Quantum gravity on foliated spacetime asymptotically safe and sound

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Outline

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  - ADM formalism
- FRG on foliated spacetime
- Results
  - Universality classes for Quantum Gravity
  - RG flow in $D = 3$ and $D = 4$
- Conclusions

Motivations

- FRG is based on Euclidean-signature computations
- The nature of our universe is Lorentzian
- In the metric formulation of QEG it is not clear how to define the Wick rotation
- The results obtained in CDT and EDT computations are totally different
- The main results of Asymptotic Safety could depend on the spacetime signature

Spacetime foliation

The idea is to introduce a preferred time direction

$$\mathcal{M}^D \longrightarrow \mathcal{M}^{D-1} \times \mathbb{R} \tag{1}$$

The covariant metric $g_{\mu\nu}$ is written in terms of the ADM fields

$$g_{\mu\nu} \longrightarrow \{N, N_i, \sigma_{ij}\} \tag{2}$$

$$ds^2 = N^2 d\tau^2 + \sigma_{ij}(dx^i + N^i d\tau)(dx^j + N^j d\tau)$$

The Wick rotation is thus implemented via

$$\tau_E \to -i\tau_L \,, \qquad N^i_E \to iN^i_L \tag{3}$$

FRGE on foliated spacetime

$$k\partial_k \Gamma_k = \frac{1}{2} \mathrm{STr}\left[\frac{k\partial_k \mathcal{R}_k}{\Gamma_k^{(2)} + \mathcal{R}_k}\right] \qquad \Gamma_k = \Gamma_k[N, N^i, \sigma_{ij}]$$

We study the RG flow of the Euclidean Einstein-Hilbert action

$$\Gamma_k^{\mathrm{grav}} = \frac{1}{16\pi G_k} \int d\tau d^{D-1}x \, N\sqrt{\sigma} \left\{K_{ij}K^{ij} - K^2 - R + 2\Lambda_k\right\} \qquad (4)$$

$$K_{ij} = \frac{1}{2N}(\partial_\tau \sigma_{ij} - D_i N_j - D_j N_i) \qquad K = \sigma^{ij} K_{ij}$$

We construct Γ_k by using the background field method

$$N = \bar{N} + \hat{N} \qquad N_i = \bar{N}_i + \hat{N}_i \qquad \sigma_{ij} = \bar{\sigma}_{ij} + \hat{\sigma}_{ij} \tag{5}$$

- We use a (Euclidean) FRW background

$$\bar{N} = 1 \qquad \bar{N}_i = 0 \qquad \bar{\sigma}_{ij} = a^2(\tau)\,\delta_{ij}$$

$$\bar{\Gamma}_k^{\text{grav}} = \frac{1}{16\pi G_k} \int d\tau d^{D-1}x \, \sqrt{\bar{\sigma}} \, \{ -\tfrac{d-1}{d} \bar{K}^2 + 2\Lambda_k \} \tag{6}$$

- Transverse-traceless decomposition of the fluctuation fields

$$\hat{N}_i = u_i + \partial_i \, \tfrac{1}{\sqrt{\Delta}} \, B \, , \qquad \partial^i \, u_i = 0$$

$$\hat{\sigma}_{ij} = h_{ij} - \left(\bar{\sigma}_{ij} + \partial_i \partial_j \, \tfrac{1}{\Delta} \right) \psi + \partial_i \partial_j \, \tfrac{1}{\Delta} \, E + \partial_i \tfrac{1}{\sqrt{\Delta}} v_j + \partial_j \, \tfrac{1}{\sqrt{\Delta}} \, v_i$$

Gauge fixing

$$S^{\rm grav} \mapsto \Gamma_k^{\rm grav}[N, N_i, \sigma_{ij}] + \Gamma_k^{\rm gf}[\hat{N}, \hat{N}_i, \hat{\sigma}_{ij}; \bar{N}, \bar{N}_i, \bar{\sigma}_{ij}] + \Gamma_k^{\rm ghost}$$

$\Gamma_k^{\rm grav}$ has to be complemented by a suitable gauge-fixing action

$$\Gamma_k^{\rm gf} = \frac{1}{16\pi G_k} \int d\tau d^{D-1}x \sqrt{\bar{\sigma}} \left\{ F_i \bar{\sigma}^{ij} F_j + F^2 \right\} \tag{7}$$

$$\begin{aligned} F &= c_1\, \partial_\tau\, \hat{N} + c_2\, \partial^i\, \hat{N}_i + c_3\, \partial_\tau\, \bar{\sigma}_{ij} \hat{\sigma}^{ij} + d\, c_8\, \bar{K}^{ij}\, \hat{\sigma}_{ij} + c_9\, \bar{K}\, \hat{N}\,, \\ F_i &= c_4\, \partial_\tau\, \hat{N}_i + c_5\, \partial_i\, \hat{N} + c_6\, \partial_i\, \bar{\sigma}_{ij} \hat{\sigma}^{ij} + c_7\, \partial^j\, \hat{\sigma}_{ji} + d\, c_{10}\, \bar{K}_{ij} \hat{N}^j \end{aligned} \tag{8}$$

There exists a unique gauge choice providing regular propagators for all fluctuation fields $\Rightarrow$ well defined Hessian $\Gamma^{(2)}$

$$32\pi G_k\left(\tfrac{1}{2}\delta^2\Gamma_k^{\rm grav}+\Gamma_k^{\rm gf}\right)=$$

$$\int\sqrt{\bar\sigma}\left\{\tfrac{1}{2}\,h^{ij}\left[\Delta_2-2\Lambda_k-\tfrac{2(d-1)}{d}\dot{\bar K}-\tfrac{d^2-d+2}{d^2}\bar K^2\right]\,h_{ij}\right.$$

$$+\,u^i\left[\Delta_1-\tfrac{d-1}{d}\dot{\bar K}-\tfrac{1}{d}\bar K^2\right]u_i+v^i\left[\Delta_1-2\Lambda_k-\dot{\bar K}-\tfrac{5d-7}{d^2}\bar K^2\right]v_i$$

$$+\,B\left[\Delta_0-\tfrac{d-1}{d}\,\dot{\bar K}-\tfrac{d-1}{d^2}\bar K^2\right]B+\hat N\left[\Delta_0-\tfrac{2(d-1)}{d}\dot{\bar K}-\tfrac{4(d-1)}{d^2}\,\bar K^2\right]\hat N$$

$$+\,\hat N\left[\Delta_0-2\Lambda_k-\tfrac{5d^2-12d+16}{4d^2}\,\bar K^2\right]\left((d-1)\psi+E\right)$$

$$-\,\tfrac{(d-1)(d-3)}{4}\,\psi\left[\Delta_0-2\Lambda_k-\tfrac{2(d-1)}{d}\,\dot{\bar K}-\tfrac{d-1}{d}\bar K^2\right]\psi$$

$$+\,\tfrac{1}{4}\,E\left[\Delta_0-2\Lambda_k-\tfrac{2(d-1)}{d}\,\dot{\bar K}-\tfrac{d-1}{d}\,\bar K^2\right]E$$

$$\left.-\,\tfrac{1}{2}(d-1)\,\psi\left[\Delta_0-2\Lambda_k-\tfrac{2(d-1)}{d}\dot{\bar K}-\tfrac{d-1}{d}\,\bar K^2\right]E\right\}d\tau\,d^{D-1}x \tag{9}$$

$\Delta_i\equiv-\bar g^{\mu\nu}\bar D_\mu\bar D_\nu$ D-dimensional Laplacian

IR regulator

$$k\partial_k\Gamma_k = \frac{1}{2}\mathrm{STr}\left[\frac{k\partial_k\mathcal{R}_k}{\Gamma_k^{(2)} + \mathcal{R}_k}\right]$$

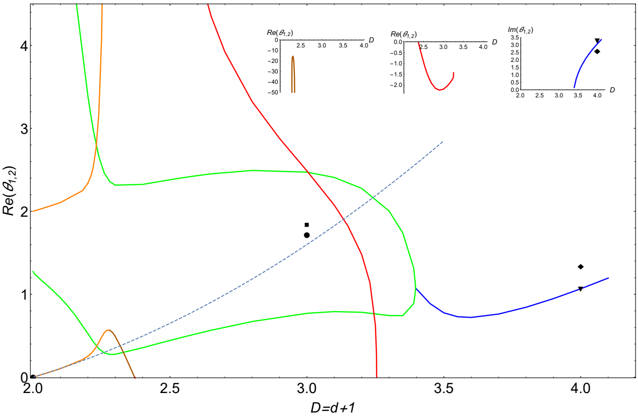
We use a Type I cutoff, defined by

$$\Delta_i \mapsto P_k = \Delta_i + R_k$$

$$R_k = (k^2 - \Delta_i)\,\theta(k^2 - \Delta_i)$$

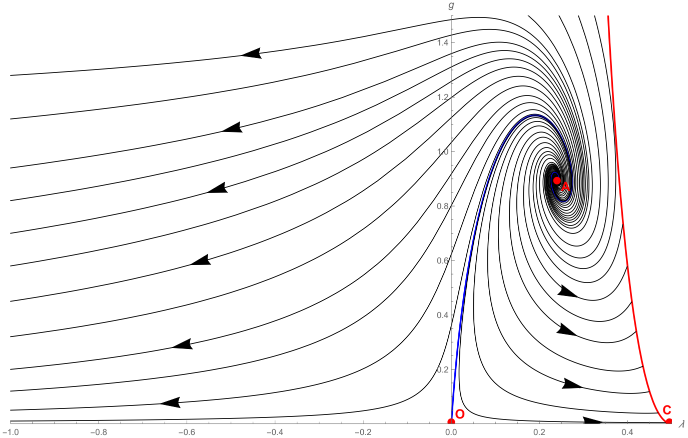
Questions?

Universality classes for Quantum Gravity



- - - **Two-loop ϵ-expansion** [T. Aida and Y. Kitazawa. Nucl. Phys. B 491 (1997) 427]
- ● **Lattice Quantum Gravity** [H. W. Hamber and R. M. Williams, Phys. Rev. D 47 (1993) 510]
- ■ **Discretized Wheeler-DeWitt equation** [H. W. Hamber and R. M. Williams, Phys. Rev. D 84 (2011)]
- ◆ **Geometrical flow equation** [I. Donkin and J. M. Pawlowski, arXiv:1203.4207]
- ▼ **Matsubara formalism** [E. Manrique, S. Rechenberger and F. Saueressig, Phys. Rev. Lett. 106 (2011)]

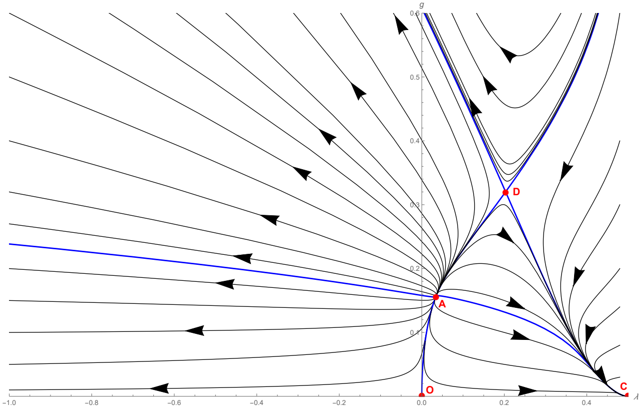
Phase portrait in $D = 4$



$$(\mathrm{A}) \equiv \text{NGFP}: \qquad g_* = 0.895\,, \quad \lambda_* = 0.239\,, \quad g_*\lambda_* = 0.215$$

$$\text{Critical exponents} \qquad \theta_\pm = 1.07 \pm 3.07\, i$$

Phase portrait in $D = 3$

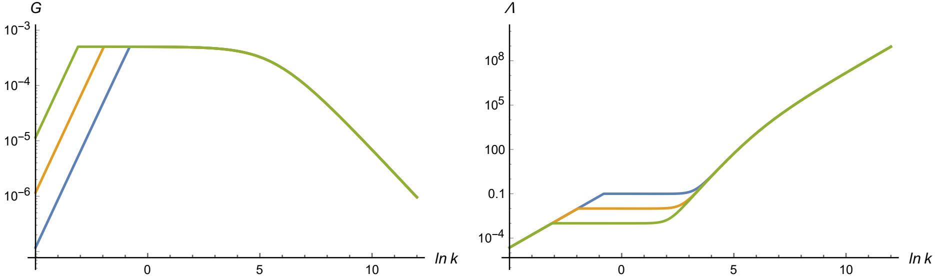


$$(\mathrm{A}) \equiv \text{UV-NGFP}: \quad g_* = 0.156 \quad \lambda_* = 0.034\,, \quad \theta_1^{\mathrm{UV}} = 2.472 \quad \theta_2^{\mathrm{UV}} = 0.770$$

$$(\mathrm{D}) \equiv \text{SP-NGFP}: \quad g_* = 0.320 \quad \lambda_* = 0.239\,, \quad \theta_1^{\mathrm{SP}} = 2.486 \quad \theta_2^{\mathrm{SP}} = -2.196$$

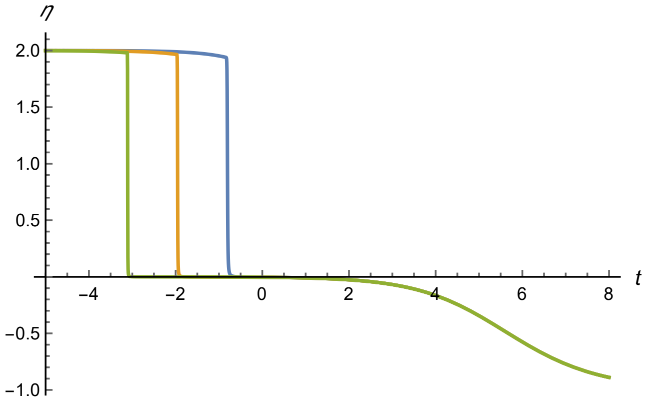
$$(\mathrm{C}) \longrightarrow \text{Quasi Fixed Point (QFP)}: \quad g = 0 \quad \lambda = \tfrac{1}{2}$$

Type IIIa trajectories: $G(k)$ and $\Lambda(k)$



- For $k \gg 1$ the RG flow is controlled by the UV-NGFP
- Crossover to classical regime (plateau)
- QFP regime: well defined limit $k \to 0$, with $G(k) \to 0$ and $\Lambda(k) \to 0$

Type IIIa trajectories: $\eta(k)$



For all the Type IIIa trajectories: $\eta \to 2$ in the infrared limit

Conclusions

- We studied the FRGE adapted to the ADM formalism;
- We introduced a novel gauge-fixing scheme, leading to a well defined Hessian: all the component fields propagate with a relativistic dispersion relation;
- In $D = 3 + 1$ spacetime dimensions the flow is controlled by a unique NGFP with complex critical exponents;
- In $D = 2 + 1$ dimensions the RG flow is characterized by two NGFPs;
- The presence of a saddle-point of the RG flow could provide a mechanism to render the flow regular in the infrared limit.