Catching ghosts

On avoiding Ostrogradski instabilities within Asymptotic Safety

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Ostrogradski instabilities

- Classical level:
  - Ostrogradski (1850): higher order time derivatives
    $\Rightarrow$ unstable equations of motion
  - Particle acceleration w/o energy increase
- Quantum level:
  - instantaneous vacuum decay
  - Negative norm states
    $\Rightarrow$unitarity violation

Example: free scalar field
Källén-Lehmann representation:

$$G(p^2) = \frac{1}{Z} \frac{1}{p^2 + Y p^4} = \frac{1}{Z} \left(\frac{1}{p^2} - \frac{1}{p^2 + \frac{1}{Y}} \right)$$

Ghost mass: $\mu^2 = \frac{1}{Y}$

Loopholes

- Higher derivatives are not really there:
  - Combine to total derivatives
  - Use gauge symmetry
- Highest derivative is not high enough: Propagator is entire function with only single order mass pole:

$$G(p^2) = \frac{e^{-\gamma(p^2)}}{p^2 + m^2}$$

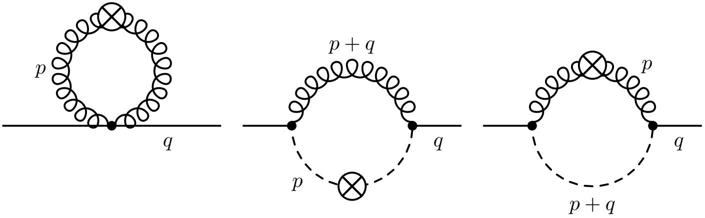
- **Use the RG!**
  - Require absence of higher derivatives in IR ($k \to 0$)
  - Higher derivatives may be generated for $k \neq 0$ (and are with gravity present)
  - Example: $Y_k = 0$ as $k \to 0$ Either by:
    - fixed point
    - attraction to $Y = 0$-hyperplane

Truncation

- Flow equation:
  $$\partial_t \Gamma_k = \frac{1}{2}\mathrm{STr}\left[\left(\Gamma_k^{(2)} + \mathcal{R}_k\right)^{-1} \partial_t \mathcal{R}_k\right]$$
- Truncation Ansatz:
  $$\Gamma_k[g, \phi, \bar{c}, c; \bar{g}] \approx \Gamma_k^{\mathrm{grav}}[g] + \Gamma_k^{\mathrm{matter}}[\phi, g] + \Gamma_k^{\mathrm{gf}}[g; \bar{g}] + S^{\mathrm{ghost}}[g, \bar{c}, c; \bar{g}]$$
  - $\Gamma_k^{\mathrm{grav}}[g] = \frac{1}{16\pi G_k} \int \mathrm{d}^4x \sqrt{g}\, [-R + 2\Lambda_k]$
  - $\Gamma_k^{\mathrm{matter}}[\phi, g] = \frac{1}{2} Z_k \sum_{i=1}^{N_s} \int \mathrm{d}^4x \sqrt{g} \phi^i \left[\Delta + Y_k \Delta^2\right] \phi^i$
- Linear split: $g = \bar{g} + h$, harmonic gauge
- Type I coarse-graining
- Optimized cutoff

Beta functions

- Projection on scalar couplings:
  - $\bar{g} = \delta$
  - Expand in background scalar momentum q



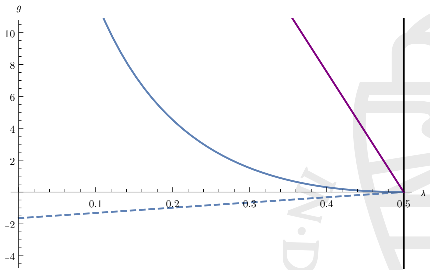
- Dimensionless couplings
  $g_k = G_k k^2$, $\lambda_k = \Lambda_k k^{-2}$, $y_k = Y_k k^2$
  $\eta_N = -\partial_t \ln G_k$, $\eta_s = -\partial_t \ln Z_k$

Singularity structure

- $\eta_N = \frac{B_1(\lambda)+N_s B_3(\lambda,y)}{1-gB_2(\lambda)}$
- $\eta_s = \frac{g}{1-gS_4(\lambda,y)}\left(S_1 + \eta_N S_2 + \beta_y S_3\right)$
- $\lambda = \frac{1}{2}$ singularity
- $y = -1$ singularity

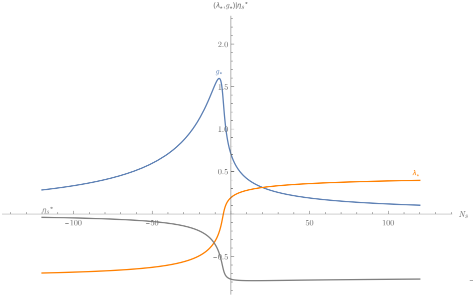
"Physical" region:

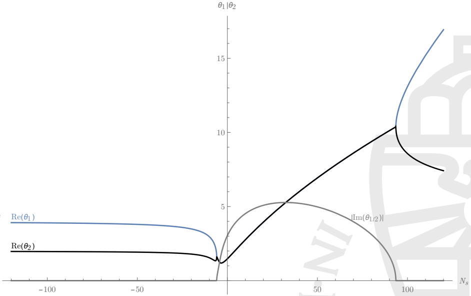
- $g \geq 0$
- $\lambda < \lambda^{\text{sing}}$
- $y > -1$



Purple: η_s^{sing}, blue: η_N^{sing}
Solid: Type I, dashed: Type II

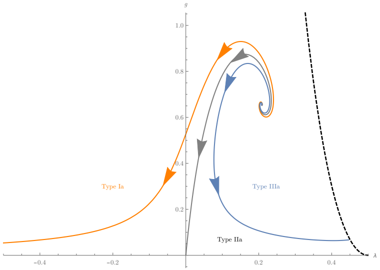
Minimal coupling: $N_s \neq 1$

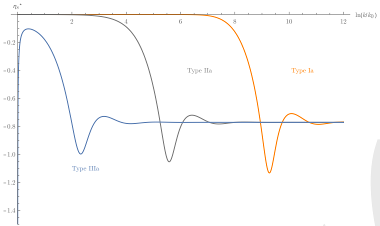




- NGFP is UV attractive
- $|N_s|$ small: complex critical exponents
- η_s^* negative

Minimal coupling: $N_s = 1$





- Phase diagram similar to Einstein-Hilbert truncation
- Type IIIa trajectories: hit η_N^{sing} line
- $\eta_s \to 0$ in the IR: classical behaviour
  (Type IIIa: diverges)

Higher derivatives: fixed point structure

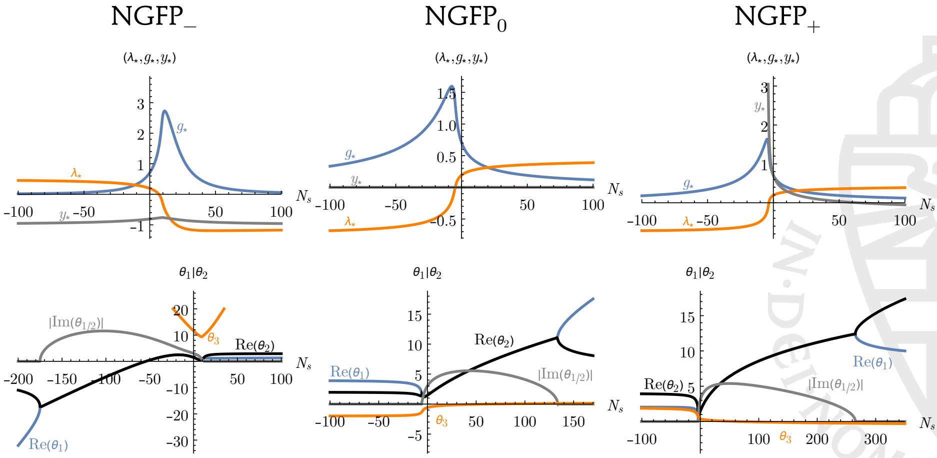
- In any dimension:

$$\beta_y|_{y=0} = -\frac{g}{6\pi}\frac{2+\eta_N}{1-2\lambda)^2}$$

$\Rightarrow$ only in $d = 4$: $y = 0$ supports a NGFP!

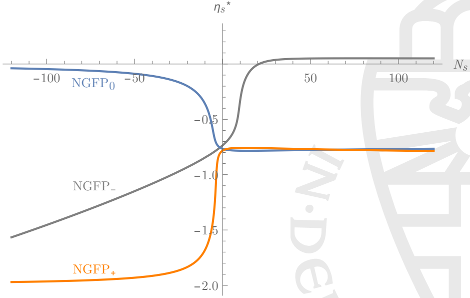
- Numerical search in $d = 4$: three NGFPs labeled by sign y_*: NGFP$_-$, NGFP$_0$, NGFP$_+$

N_s dependence

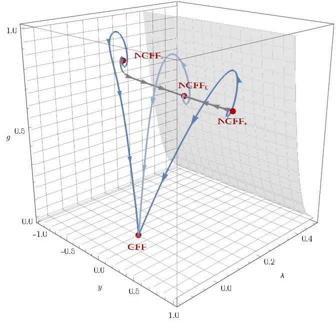


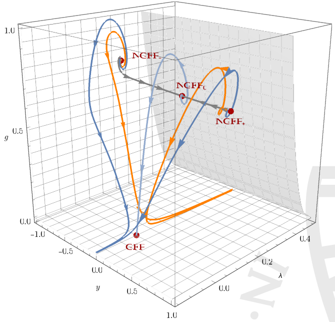
Anomalous dimension

- Anomalous dimension mostly negative
- NGFP_: crossover at $N_s \approx 20$ to (small) positive anomalous dimension



$N_s = 1$: phase diagram





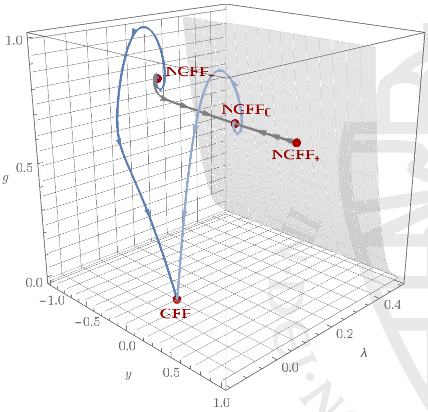
	g_*	λ_*	y_*	$g_*\lambda_*$	η_s^*	θ_1	θ_2	θ_3
GFP	0	0	0	0	0	$+2$	-2	-2
NGFP_-	0.776	0.176	-0.804	0.137	-0.721	$1.34 \pm 2.92i$		11.3
NGFP_0	0.655	0.208	0	0.136	-0.771	$1.59 \pm 3.28i$		-0.529
NGFP_+	0.646	0.211	0.621	0.136	-0.775	$1.67 \pm 3.32i$		0.357

Ghost-free flows

- All trajectories flow to $y = 0$
- Ghost mass $\mu^2 = \frac{1}{Y_0}$
- Dimensionful coupling $Y_k = y_k k^{-2}$
- Three IR scenarios:
  - $y_k \to 0$ **slower** than quadratically
    $\Rightarrow \mu^2 \to 0$, ghost eats scalar degree of freedom
  - $y_k \to 0$ **faster** than quadratically
    $\Rightarrow \mu^2 \to \infty$, stable theory
  - $y_k \to 0$ **exactly** quadratically
    $\Rightarrow Y_0 =$ const, which may be zero

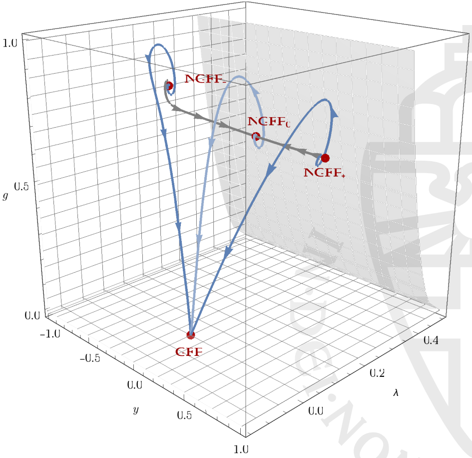
IR limits: Type IIa

- IR attractive hypersurface of GFP spanned by: $\hat{y}$ and $\frac{2+N_s}{16\pi}\hat{\lambda} + \hat{g}$
- Critical exponent -2:
  $y_k = y_{k_0}\left(\frac{k^2}{k_0^2}\right) \Leftrightarrow Y_k = Y_{k_0}$
- UV limit: NGFP_



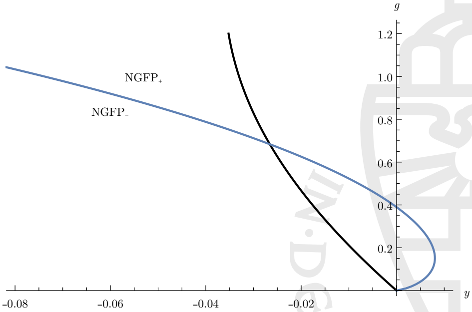
IR limits: to NGFP$_0$

- Linearized flow: $y_k = y_{k_0} \left(\frac{k}{k_0}\right)^{0.529}$
- $Y_k = Y_{k_0} \left(\frac{k_0}{k}\right)^{1.471}$
- $\mu^2 \to 0$: scalar does not propagate
- Requires higher order derivative truncation



IR limits: Type Ia/IIIa

- Numerical integration of trajectories
- Y_k flows to constant Y_{k_0}
- Fine-tuning Y_{k_0} to zero removes ghost from IR
- Fixes one free parameter



Slice of phase diagram at $\lambda = -0.1$

Conclusion and outlook

- Higher order derivatives may introduce Ostrogradski ghosts
- NGFPs compatible with Asymptotic Safety
- IR limits: ghosts are exorcised,
  introducing extra condition on physical RG trajectories

Ways to proceed:

- Higher order derivatives: p^6, p^8, ...
- Non-polynomial kinetic functions

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