One force to rule them all: Asymptotic safety of gravity with matter

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Based on

Christiansen, Litim, Pawlowski, MR: arXiv:1710.04669

Asymptotic Safety Seminar

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IMPRS PTFS

Motivation

- Investigate effect of gravity on matter and vice versa, guided by the example of Yang-Mills theory
- Try to understand the basic mechanisms that apply for all weakly coupled matter-gravity systems
- Manifest the results with explicit computations in high-order vertex expansion

Setup

Gauge-fixed Yang-Mills action with $A_\mu = \bar{A}_\mu + a_\mu$ and $\bar{A}_\mu = 0$

$$S_A = \frac{1}{2} \int \mathrm{d}^4 x \sqrt{g}\, g^{\mu\mu'} g^{\nu\nu'} \,\mathrm{tr}\, F_{\mu'\nu'} F_{\mu\nu} + S_{A,\mathrm{gf}} + S_{A,\mathrm{gh}}$$

Gauge-fixed Einstein-Hilbert action with $g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$ and $\bar{g}_{\mu\nu} = \delta_{\mu\nu}$

$$S_{\mathrm{EH}} = \frac{1}{16\pi G} \int \mathrm{d}^4 x \sqrt{g}\, (2\Lambda - R) + S_{g,\mathrm{gf}} + S_{g,\mathrm{gh}}$$

We employ a hard gauge fixing condition $\xi, \alpha \to 0$ and $\beta = 1$

Formal argument I

- Graviton fluctuations support asymptotic freedom in the gauge sector

see later in this talk & Daum, Harst, Reuter (2009) & Folkerts, Litim, Pawlowski (2011)

$$\beta_{\alpha_s} = \beta_{\alpha_s,a} + \beta_{\alpha_s,h} \qquad \text{with} \qquad \beta_{\alpha_s,h} \leq 0$$

- First integrate out the gauge field, afterwards the graviton
- One-loop approximation with IR and UV regularisation

$$S_{\text{gravity,eff}} = S_{\text{gravity}} + S_A \underbrace{- \frac{1}{2} \mathrm{Tr} \ln \left[\Delta_1 \delta_{\mu\nu} + \left(1 - \frac{1}{\xi} \right) \nabla_\mu \nabla_\nu \right]_{k_a^{\text{IR}}}^{k_a^{\text{UV}}}}_{=(*)}$$

- Higher loop orders and higher order couplings such as $\left(\mathrm{tr}F^2\right)^2$ and $\mathrm{tr}F^4$ are suppressed by the asymptotically free gauge coupling

Formal argument II

- Expand $(*)$ in orders of curvature ($k_a = k_a^{\text{UV}}$)

$$(N_c^2 - 1)\left[c_{g,a} k_a^2 \int \mathrm{d}^4x \sqrt{g}\left(2c_{\lambda,a} k_a^2 - R\right)\right.$$

$$\left. + c_{R^2,a} \int \mathrm{d}^4x \sqrt{g}\ \left(R^2 + z_a R_{\mu\nu}^2\right) \ln \frac{R + k_a^{\text{IR}\,2}}{k_a^2}\right] + \mathcal{O}\left(\frac{R^3}{k_a^2}\right)$$

- Redefinition of Einstein-Hilbert couplings

$$G_{\text{eff}} = \frac{G}{1 + (N_c^2 - 1) c_{g,a} k_a^2 G}\,, \qquad \frac{\Lambda_{\text{eff}}}{G_{\text{eff}}} = \frac{\Lambda}{G} + (N_c^2 - 1) c_{g,a} c_{\lambda,a} k_a^4$$

- Redefinition of marginal couplings

$$g_{R^2,\text{eff}} = g_{R^2} + (N_c^2 - 1) c_{R^2,a} \ln \frac{k_a^{\text{IR}\,2}}{k_a^2}$$

$$g_{R_{\mu\nu}^2,\text{eff}} = g_{R_{\mu\nu}^2} + (N_c^2 - 1) c_{R^2,a} z_a \ln \frac{k_a^{\text{IR}\,2}}{k_a^2}$$

Formal argument III

- Demanding cutoff independence leads to N_c independence, e.g.

$$\partial_{k_a^2} G_{\text{eff}} = 0 \qquad \Rightarrow \qquad \partial_{(N_c^2-1)} G_{\text{eff}} = 0$$

- Remaining N_c dependence (analogous for $R_{\mu\nu}^2$)

$$\left(N_c^2 - 1\right) c_{R^2,a} \int \mathrm{d}^4 x \sqrt{g}\, R^2 \ln\left(1 + \frac{R}{k_a^{\mathrm{IR}\,2}}\right)$$

- Only non-removable N_c dependence: from the logarithmic divergences
- Neglected in all asymptotic safety computations so far
- If neglected, integrating out the graviton gives UV-FP "as usual"
- Argument applies also to minimally coupled matter fields

Computation with a vertex expansion

We should be able to observe this in explicit computations

- We evaluate the flow of

$$\Gamma,\ \Gamma^{(hh)},\ \Gamma^{(aa)},\ \Gamma^{(c\bar{c})},\ \Gamma^{(hhh)},\ \Gamma^{(aah)}$$

- Couplings evaluated at $p = k$

$$g,\ g_a,\ \eta_h,\ \eta_a,\ \eta_c$$

- Couplings evaluated at $p = 0$

$$\mu,\ \lambda_3$$

- Background couplings

$$\bar{g},\ \bar{\lambda}$$

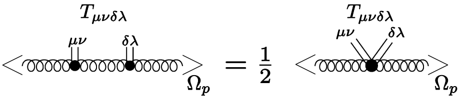
Gravity contribution to Yang-Mills theory

- Running of α_s can be split in gravity and gluon part

$$\partial_t \alpha_s = (\eta_{a,a} + \eta_{a,h})\alpha_s$$

- If $\eta_{a,h} \leq 0$ then asymptotic freedom is preserved

- $\eta_{a,h} = 0 \quad \Leftrightarrow \quad \frac{r_a}{1+r_a}\frac{1}{1+r_h} = 0 \quad$ due to

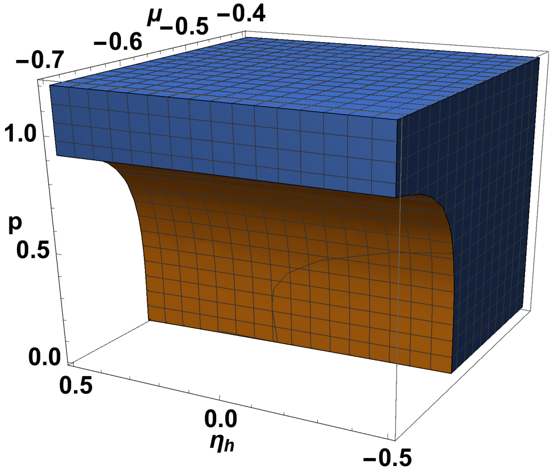


Folkerts, Litim, Pawlowski (2011)

- One can either choose regulators that preserve this identity or work in the momentum regime $p \gtrsim k$, i.e.

$$\partial_t \alpha_s = (\eta_{a,a}(k^2) + \eta_{a,h}(k^2))\alpha_s$$

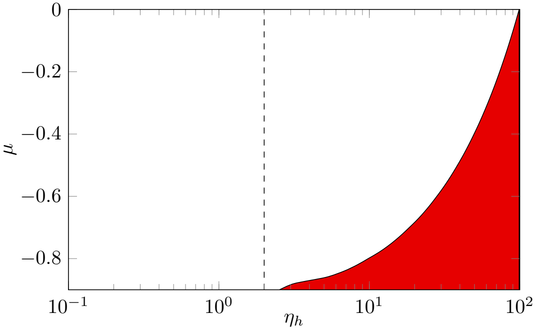
Sign of $\eta_{a,h}(p^2)$



Filled block: $\eta_{a,h}(k^2) < 0$

Outside: $\eta_{a,h}(k^2) > 0$

Sign of $\eta_{a,h}(p^2)$ at $p = k$

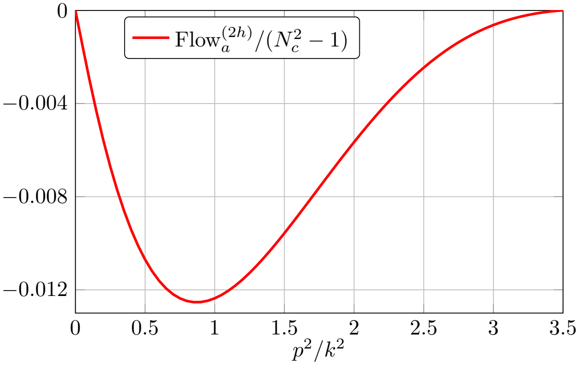


White region: $\eta_{a,h}(k^2) < 0$

Red region: $\eta_{a,h}(k^2) > 0$

Dashed line: $\eta_h = 2$

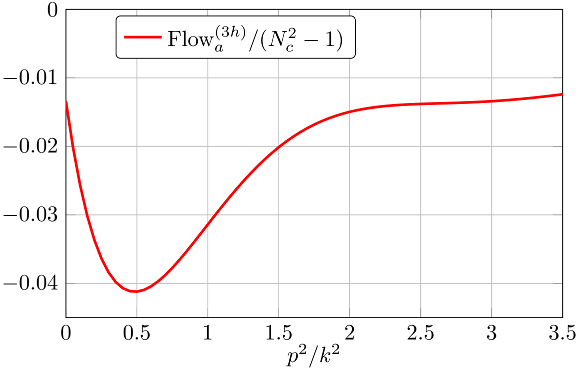
Yang-Mills contribution to gravity



$$\partial_t \mu \Big|_a = (N_c^2 - 1)\frac{g_a \eta_a}{60\pi} \qquad \eta_{h,a} > 0$$

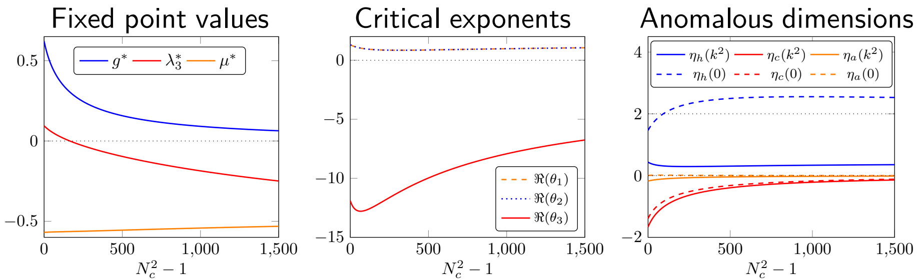
Zero at leading order!

Yang-Mills contribution to gravity



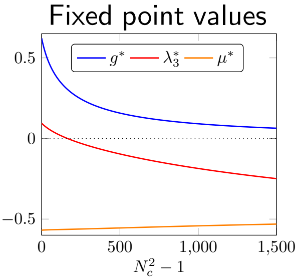
$$\partial_t \lambda_3\Big|_a = g_a^{\frac{3}{2}} (N_c^2 - 1) \frac{3-\eta_a}{60\pi} \qquad \partial_t g\Big|_a < 0$$

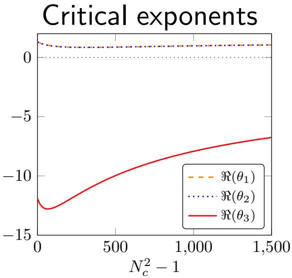
Fixed point properties with $g = g_a$

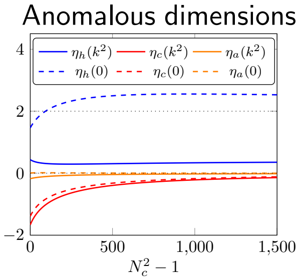


Reliable attractive UV fixed point for all N_c

Fixed point properties with $g = g_a$







$$g^* \to \frac{89}{N_c^2} + \frac{8.0 \cdot 10^4}{N_c^4}$$

$$\mu^* \to -0.45 - \frac{3.3 \cdot 10^2}{N_c^2}$$

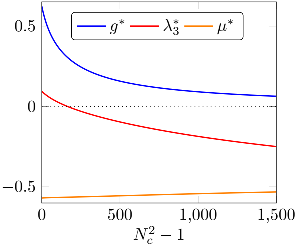
$$\lambda_3^* \to -0.71 + \frac{2.4 \cdot 10^3}{N_c^2}$$

$$\theta_{1,2} \to 1.2 \pm 2.1i + \frac{(1.1 \mp 5.6i) \cdot 10^3}{N_c^2}$$

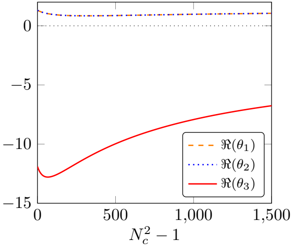
$$\theta_3 \to -2.3 - \frac{14 \cdot 10^3}{N_c^2}$$

Fixed point properties with $g = g_a$

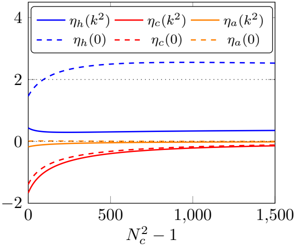
Fixed point values



Critical exponents



Anomalous dimensions

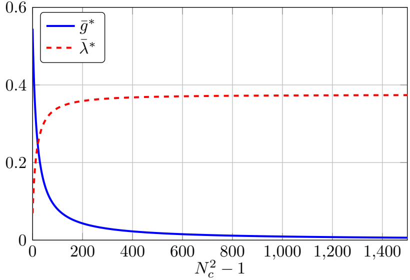


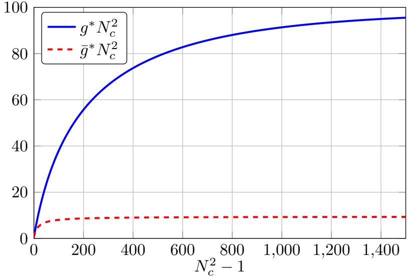
$$g^* \to \frac{89}{N_c^2} + \frac{8.0 \cdot 10^4}{N_c^4} \qquad \eta_h(0) \to 2 + \frac{2.7 \cdot 10^3}{N_c^2} \qquad \eta_h(k^2) \to 0.36 + \frac{2.9 \cdot 10^2}{N_c^2}$$

$$\mu^* \to -0.45 - \frac{3.3 \cdot 10^2}{N_c^2} \qquad \eta_c(0) \to -\frac{1.3 \cdot 10^2}{N_c^2} \qquad \eta_c(k^2) \to -\frac{1.5 \cdot 10^2}{N_c^2}$$

$$\lambda_3^* \to -0.71 + \frac{2.4 \cdot 10^3}{N_c^2} \qquad \eta_a(0) \to -\frac{8.7}{N_c^2} \qquad \eta_a(k^2) \to -\frac{22}{N_c^2}$$

Background and 't Hooft couplings





$$\bar{g}^* \to \frac{9.4}{N_c^2} - \frac{1.3 \cdot 10^2}{N_c^4}$$

$$\bar{\lambda}^* \to 0.38 - \frac{1.4}{N_c^2}$$

Everything alright?

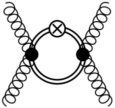
Problem: solution uses $g_a = g \sim 1/N_c^2$ but that's not compatible with

$$\partial_t g_a = (2 + 2\eta_a + \eta_h)\, g_a - g_a^2 \left[c_{g_a,a}(\mu) + c_{g_a,h}(\mu) \left(\frac{g}{g_a} \right)^{\frac{1}{2}} \right]$$

$$\to 4 g_a - \tilde{c}_{g_a}(\mu) g_a^2$$

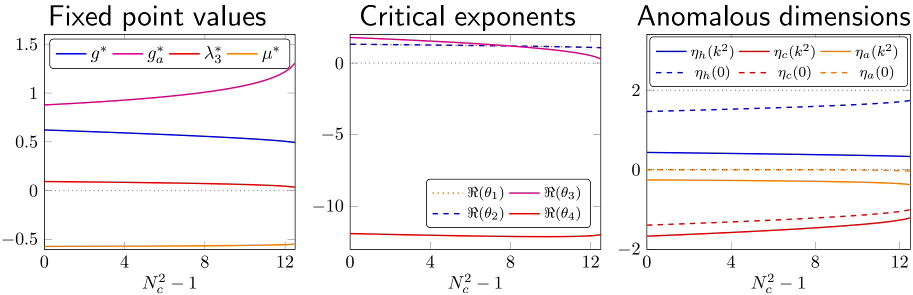
if $\mu \sim$ const

Cannot be cured by higher-order terms like $\left(\mathrm{tr} F^2\right)^2$ or $\mathrm{tr} F^4$ since



$$\sim \frac{g_a^2}{(1+\mu)^3} \sim \frac{1}{N_c^4} \to 0$$

Fixed point properties with g_a



Fixed point disappears at $N_c \approx 12$

Not compatible with the formal argument

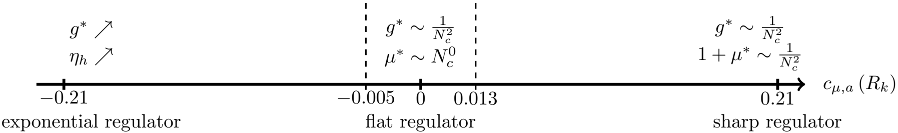
Regulator dependence of $c_{\mu,a}$

Crucial observation:

The leading order gluon contribution to $\partial_t \mu$ is strongly regulator dependent and vanishing for the flat regulator

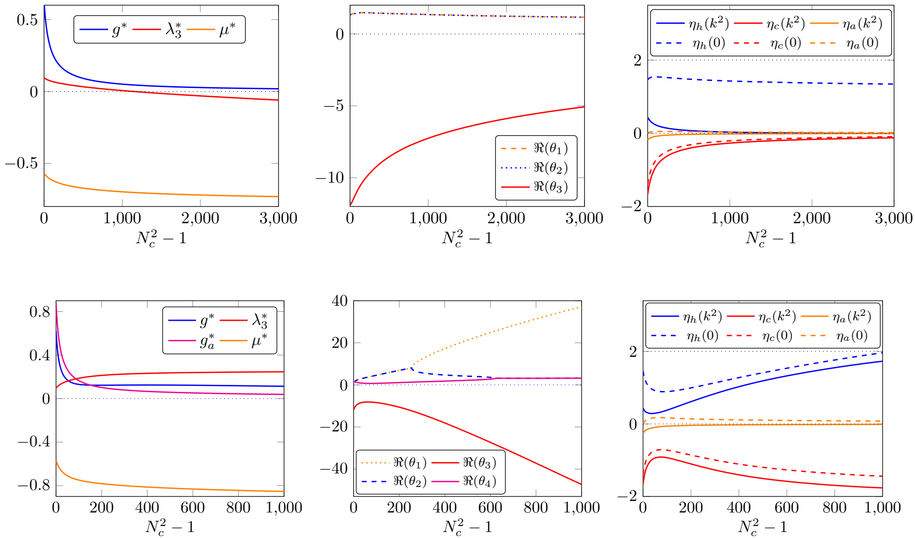
Regulator	$c_{\mu,a}$
Exponential: $r(x) = \frac{1}{\exp(x)-1}$	-0.21
$r(x) = \frac{1}{x}\exp(-x^2)$	-0.027
Flat: $r(x) = (\frac{1}{x} - 1)\Theta(1-x)$	0
$r(x) = \frac{1}{x}\Theta(1-x)$	0.034
$r(x) = \frac{10}{x}\Theta(1-x)$	0.17
Sharp: $r(x) = \frac{1}{\Theta(x-1)} - 1$	$\frac{2}{3\pi} \approx 0.21$

Regulator dependence of the fixed point with $g = g_a$



- The flat regulator triggers a rather exceptional behaviour
- Generic feature is the enhancement of the graviton propagator $(k^2 G_h(p^2 = 0) = \frac{1}{Z_h} \frac{1}{1+\mu})$
  - via $\eta_h^* \nearrow$ (exponential regulator)
    not accessible, due to regulator bound $\eta_h < 2$ Meibohm, Pawlowski, MR (2015)
  - via $\mu^* \to -1$ (sharp regulator)

Fixed point behaviour with $c_{\mu,a} < 0$



Resumé

With the regulator we can access different versions of the same physics:

- g^* and η_h^* increase

  $\longrightarrow$ analogous to scalar-gravity system

- $\mu^* \to -1$ and $g^* \sim 1/N_c^2$

  $\longrightarrow$ analogous to fermion-gravity system

In all cases the graviton propagator gets enhanced and dominates

$\longrightarrow$ One force to rule them all

Summary and Outlook

- We investigated quantum gravity with weakly coupled matter systems
- We provided a formal argument for its asymptotic safety
- The special rôle of the marginal couplings was highlighted
- We further manifested that gravity supports asymptotic freedom
- Asymptotic safety for all N_c was confirmed with explicit computations
- However: strong scheme dependence
- R^2 and $R_{\mu\nu}^2$ must be included in future computations