Towards apparent convergence in asymptotically safe quantum gravity

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Based on

TD, J. M. Pawlowski, M. Reichert: arXiv:1612.07315

Asymptotic safety seminar
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Motivation

Examine asymptotic safety (AS) scenario using a systematic vertex expansion

- extend the expansion to the dynamical graviton four-point function
- obtain closed flow equation for the graviton propagator
- investigate convergence properties of the vertex expansion
- access different tensor structures in the flow
- technical improvement towards more quantitative results

Outline

1. Systematic vertex expansion
2. Accessing tensor structures
3. Apparent convergence
4. Summary and Outlook

Quantum Einstein gravity

Einstein-Hilbert action:

$$S_{\mathrm{EH}}(g) = \frac{1}{16\pi G_N} \int \mathrm{d}^4 x \, \sqrt{g} \, (2\Lambda - R(g)) \tag{1}$$

gauge symmetry: diffeomorphism invariance
gauge fixing necessitates background metric $\bar{g}$:

- use a linear metric split $g = \bar{g} + h$
- with a flat Euclidean background $\bar{g} = \mathbb{1}$
- gauge-fixed action (with Fadeev-Popov ghosts $\bar{c}, c$):

$$S_{\mathrm{gf}}(\bar{g}, h) = \frac{1}{2\xi_1} \int \mathrm{d}^4 x \, \sqrt{\bar{g}} \, \bar{g}^{\mu\nu} F_\mu F_\nu$$

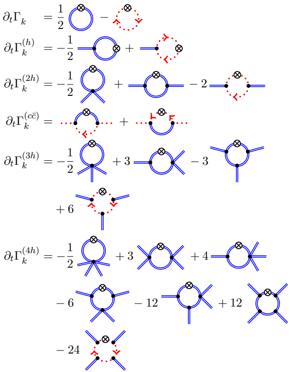
$$F_\mu = \bar{\nabla}^\nu h_{\mu\nu} - \frac{1+\xi_2}{4} \bar{\nabla}_\mu h^\nu{}_\nu$$

$$S(\bar{g}, h, \bar{c}, c) = S_{\mathrm{EH}}(\bar{g} + h) + S_{\mathrm{gf}}(\bar{g}, h) + S_{\mathrm{gh}}(\bar{g}, h, \bar{c}, c) \tag{2}$$

Systematic vertex expansion

Solve Wetterich equation via vertex expansion ($\phi = (h, c, \bar{c})$):

$$\Gamma_k(\bar{g}, \phi) \approx \sum_{n=0}^{N} \frac{1}{n!} \, \Gamma_k^{(n)}(\bar{g}, \phi)\Big|_{\phi=0} \phi^n \quad (3)$$



$\Rightarrow$ obtain tower of coupled differential equations

- compute flow equations up to $N = 4$
- closed flow for graviton propagator

Flows of fluctuation correlation functions are closed

Background flows

Non-trivial Nielsen identities (NI) connect derivatives w.r.t. background and fluctuation field:

$$\frac{\delta\Gamma_k(\bar{g},h)}{\delta\bar{g}_i} = \frac{\delta\Gamma_k(\bar{g},h)}{\delta h_i} + \mathcal{N}^{\text{gf}}_{k,i}(\bar{g},h) + \mathcal{N}^{\text{reg}}_{k,i}(\bar{g},h) \tag{4}$$

Obtain flow equations for background couplings using heat kernel methods:

$$\partial_t\bar{g} = 2\bar{g} - \bar{g}^2 f_{R^1}(\lambda_2;\eta_h,\eta_c) \tag{5}$$

$$\partial_t\bar{\lambda} = -4\bar{\lambda} + \bar{\lambda}\frac{\partial_t\bar{g}}{\bar{g}} + \bar{g} f_{R^0}(\lambda_2;\eta_h,\eta_c) \tag{6}$$

- flow of background couplings depends on couplings of the dynamical two-point functions

n-point functions

Ansatz for n-point functions ($\boldsymbol{p} = (p_1, \dots, p_n)$):

Fischer & Pawlowski 2009, Christiansen et.al. 2012, 2014, 2015

$$\Gamma^{(\phi_1,\dots,\phi_n)}(\boldsymbol{p}) = \left(\prod_{i=1}^{n} Z_{\phi_i}^{1/2}(p_i^2)\right)\left(k^{-2} g_n(\boldsymbol{p})\right)^{\frac{n-2}{2}} \mathcal{T}^{(\phi_1,\dots,\phi_n)}(\boldsymbol{p}) \quad (7)$$

$$\mathcal{T}^{(\phi_1,\dots,\phi_n)}(\boldsymbol{p}) = G_N S^{(\phi_1,\dots,\phi_n)}(\boldsymbol{p}; \Lambda \to k^2 \lambda_n)$$

- disentangle background and fluctuation couplings
- concentrate on tensor structures generated by gauge-fixed EH-action
- introduce level-n fluctuation couplings λ_n, $g_n(\boldsymbol{p})$ for each n-point function
  - λ_n describes momentum-independent part of $\Gamma_k^{(n)}$
  - $g_n(\boldsymbol{p})$ carries global scale- and momentum dependence
- RG-running carried by wave function renormalisations $Z_{\phi_i}(p_i^2)$
  - only appear via anomalous dimensions $\eta_{\phi_i}(p^2) = -\dot{Z}_{\phi_i}(p^2)/Z_{\phi_i}(p^2)$

Systematic vertex expansion

- concentrate on transverse-traceless (TT) parts of the flow equations
  - spin-two, numerically dominant, gauge-independent
  - assume uniform wave function renormalisations
  - apply TT projectors to all external graviton legs
- evaluate momenta at the symmetric point:
  - $(n-1)$-simplex configuration where all angles and absolute values are the same
  - $\langle p_i, p_j \rangle = \frac{n\delta_{ij} - 1}{n-1} p^2$
  - $\boldsymbol{p} \to p^2$
- use Litim regulator $r(x) = (x^{-1} - 1)\Theta(1-x)$
  - analytic flow equations for couplings at $p^2 = 0$

Systematic vertex expansion

Summary

- vertex expansion around $\bar{g} = \mathbb{1}$ up to the graviton four-point function
- ansatz for n-point vertices:
  - tensor structures generated from gauge-fixed EH action
  - RG-running carried by Z_{ϕ_i}
  - overall momentum and scale dependence carried by $g_n(\boldsymbol{p})$
  - momentum independent part described by λ_n
- concentrate on transverse-traceless (TT) parts of the flow equations
- evaluate momenta at the symmetric point

Accessing tensor structures

In principle, the flow generates all tensor structures:

- tensor structures of higher curvature invariants contribute to momentum dependence of $\Gamma^{(n)}(p^2)$ via g_n and Z_{ϕ_i}

$\Rightarrow$ resolving momentum dependence allows to identify these invariants

Schematically, $R \sim h_{\mathrm{TT}}^2 + \mathcal{O}(h_{\mathrm{TT}}^3)$ and $R_{\mu\nu} \sim h_{\mathrm{TT}} + \mathcal{O}(h_{\mathrm{TT}}^2)$
For our projection scheme:

$\Rightarrow$ graviton three-point function:

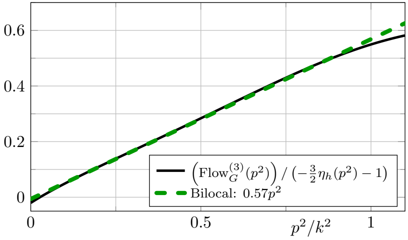
- overlap with R, $R_{\mu\nu}^2$, $R_{\mu\nu}^3$, $R^{\mu\nu} f^{(3)}_{\mu\nu\rho\sigma}(\nabla) R^{\rho\sigma}$, $R^{\mu\nu} R^{\rho\sigma} f^{(3)}_{\mu\nu\rho\sigma\omega\zeta}(\nabla) R^{\omega\zeta}$ tensor structures
- no overlap with $R^{n\geq 2}$ tensor structures

$\Rightarrow$ graviton four-point function:

- analogous to three-point function plus R^2, $R_{\mu\nu}^4$, $R^{\alpha\beta} R^{\mu\nu} R^{\rho\sigma} f^{(4)}_{\alpha\beta\mu\nu\rho\sigma\omega\zeta}(\nabla) R^{\omega\zeta}$ tensor structures
- no overlap with $R^{n\geq 3}$ tensor structures

Graviton 3-point function

momentum dependence of the flow of the normalised three-point function ($p^2 \in [0, k^2]$, $|\lambda_n| \lesssim 1$):

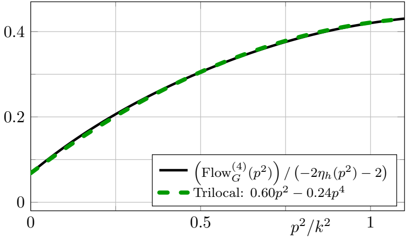


- polynomial structure up to p^2
  - no signature of $R^2_{\mu\nu}$ tensor structures or others that would result in p^4 contribution

$\Rightarrow$ $R^2_{\mu\nu}$ can be excluded as a UV-relevant direction

Graviton 4-point functions

momentum dependence of the flow of the normalised four-point function ($p^2 \in [0, k^2]$, $|\lambda_n| \lesssim 1$):



- polynomial structure up to p^4
  - generation of p^6 contributions non-trivially suppressed
  - $R_{\mu\nu}^2$ was excluded as a UV-relevant direction

  $\Rightarrow$ UV-relevant part can be attributed to R^2 tensor structures
- suggests strategy to disentangle contributions from R, R^2

In our work, we include the contribution of R^2 via the momentum dependence of $g_4(p^2)$.

UV fixed point

Look for a fixed point for $\mu = -2\lambda_2$, λ_3, λ_4, $g_3(p^2)$, and $g_4(p^2)$

- project onto $\Gamma^{(n)}(p^2 = 0)$ for μ, $\lambda_{3,4}$
- project bilocally at $p^2 = 0$ and $p^2 = k^2$ for $g_{3,4} = g_{3,4}(p^2) \approx g_{3,4}(k^2)$

Fixed point values and critical exponents:

$$(\mu^*, \lambda_3^*, \lambda_4^*, g_3^*, g_4^*) = (-0.45,\ 0.12,\ 0.028,\ 0.83,\ 0.57) \tag{8}$$

$$\theta_i = (-4.7,\ -2.0 \pm 3.1\mathrm{i},\ 2.9,\ 8.0) \tag{9}$$

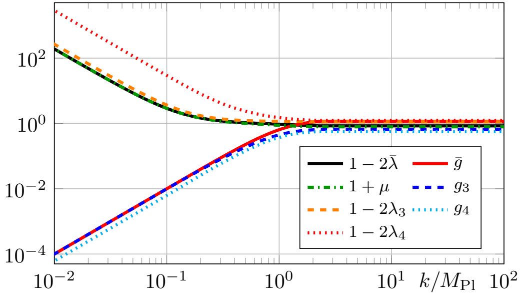
Three relevant directions, can be attributed to Λ, R, and R^2

- in agreement with previous background field approximation studies

e.g. Lauscher & Reuter 2002, Codello et.al. 2007 & 2009, Machado & Saueressig 2007, Falls et.al. 2013

IR behaviour

example trajectory connecting UV fixed point with GR:



tune $\bar{g}$ and $\bar{\lambda}$ via solving NI in the IR:

$$\frac{\delta\Gamma(\bar{g},h)}{\delta\bar{g}} = \frac{\delta\Gamma(\bar{g},h)}{\delta h} \quad \text{for} \quad \mu\to\infty \quad \& \quad k\to 0 \qquad (10)$$

- couplings scale classically in the IR
- diffeomorphism invariance corresponds to $g_n = g_3$ for $k \to 0$

$\Rightarrow$ fine-tuning problem

Apparent convergence

We want to estimate the quality of our truncation of the vertex expansion up to $N = 4$.

$\Rightarrow$ look for signatures of apparent convergence:

- compare results for different levels of the expansion, here $N = 3$ and $N = 4$
- Does the system become more stable?

UV fixed point

Compare fixed point values and critical exponents:

$N = 3$:

Christiansen, Knorr, Meibohm, Pawlowski, Reichert 2015

$$(\mu^*, \lambda_3^*, g_3^*) = (-0.57,\ 0.095,\ 0.62) \tag{11}$$

$$\theta_i = (-1.3 \pm 4.1\mathrm{i},\ 12) \tag{12}$$

$N = 4$:

$$(\mu^*, \lambda_3^*, \lambda_4^*, g_3^*, g_4^*) = (-0.45,\ 0.12,\ 0.028,\ 0.83,\ 0.57) \tag{13}$$

$$\theta_i = (-4.7,\ -2.0 \pm 3.1\mathrm{i},\ 2.9,\ 8.0) \tag{14}$$

- fixed point values are comparable
- four-point truncation has one relevant direction more due to inclusion of R^2

Stability matrix approximation

The critical exponents are the eigenvalues of the stability matrix $J_{ij} = \partial_{\alpha_j} \dot{\alpha}_i$

- need to make ansatz for higher couplings
  - here: $g_{5,6} = g_4$ and $\lambda_{5,6} = \lambda_3$ for $N = 4$
  - for $N = 3$: $g_{4,5} = g_3$ and $\lambda_{4,5} = \lambda_3$

⇒ approximation for stability matrix of the truncated flow is ambiguous

- details of the approximation should become less important for improved truncations

θ_i: identify higher couplings first, then take derivatives
$\tilde{\theta}_i$: first take derivatives, then identify higher couplings

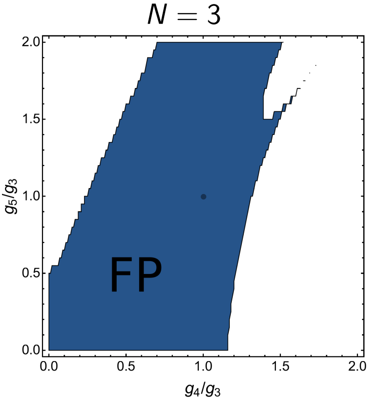
	three-point function	four-point function
θ_i	$-1.3 \pm 4.1\mathrm{i},\ 12$	$-4.7,\ -2.0 \pm 3.1\mathrm{i},\ 2.9,\ 8.0$
$\tilde{\theta}_i$	$-7.3,\ 3.4,\ 7.4$	$-5.0,\ -0.37 \pm 2.4\mathrm{i},\ 5.6,\ 7.9$

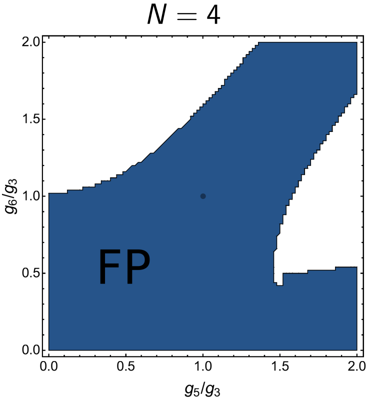
⇒ critical exponents are more stable for the improved truncation

Identification of higher couplings

Identification of higher order couplings is not fixed

- Which choices still allow for the existence of the UV FP?





- identifying higher order g_n with g_3 possible in both cases
- existence of UV FP in the four-point truncation depends less on closure of the flow equations

Apparent convergence

Summary

- fixed point values change little from $N = 3$ to $N = 4$
- one additional relevant direction for the improved truncation due to R^2
- UV FP becomes generally more stable
  - less dependence on approximation of the stability matrix
  - more freedom for the identification of higher couplings

Conclusion: promising hints towards apparent convergence

Summary and Outlook

systematic vertex expansion up to the graviton four-point function

- closed graviton propagator flow
- identified different diffeomorphism-invariant structures via momentum dependence of n-point functions
- non-trivial UV fixed point
  - three relevant directions corresponding to Λ, R, and R^2
  - connected to GR in the IR
- found promising hints towards apparent convergence

potential next steps:

- include further tensor structures in the vertices, especially of R^2
- look at all graviton modes
- improve bounds for gravity-matter systems

Thank you for your attention!