Correlation functions on a curved background

Benjamin Knorr and Stefan Lippoldt

UNIVERSITÄT HEIDELBERG
Zukunft. Seit 1386.

Institut für Theoretische Physik

Ruprecht-Karls-Universität Heidelberg

based on

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Motivation

- the light of the CMB is polarized (Thomson scattering)

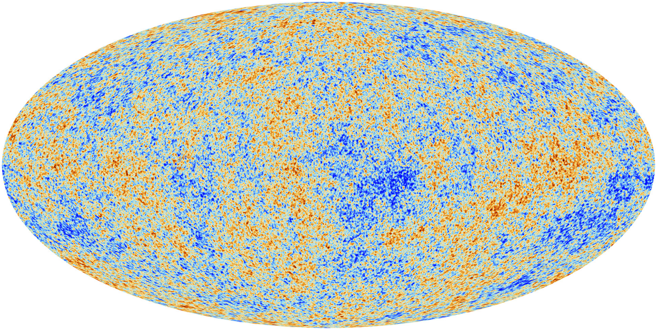


Image: ESA and the Planck Collaboration

Motivation

- the light of the CMB is polarized (Thomson scattering)
- the propagation of this light is influenced by metric perturbations $\delta g_{\mu\nu}$ (scalar and tensor perturbations)
- by measuring the polarization we can infer properties of $\delta g_{\mu\nu}$
- on the other hand, we can calculate these metric perturbations:

$$\delta g_{..}(x)\,\delta g_{..}(y) \;\sim\; \langle h_{..}(x)\,h_{..}(y)\rangle \;\sim\; \Gamma^{(2)}[\bar{g}_{\rm os}] \;\sim\; \Gamma^{(0;2)}_{k=0}[0;\bar{g}_{\rm os}]$$

Bi-Metric Nature of Γ_k

Split Symmetry

- linear split: $g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$
- Einstein-Hilbert action: $S_{\mathrm{EH}}[g] = S_{\mathrm{EH}}[\bar{g} + h]$
- **split symmetry**: $\delta_\varepsilon \bar{g} = -\varepsilon$, $\delta_\varepsilon h = \varepsilon$
  $\Rightarrow \quad \delta_\varepsilon S_{\mathrm{EH}}[g] = 0$
- regulator/gauge-fixing: $\Delta S_k = \Delta S_k[h; \bar{g}]$, $S_{\mathrm{gf}} = S_{\mathrm{gf}}[h; \bar{g}]$
  $\Rightarrow \quad \delta_\varepsilon \Delta S_k[h; \bar{g}] \neq 0, \quad \delta_\varepsilon S_{\mathrm{gf}}[h; \bar{g}] \neq 0$

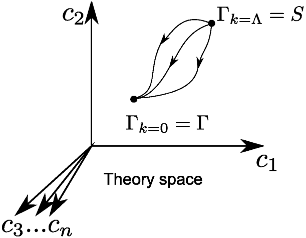
Vertex Expansion

- flow equation:

$$\partial_k \Gamma_k[h;\bar{g}] = \frac{1}{2}\,\mathrm{STr}\left[\left(\Gamma_k^{(2;0)}[h;\bar{g}] + \mathcal{R}_k[\bar{g}]\right)^{-1} \partial_k \mathcal{R}_k[\bar{g}]\right]$$

$\Rightarrow$ $\partial_k \Gamma_k^{(n;m)}$ obeys hierarchy:

$$\begin{array}{ccccc}
\Gamma_k^{(2;0)} & \Gamma_k^{(3;0)} & \Gamma_k^{(4;0)} & \ldots & \Gamma_k^{(n+2;0)} \\
\Gamma_k^{(2;1)} & \Gamma_k^{(3;1)} & \ldots & \ldots & \Gamma_k^{(n+2;1)} \\
\vdots & \vdots & \ddots & \ddots & \vdots \\
\Gamma_k^{(2;m)} & \Gamma_k^{(3;m)} & \ldots & \ldots & \Gamma_k^{(n+2;m)}
\end{array}$$



Truncation

- start with Einstein-Hilbert action:

$$S_{\mathrm{EH}}[g] = \frac{1}{16\pi G_{\mathrm{N}}} \int (-R + 2\Lambda)$$

- amend with gauge fixing ($\alpha \to 0$):

$$S_{\mathrm{gf}}[h;\bar{g}] = \frac{1}{16\pi G_{\mathrm{N}}\alpha} \int \bar{g}^{\mu\nu} F_\mu[h;\bar{g}] F_\nu[h;\bar{g}]$$

$$F_\mu[h;\bar{g}] = \left(\delta_\mu^{(\alpha}\bar{D}^{\beta)} - \frac{1+\beta}{4}\bar{g}^{\alpha\beta}\bar{D}_\mu\right) h_{\alpha\beta}$$

- use them together as generator for vertices:

$$S_{\mathrm{gen}}[h;\bar{g}] = S_{\mathrm{EH}}[\bar{g}+h] + S_{\mathrm{gf}}[h;\bar{g}]$$

- rescale the graviton $h \to \sqrt{G}\,h$:

$$S^{(n)}_{\mathrm{EH}}[\bar{g}] \to G_n^{\frac{n}{2}-1} G_{\mathrm{N}}\; S^{(n)}_{\mathrm{EH}}[\bar{g}]\Big|_{\Lambda\to\Lambda_n}$$

Truncation

- $\Gamma_k^{(2;0)}$: $\frac{1}{32\pi}\int h_{\mu\nu}\Big(\big(\frac{(\beta-3)^2}{6}\tilde{\Lambda}+\frac{\beta^2}{3}\Lambda_{\mathrm{TL}}\big)\Pi_{\mathrm{Tr}}{}^{\mu\nu}{}_{\alpha\beta}-\Lambda_{\mathrm{TL}}\Pi_{\mathrm{TL}}{}^{\mu\nu}{}_{\alpha\beta}\Big)h^{\alpha\beta}$

- $\Gamma_k^{(3;0)}$: $\frac{\sqrt{G_3}}{384\pi}\int h^{\mu\nu}h^{\rho\sigma}\left(\bar{D}_\mu\bar{D}_\nu h_{\rho\sigma}-14\bar{D}_{(\mu}\bar{D}_{\rho)}h_{\nu\sigma}-6\bar{g}_{\nu\rho}\bar{\Delta}h_{\mu\sigma}\right)$
  $+\frac{\sqrt{G_3}\Lambda_3}{48\pi}\int h_\mu{}^\nu h_\nu{}^\rho h_\rho{}^\mu$

- $\Gamma_k^{(2;1)}$: $\frac{1}{32\pi}\int h_{\mu\nu}\Big(\frac{1}{3}(1+3\mathcal{R}_{R\mathrm{TL}})\bar{R}\Pi_{\mathrm{TL}}{}^{\mu\nu}{}_{\alpha\beta}+(\mathcal{R}_C-1)\bar{C}^\mu{}_\alpha{}^\nu{}_\beta$
  $+\mathcal{R}_{S\mathrm{TL}}\Pi_{\mathrm{TL}}{}^{\mu\nu}{}_{\rho\sigma}\bar{S}^\rho_\kappa\Pi_{\mathrm{TL}}{}^{\sigma\kappa}{}_{\alpha\beta}+2\mathcal{R}_{S\mathrm{Tr}}\bar{g}^{\mu\nu}\bar{S}_{\alpha\beta}$
  $+\mathcal{R}_{\mathrm{Tr}}\bar{R}\Pi_{\mathrm{Tr}}{}^{\mu\nu}{}_{\alpha\beta}\Big)h^{\alpha\beta}$

Regulator

- interplay of Landau limit and degeneracy of scalar sector:

$$\mathcal{P}_{(\sigma h)} = \frac{\bar{\Delta}}{16\alpha} \begin{pmatrix} 12(3-\alpha)\bar{\Delta}^2 & 6(\beta-\alpha)\bar{\Delta} \\ 6(\beta-\alpha)\bar{\Delta} & \beta^2 - 3\alpha \end{pmatrix}$$

$$\mathcal{P}^{-1}_{(\sigma h)} \xrightarrow{\alpha \to 0} -\frac{4}{3(3-\beta)^2 \bar{\Delta}^3} \begin{pmatrix} \beta^2 & -6\beta\bar{\Delta} \\ -6\beta\bar{\Delta} & 36\bar{\Delta}^2 \end{pmatrix}$$

- in Landau limit propagator is of the form:

$$G_{(\sigma h)} = \begin{pmatrix} \mathcal{O}(\beta^2) & \mathcal{O}(\beta) \\ \mathcal{O}(\beta) & \mathcal{O}(1) \end{pmatrix}$$

- spectrally adjusted regulator:

$$\mathfrak{R}_{(\sigma h)} = \frac{1}{\alpha} \begin{pmatrix} \mathcal{O}(1) & \mathcal{O}(\beta) \\ \mathcal{O}(\beta) & \mathcal{O}(\beta^2) \end{pmatrix}$$

Regulator

- flow equation in Landau limit:

$$\mathrm{Tr}[G_{(\sigma h)}\mathfrak{R}_{(\sigma h)}] = \frac{1}{\alpha}\mathcal{O}(\beta^2) + \mathcal{O}(1)$$

- when everything commutes it is straight forward to ensure that the $\frac{1}{\alpha}\mathcal{O}(\beta^2)$ term vanishes

- but in curved background $[f(\bar{\Delta}), \bar{D}] \neq 0$

  $\Rightarrow$ vanishing of $\frac{1}{\alpha}\mathcal{O}(\beta^2)$ is nontrivial

- well behaved regulator $\big(\mathfrak{R} \sim S^{(2)}_{\mathrm{gen}}\mathfrak{r}(\frac{\bar{\Delta}}{k^2})\big)$:

$$\mathfrak{R} = \frac{1}{32\pi}\Big(-\mathbb{1}\,\bar{D}^\rho\mathfrak{r}(\tfrac{\bar{\Delta}}{k^2})\bar{D}_\rho + \frac{8\alpha-(1+\beta)^2}{2\alpha}\Pi_{\mathrm{Tr}}\bar{D}^\rho\mathfrak{r}(\tfrac{\bar{\Delta}}{k^2})\bar{D}_\rho + \frac{1-2\alpha+\beta}{2}\big(\bar{g}^{\cdot\cdot}\bar{D}_{(\cdot}\mathfrak{r}(\tfrac{\bar{\Delta}}{k^2})\bar{D}_{\cdot)} + \bar{D}^{(\cdot}\mathfrak{r}(\tfrac{\bar{\Delta}}{k^2})\bar{D}^{\cdot)}\bar{g}_{\cdot\cdot}\big) + \delta^{(\cdot}_{(\cdot}\bar{D}^{\cdot)}\mathfrak{r}(\tfrac{\bar{\Delta}}{k^2})\bar{D}_{\cdot)}\Big)$$

Results

Fixed Points of the Gauge Parameters

- a direct computation gives:

$$\dot{\alpha} = \mathcal{O}(\alpha^2)$$
$$\dot{\beta} = \mathcal{C}(\alpha,\beta)\cdot\alpha + \mathcal{O}(\alpha^2), \qquad \lim_{\alpha\to 0}|\mathcal{C}(\alpha,\beta)| < \infty$$

- Landau gauge is a fixed point for arbitrary β

- both gauge parameters are exactly marginal couplings

$$\begin{pmatrix} \frac{\partial\dot{\alpha}}{\partial\alpha} & \frac{\partial\dot{\alpha}}{\partial\beta} \\ \frac{\partial\dot{\beta}}{\partial\alpha} & \frac{\partial\dot{\beta}}{\partial\beta} \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ \lim_{\alpha\to 0}\mathcal{C}(\alpha,\beta) & 0 \end{pmatrix} + \mathcal{O}(\alpha)$$

Fixed Points of the Couplings

- switch to dimensionless variables:

$$g_n = G_n k^2, \quad \lambda_n = \frac{\Lambda_n}{k^2}, \quad \lambda_{\mathrm{TL}} = \frac{\Lambda_{\mathrm{TL}}}{k^2}, \quad \tilde{\lambda} = \frac{\tilde{\Lambda}}{k^2}$$

- note the hierarchy:

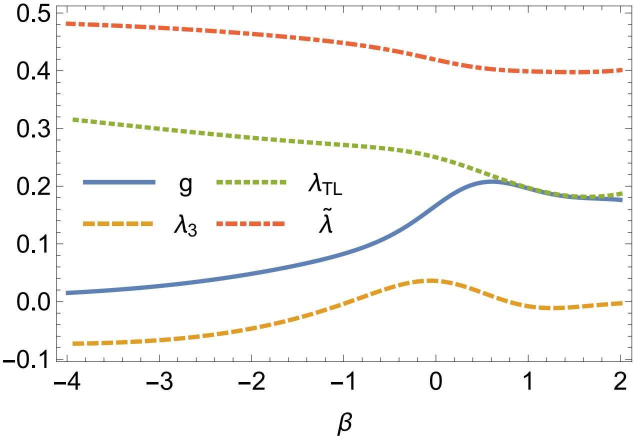
0	$\Gamma_k^{(2;0)}$	$\Gamma_k^{(3;0)}$	$\Gamma_k^{(4;0)}$	$\dots$	$\Gamma_k^{(n+2;0)}$
1	$\Gamma_k^{(2;1)}$	$\Gamma_k^{(3;1)}$	$\dots$	$\dots$	$\Gamma_k^{(n+2;1)}$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\ddots$	$\vdots$
m	$\Gamma_k^{(2;m)}$	$\Gamma_k^{(3;m)}$	$\dots$	$\dots$	$\Gamma_k^{(n+2;m)}$

- close the flow equation:

$$g_5 = g_4 = g_3 = g, \quad \lambda_5 = \lambda_4 = \lambda_3$$

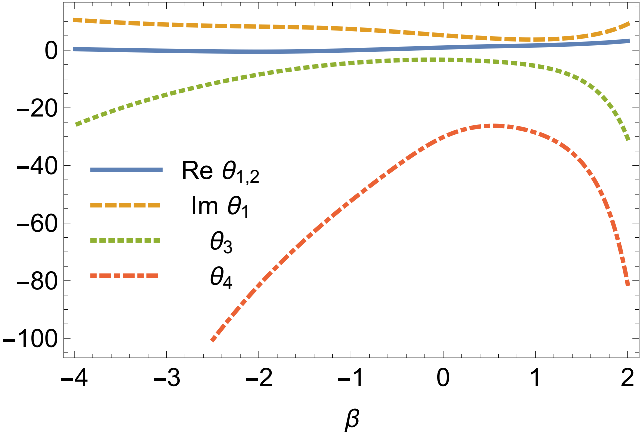
0th Order Curvature Couplings

- $\Gamma_k^{(2;0)}$ and $\Gamma_k^{(3;0)}$



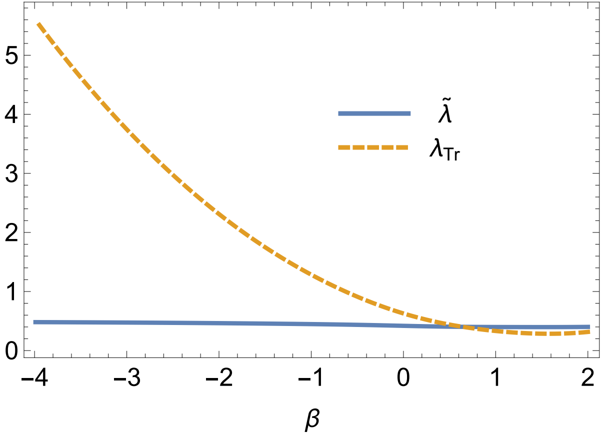
0th Order Curvature Couplings

- $\Gamma_k^{(2;0)}$ and $\Gamma_k^{(3;0)}$



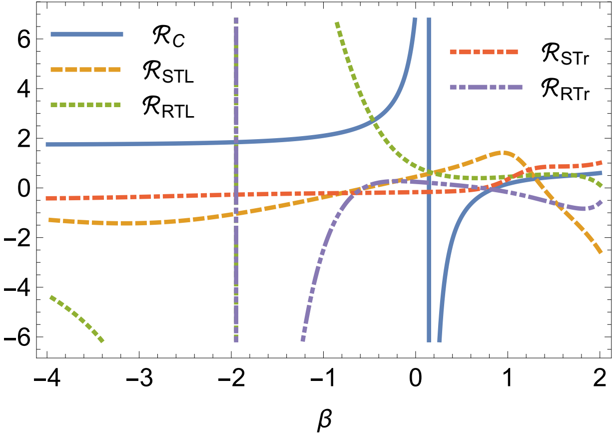
0th Order Curvature Couplings

- $\tilde{\lambda}$ vs. λ_{Tr}



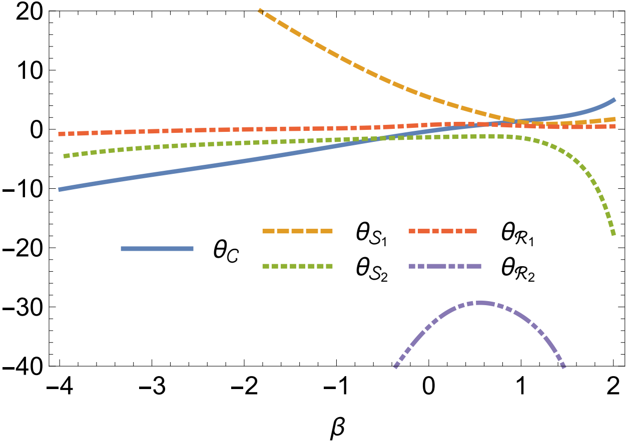
1st Order Curvature Couplings

- $\Gamma_k^{(2;1)}$



1st Order Curvature Couplings

- $\Gamma_k^{(2;1)}$



Summary & Outlook

- existence of unique UV fixed point persists under inclusion of background curvature
- Landau gauge implies fixed point also for β
- regulator choice in curved background
- cosmological post-/predictions
- calculations on self-consistent backgrounds
- inclusion of matter

Thank you for your attention!