i$ irrelevant for a given regime in the $g$-$\mu_h$-plane
- fixed-point annihiliations require $\frac{g}{1+\mu_h}$ to exceed a critical value which is well beyond that reached in results in the literature, see, e.g., [3, 6, 7]

# Summary Part II

- induced $\mathcal{X}_i$-couplings, i.e. **necessarily non-Gaussian** under gravity
- **excluded gravitational parameter space**, if demanding a shifted GFP

# Summary

# Summary & Outlook

## Summary

- an **irrelevant Gaussian Yukawa coupling** persists in gravity
- identification of **background and fluctuating quantities** can qualitatively change FP properties
- demanding all $\mathcal{X}_i$-couplings to be irrelevant, we can **exclude gravitational parameter space**

## Outlook

- improve anomalous dimensions
- induced matter channels influencing $\theta_y$
- upgrade to a Higgs-top-bottom model closer to the SM

# Thank you!

# References I


M. Reuter and Frank Saueressig.
Renormalization group flow of quantum gravity in the Einstein-Hilbert truncation.
*Phys. Rev.*, D65:065016, 2002.

K. Falls, D.F. Litim, K. Nikolakopoulos, and C. Rahmede.
A bootstrap towards asymptotic safety.
2013.

Jan Meibohm, Jan M. Pawlowski, and Manuel Reichert.
Asymptotic safety of gravity-matter systems.
2015.

Holger Gies and Michael M. Scherer.
Asymptotic safety of simple Yukawa systems.
*Eur. Phys. J.*, C66:387–402, 2010.

# References II


Kin-ya Oda and Masatoshi Yamada.
Non-minimal coupling in Higgs-Yukawa model with asymptotically safe gravity.
2015.

Pietro Donà, Astrid Eichhorn, and Roberto Percacci.
Matter matters in asymptotically safe quantum gravity.
*Phys.Rev.*, D89:084035, 2014.

Daniel Becker and Martin Reuter.
En route to Background Independence: Broken split-symmetry, and how to restore it with bi-metric average actions.
*Annals Phys.*, 350:225–301, 2014.