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The Interplay of Quantum Gravity and Gauge Theories

NC, Eichhorn: arXiv:1702.07724

NC, Litim, Pawlowski: in prep

NC, Eichhorn, Held: in prep

Asymptotic Safety Seminar,
March 20, 2017

Outline

- Gravity Coupled to Gauge Theories
  - General Structure: „Interacting Asymptotic Freedom"
  - Quantum Gravity Corrections to U(1): A solution to the triviality problem and the role of higher order operators
  - Gravity and SU(N)
  - UV-safe Gauge-Yukawa Models
- Summary and Outlook

...known territory ...

- UV behavior of gauge couplings

Gockeler et al 98,
Gies & Jaeckel 04

QED: $\beta_\alpha = \#\alpha^2 + \ldots \quad , \quad \# > 0$ → Triviality (non-pert)

QCD/YM: $\beta_\alpha = \#\alpha^2 + \ldots \quad , \quad \# < 0$ → Asymptotic Freedom

- General Relativity:

$$S_{\rm EH} = \frac{1}{16\pi G_{\rm N}} \int {\rm d}^4x \sqrt{g}(-R(g) + 2\Lambda)$$

→ (perturbatively) non-renormalizable → **Asymptotic Safety**

- Ultimate goal:

UV-completion of: Standard Model (+ something?) + Gravity

Gauge Theories & Gravity

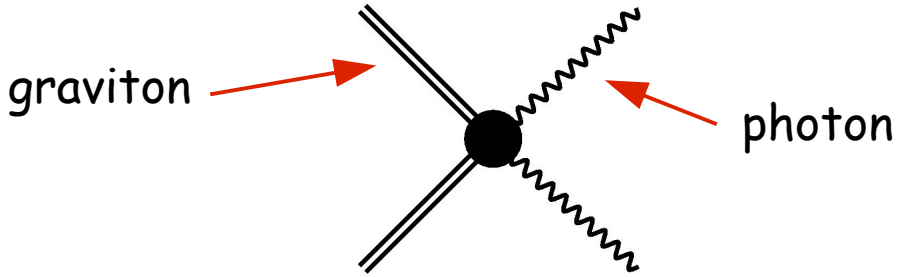
- Can we find Asymptotic Safety in coupled gauge-gravity systems?

$$\Gamma[g, A] = \Gamma_{\rm grav}[g] + \Gamma_{\rm gauge}[g, A]$$

- Minimal coupling of gravity to gauge theories already in F^2 term

$$\int \mathrm{d}^4x \sqrt{g}\, g^{\mu\alpha} g^{\nu\beta} F_{\alpha\beta} F_{\mu\nu}$$

→ vertices with gauge bosons and gravitons. e.g.:



→ What is the possible UV-structure of such theories?

„Interacting Asymptotic Freedom" I

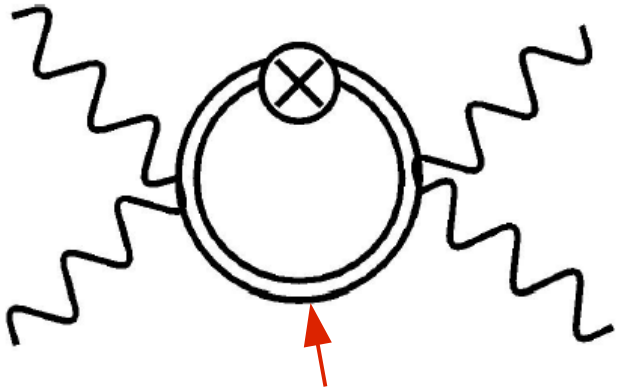
- Assume non-vanishing gravitational coupling:

$$g > 0$$

- gravity contr. to running of higher order operators in the gauge sector: $w_n F^{2n}$

coupling of F^4 operator

$$k\frac{\mathrm{d}}{\mathrm{d}k}w_2 \supset$$



$$\sim g^2 > 0$$

independent of w_2

4 external gauge-fields

internal graviton

→ Gravity induced photon-photon scattering

Eichhorn 12,
Christiansen & Eichhorn 17

→ Even if w_2 is zero at some scale:

$\beta_{w_2} \neq 0$ due to gravity fluctuations!

„Interacting Asymptotic Freedom" II

- Structure of the beta-function

$$\beta_{w_n} = A + Bg^2$$

gravitational coupling

contains gauge couplings

$A\left((g_{\rm gauge}, \vec{w}) = 0\right) = 0$

independent of gauge couplings

minimal gauge coupling

→ $(g_{\rm gauge}, \vec{w}) = 0$ is not a fixed point

→ For $g > 0$ only non-Gaussian FP possible: $\vec{w}^* \neq 0$

- Fixed point can be fully interacting $g^*_{\rm gauge} > 0$ or

$\left(g^*_{\rm gauge} = 0, \vec{w}^* \neq 0\right)$ „Interacting Asymptotic Freedom"

U(1) & Quantum Gravity

- Beta function of U(1) gauge coupling using WTI

Harst & Reuter 11

$$\beta_{g^2_{U(1)}} = g^2_{U(1)}\,\eta_A$$ ← anomalous dimension

- anomalous dimension of the photon

$\eta_A \equiv 0$ without fermions and gravitons (free theory)

$\eta_A > 0$ with fermions only (triviality)

- As argued: Gravity and higher order operators are important!

$$\Gamma[g,A] = \int (R - 2\Lambda) + \frac{1}{g^2_{U(1)}} \int F^2 + \bar{w}_2 \int F^4 + \text{gauge}$$

gravity min. term higher order op.

- Can gravity cure UV-problems?

Christiansen, Eichhorn 17

→ Beta functions using the Wetterich equation (FRG)

U(1) & Quantum Gravity

- Approximation

Z_A : wave function renorm. of the photon

$$w_2 = \frac{\bar{w}_2}{Z_A^2} k^4$$ dimensionless vertex coupling

$$\Gamma_k = \frac{Z_A}{4} \int d^4x \sqrt{g}\, g^{\mu\nu} g^{\kappa\lambda} F_{\mu\kappa} F_{\nu\lambda}$$
$$+ \frac{\bar{w}_2}{8} \int d^4x \sqrt{g} \left(g^{\mu\nu} g^{\kappa\lambda} F_{\mu\kappa} F_{\nu\lambda} \right)^2$$
$$+ \Gamma_{k\,\mathrm{EH}} + S_{\mathrm{gf},\,h} + S_{\mathrm{gf},\,A}$$

Gauge fixing:

$$S_{\mathrm{gf},\,h} = \frac{Z_h}{\alpha_h\, 32\pi} \int d^4x \sqrt{\bar{g}}\, \bar{g}^{\mu\nu} \mathcal{F}_\mu \mathcal{F}_\nu \,, \quad \mathcal{F}_\mu = \bar{D}^\nu h_{\mu\nu} - \frac{1+\beta}{4} \bar{D}_\mu h$$

$$S_{\mathrm{gf},\,A} = \frac{Z_A}{2\alpha_A} \int d^4x \sqrt{\bar{g}} \left(\bar{D}^\mu A_\mu \right)^2$$

→ $\alpha_A = \alpha_h = 0$ and $\beta = 1$

- Extract beta-functions from vertex flows

U(1) & Quantum Gravity

Two-point function : $\Gamma_k^{(2)}$

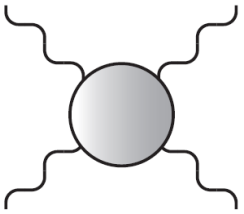
→ anomalous dimension of the photon η_A

$$\eta_A = -\lim_{p^2\to 0} \frac{1}{Z_A p^2} P_{\mu\nu}(p) \left(\frac{\delta}{\delta A_\mu(p)} \frac{\delta}{\delta A_\nu(-p)} \partial_t \Gamma_k \right) \Big|_{A=0,h=0}$$

transverse projector

→ Running gauge coupling via WTI: $\beta_{g^2_{U(1)}} = g^2_{U(1)}\, \eta_A$

Four-point function : $\Gamma_k^{(4)}$



higher order coupling w_2

$$\partial_t \bar{w}_2 \sim \lim_{p^2\to 0} \frac{1}{(p^2)^2} P_{\mu\nu\kappa\lambda} \left(\frac{\delta}{\delta A_\mu(p_1)} \frac{\delta}{\delta A_\nu(p_2)} \frac{\delta}{\delta A_\kappa(p_3)} \frac{\delta}{\delta A_\lambda(p_4)} \partial_t \Gamma_k \right) \Big|_{p_1=p_2=-p_3=-p_4=p, A=0}$$

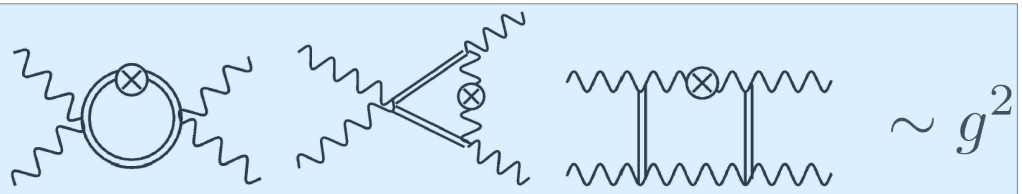
tensor strucutre of classical vertex $\sim \frac{\delta^4}{\delta A^4} (F^2)^2$

U(1) & Quantum Gravity

- Beta function of the higher order coupling w_2

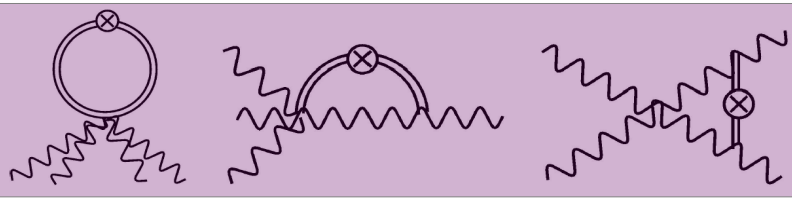
$$\beta_{w_2} = 4w_2 + 2\eta_A\, w_2 + \beta_{w_2}\big|_{\text{induced}} + \beta_{w_2}\big|_{\text{mixed}} + \beta_{w_2}\big|_{\text{pure-photon}}$$

induced: shift the GFP!



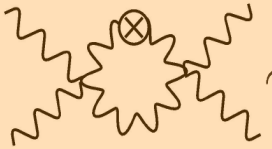
$\sim g^2$

mixed:



$\sim g w_2$

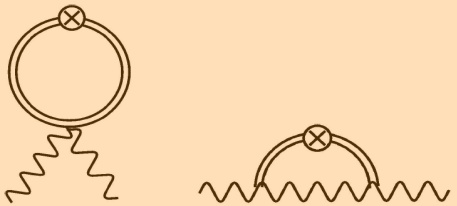
pure photon



$\sim w_2^2$

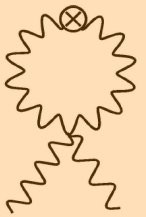
- Beta function of the gauge coupling $g^2_{U(1)}$

direct gravity contr.



$\sim g^2_{U(1)} g$

mediated contr.



$\sim g^2_{U(1)} w_2$

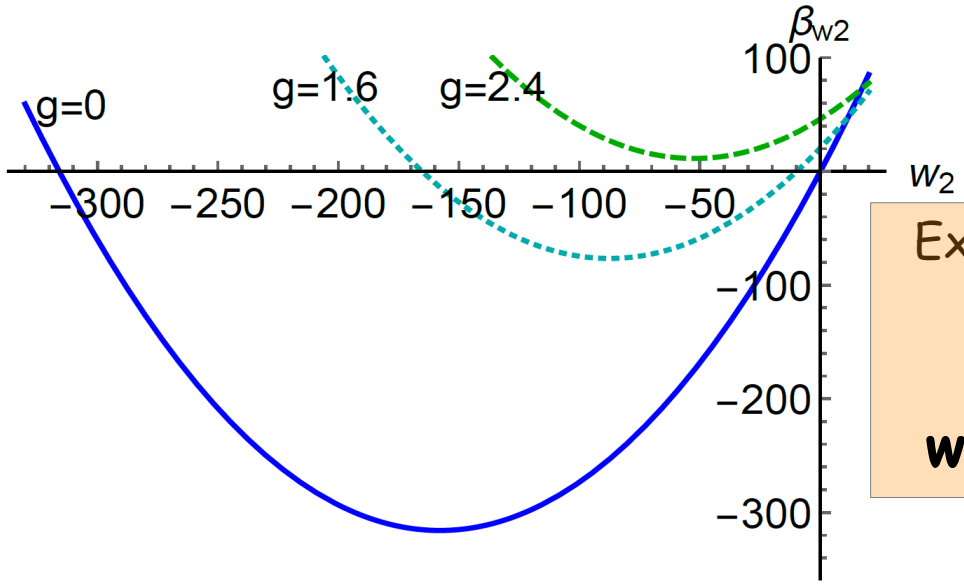
U(1) & Quantum Gravity

- Qualitative effects all visible in approximated beta-functions

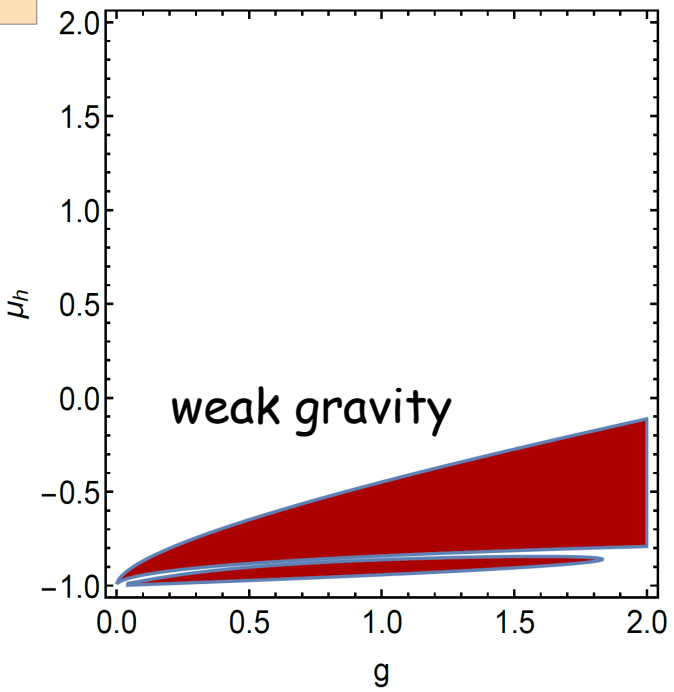
$$\beta_{g^2_{U(1)}}|_{\text{loops: }\eta=0} = \left(-\frac{g}{2\pi} - \frac{w_2}{3\pi^2}\right) g^2_{U(1)},$$

$$\beta_{w_2}|_{\text{loops: }\eta=0} = 4w_2 + 8g^2 - \frac{7}{2\pi} g\, w_2 + \frac{1}{8\pi^2} w_2^2$$

Fixed Point $g^*_{U(1)} = 0 \quad , \quad w_2^* < 0$ ← shifted Gaussian FP



Existence of FP:
$g^* < g_{\text{crit}}$
↓
weak gravity



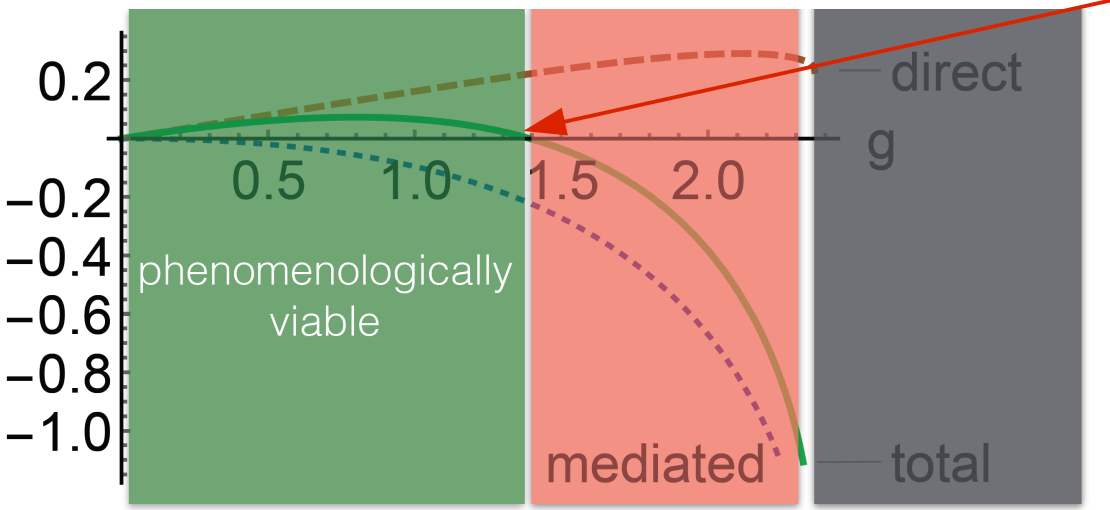
U(1) & Quantum Gravity

- Critical exponents and stability: Interacting Asymptotic Freedom?

$$\theta_{w_2} = -\frac{\partial \beta_{w_2}}{\partial w_2}\Big|_{w_2 = w_{2\,\mathrm{sGFP}}}$$

remains negative, i.e. irrelevant

contributions to $\theta_{g_{U(1)}{}^2}$



g_{zero}

g_{zero} depends on FP-values in the gravity sector.

$g_{U(1)}$ relevant if $g < g_{\mathrm{zero}}$

gravity-induced interactions destroy asymptotic freedom

gravity too strong to support fixed point in gauge sector

Interacting Asymptotic Freedom .....for weak gravity

U(1) & Quantum Gravity

- scenario based on interacting asymptotic freedom:
  - In the high energy limit $k \longrightarrow \infty$:
    - → gravitational couplings approach NGFP
    - → Gauge couplings are interacting asymptotically free
    - → higher order gauge couplings are irrelevant:
      no new free parameters
  - Flowing away from th UV-FP towards smaller k:
    - → minimal gauge coupling $g^2_{U(1)}$ can be tuned such that $g^2_{U(1)}(k \approx M_{\rm Pl})$ has the value compatible with the SM (+X)
- If the gauge coupling is irrelevant → FP becomes IR-attractive
  - → Therefore $g^2_{U(1)}(k \approx M_{\rm Pl}) \approx 0$, but below Planck mass gravity contr. switched off
  - → SM predicts $g^2_{U(1)}(k \approx M_{\rm Pl}) \gg 0$

U(1) & Quantum Gravity

- Adding Fermions does not change that picture

→ Fermions contribute at $\mathcal{O}\left(g_{U(1)}^4\right)$

→ Leading contribution from gravity!

$$\beta_{g^2_{U(1)}} = A\, g g^2_{U(1)} + B\, w_2 g^2_{U(1)} + C g^4_{U(1)} + \dots$$

gravity, direct — gravity, mediated — fermions

→ At $g^*_{U(1)} = 0$ fermions do not contribute to the critical exp.

→ If $g < g_{\text{zero}}$ QED coupled to QG exhibits

→ Interacting Asymptotic Freedom

- With fermions → also NGFP possible:

Harst, Reuter 11
Eichhorn, Versteegen in prep

SU(N) & Quantum Gravity

- YM-theory: Does Asymptotic Freedom survive when coupled to gravity?
- Calculation is similar to U(1)

→ direct gravity contributions support Asymptotic Freedom

Daum, Harst, Reuter 09
Folkerts, Litim, Pawlowski 12
....

↓

Also in extended approximation

Christiansen, Litim, Pawlowski in prep.

- Gluon contributions to the gravity sector: Fully coupled system

→ stability of the NGFP in gravity

Preliminary!

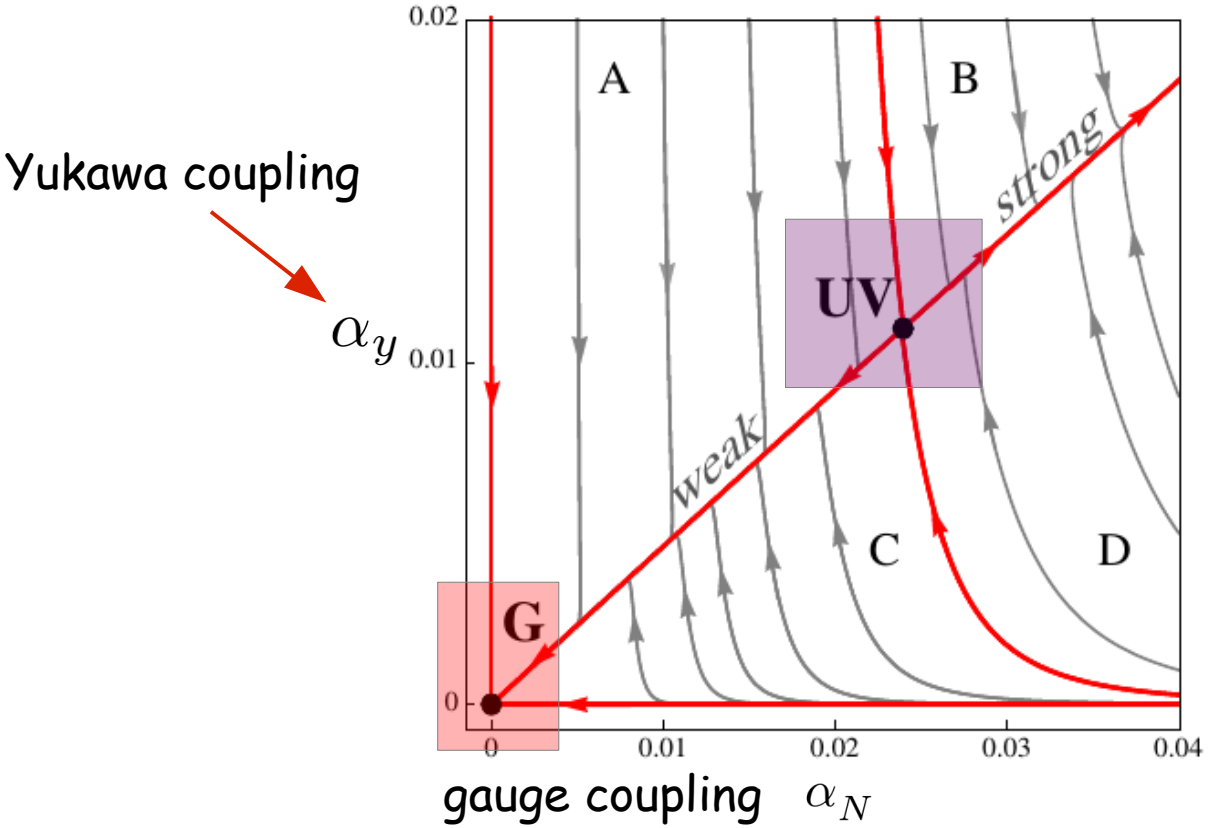
Christiansen, Litim, Pawlowski in prep.

UV safe Gauge-Yukawa models

- Perturbative asymptotic safety in gauge-Yukawa models:

→ Veneziano Limit: $\epsilon = \frac{N_F}{N_c} - \frac{11}{2} \ll 1$ Litim & Sannino 14

- Fixed points and phase diagram without gravity



Non-Gaussian UV-Fixed Point

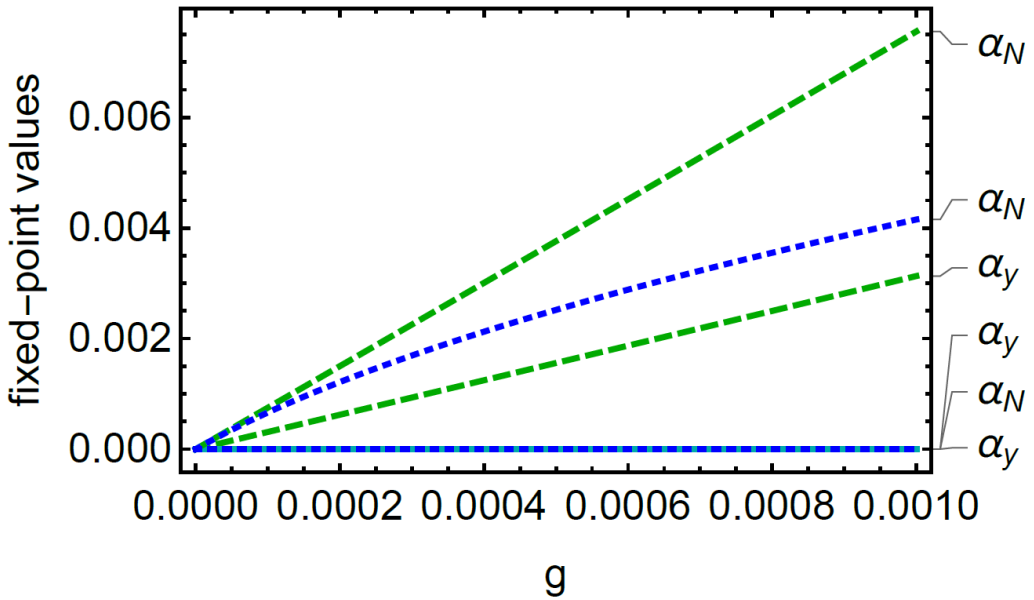
Gaussian IR-Fixed Point

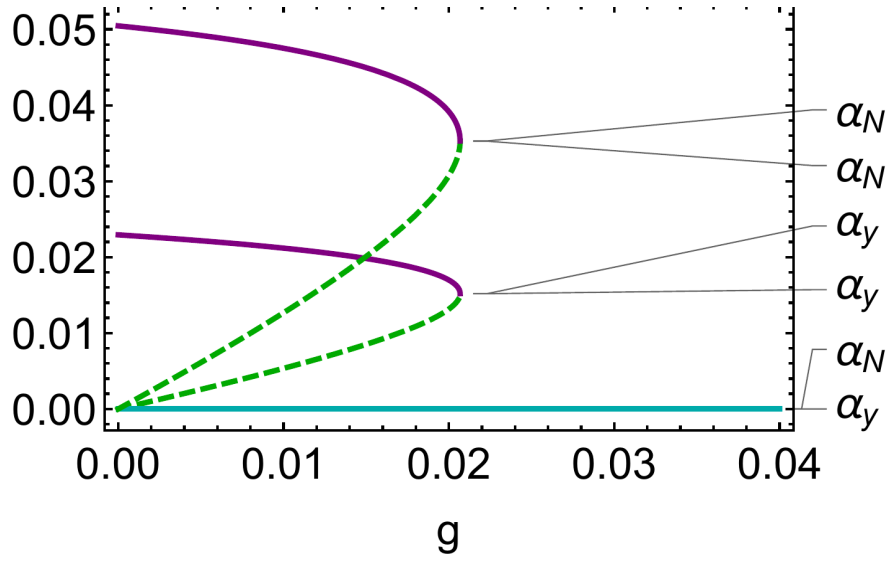
UV safe matter models

- Adding gravity contributions to the beta-functions

Christiansen, Eichhorn, Held in prep

Preliminary!





Gaussian FP splits into 3 FP's if $g > 0$

New non-Gaussian IR fixed point

Annihilation of IR -NGFP with UV- NGFP at some $g_{\rm crit}$

Annihilation sensitive to ratio of gravity contr. to α_y and α_N

Summary and Outlook

- Quantum Gravity coupled to gauge theories:

  → gravity induces higher order operators:

  Interacting Asymptotic Freedom

- gravity and U(1) gauge theories:

  → gravity induces non-Gaussian FP in F^4 coupling

  → „weak gravity": gravity induces Asymptotic Freedom in F^2 coupling

  solution to the triviality problem

  weak gravity bound!

Outlook

- Non-Abelian gauge theories
- Fully coupled gauge-gravity system
- More general matter sector!

Thank You!!!