Upper bound on the Abelian gauge coupling from asymptotic safety

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Based on:

A. Eichhorn and FV, JHEP **1801**, 030 (2018), arXiv:1709.07252

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Outline

- Triviality problem QED
- Inclusion of gravity
- Asymptotic safety/freedom
- U(1) hypercharge
- Conclusions

Triviality problem

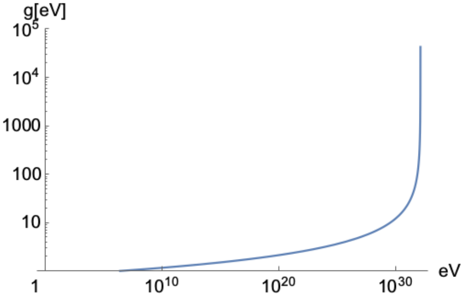
[M. Baig, H. Fort, J. B. Kogut and S. Kim, Phys. Rev. D **51**, 5216 (1995)]

- QED at one loop

$$g = \frac{g_{obs}}{1 - \beta g_{obs} \ln \frac{k}{m}}$$

g_{obs} : observable charge

g : microscopic charge

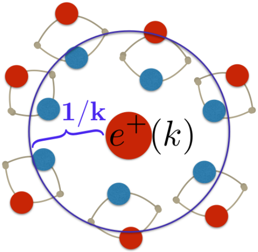


$\Rightarrow$ Landau pole

Triviality problem

- What is physically going on?

$\Rightarrow$ Virtual particle-antiparticle pairs (e^-, e^+) turn vacuum into screening "medium"



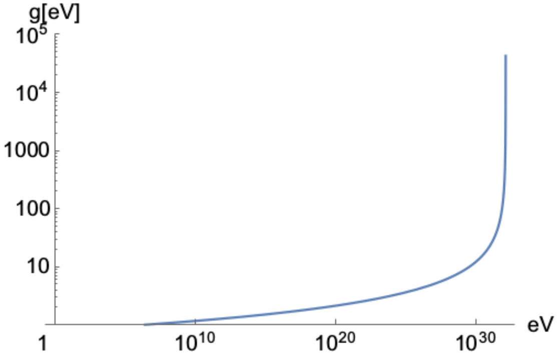
credits: Astrid Eichhorn

Moving closer to central charge (i.e. increase energy) increases effective charge

Landau pole $\Rightarrow$ Theory breaks down at high energies

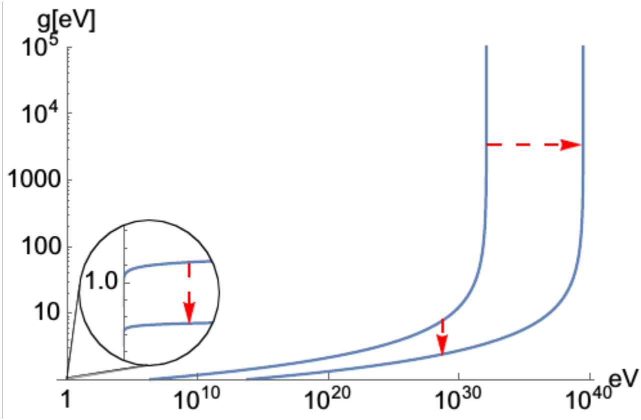
Triviality problem

- Push Landau pole to infinity



Triviality problem

- Push Landau pole to infinity



- In limit where Landau pole is removed, coupling tuned to 0 at all k

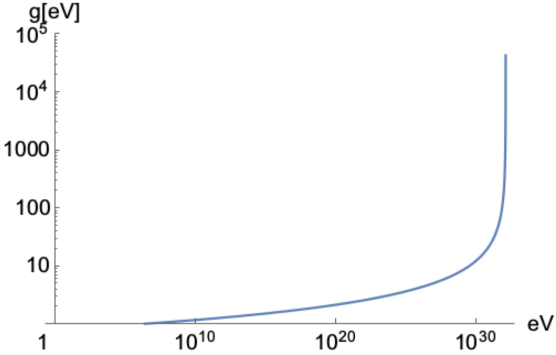
$\Rightarrow$ Triviality

[H. Gies and J. Jaeckel, Phys. Rev. Lett. **93**, 110405 (2004)]

[M. Gockeler, R. Horsley, V. Linke, P. E. L. Rakow, G. Schierholz and H. Stuben, Phys. Rev. Lett. **80**, 4119 (1998)]

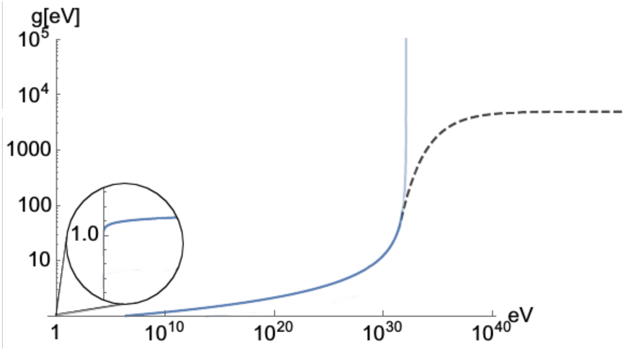
Triviality problem

- At large scales, 1-loop result not sufficient
  $\Rightarrow$ Need non-perturbative tools



Triviality problem

- If for $k \to \infty$ there is a UV fixed point s.t. in the IR $g_{obs} \to g_*$



$\Rightarrow$ Interacting theory

Scalar QED

- Action:

$$\mathcal{S} = \underbrace{\boxed{\int d^4x \sqrt{g} g^{\mu\nu} \left(D_\mu \phi\right) \left(D_\nu \phi\right)^\dagger}}_{\text{Kinetic term } \phi} + \underbrace{\boxed{\frac{1}{4} \int d^4x \sqrt{g} g^{\mu\nu} g^{\alpha\beta} F_{\mu\alpha} F_{\nu\beta}}}_{\text{Kinetic term } A} + S_{gf,A}$$

- where:

$$F_{\mu\nu} = (\partial_\mu A_\nu - \partial_\nu A_\mu)$$
$$D_\mu = \partial_\mu - i\rho A_\mu$$
$$\rho = \rho(k)$$

Scalar QED: Matter contribution

[Wetterich '93]

- Wetterich equation:

$$\partial_t \Gamma_k = \frac{1}{2} \mathrm{Tr} \left[\Gamma_k^{(2)} + \mathcal{R}_k \right]^{-1} \partial_t \mathcal{R}_k$$

- 3-point VS 4-point vertex

$$\begin{aligned} \Gamma_k \big|_\rho = & \quad i\bar{\rho}_3 \int d^4x \sqrt{g} g^{\mu\nu} \left(\phi^\dagger \partial_\nu \phi - (\partial_\nu \phi^\dagger)\phi \right) A_\mu \\ + & \quad \bar{\rho}_4{}^2 \int d^4x \sqrt{g} g^{\mu\nu} A_\mu A_\nu \phi^\dagger \phi \end{aligned}$$

- differ in normalization

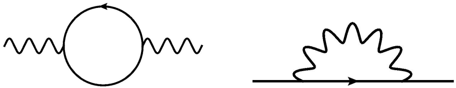
$$\rho_3 = \frac{\bar{\rho}_3}{Z_\phi Z_A^{1/2}}, \qquad \rho_4 = \frac{\bar{\rho}_4}{Z_\phi^{1/2} Z_A^{1/2}} \qquad \eta_{\phi,A} = -\partial_t \ln Z_{\phi,A}$$

indicating two projections

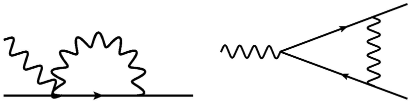
$$\begin{aligned} \beta_{\rho_3} &= \rho_3 \left(\eta_\phi + \frac{\eta_A}{2} \right) + \text{3-point matter contributions} \\ \beta_{\rho_4} &= \rho_4 \left(\frac{\eta_\phi}{2} + \frac{\eta_A}{2} \right) + \text{4-point matter contributions} \end{aligned}$$

Scalar QED: Matter contribution

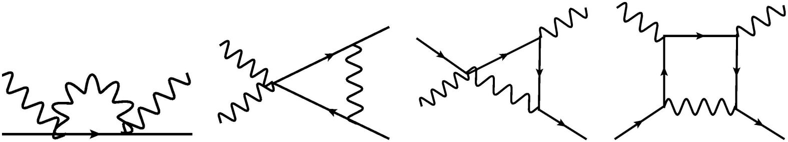
- Matter contribution anomalous dimension:



- Matter contribution 3-point vertex:



- Matter contribution 4-point vertex:



UV completion

- Identifying $\rho_3 = \rho_4$ (gauge invariance) gives universal result for the flow

$$k\partial_k \rho \mid_{matter} = \beta_\rho \mid_{matter} = \frac{\eta_A}{2} = \frac{\rho^3}{48\pi^2}$$

- Adding gravity:

$$\mathcal{S} \quad = \quad \underbrace{\boxed{\frac{1}{2}\int d^4x\sqrt{g}g^{\mu\nu}\left(D_\mu\phi\right)\left(D_\nu\phi\right)^\dagger}}_{\text{Kinetic term } \phi} + \underbrace{\boxed{\frac{1}{4}\int d^4x\sqrt{g}g^{\mu\nu}g^{\alpha\beta}\,F_{\mu\alpha}\,F_{\nu\beta}}}_{\text{Kinetic term } A}$$

$$+S_{gf,A} - \underbrace{\boxed{\frac{1}{16\pi\,\bar{G}}\int d^4x\sqrt{g}\,\left(R - 2\bar{\Lambda}\right)}}_{\text{Einstein-Hilbert}} + S_{gf,h}$$

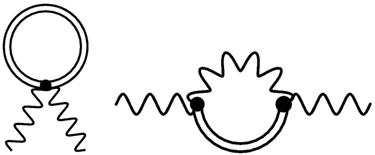
[U. Harst and M. Reuter, JHEP **1105**, 119 (2011)]

- where:

$F_{\mu\nu} = (\partial_\mu A_\nu - \partial_\nu A_\mu)$
$D_\mu = \partial_\mu - i\bar{\rho}A_\mu$
$g^{\mu\nu} = \delta^{\mu\nu} + \eta^{\mu\nu}$
$\bar{\rho} = \bar{\rho}(k)$ $\boxed{\bar{G} = \bar{G}(k),\ \bar{\Lambda} = \bar{\Lambda}(k)}$ $\quad \rightarrow \rho = \bar{\rho},\ G = \bar{G}\,k^2,\ \Lambda = \bar{\Lambda}\,k^{-2}$

UV completion: TT approximation

- project on transverse-traceless mode of $h_{\mu\nu}$
- Gravity contribution anomalous dimension



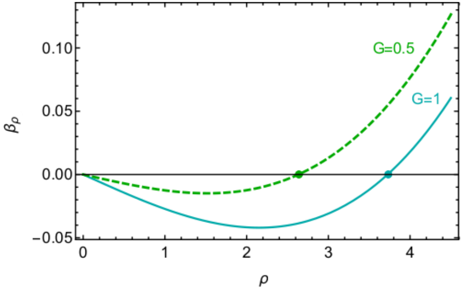
- Contribution to the flow

$$\beta_\rho \,|_{TT} = \frac{\eta_A}{2}\rho = \frac{1}{48\pi^2}\rho^3 - G\,\rho\frac{5}{36\pi}\left(\frac{16(1-\Lambda)-\eta_h}{(1-2\Lambda)^2}\right).$$

[A. Eichhorn and FV, (2017)]

UV completion: TT approximation

$$\beta_\rho\,|_{TT} = \frac{\eta_A}{2}\rho = \frac{1}{48\pi^2}\rho^3 - G\,\rho\frac{5}{36\pi}\left(\frac{16(1-\Lambda)-\eta_h}{(1-2\Lambda)^2}\right).$$



- Quantum gravity contribution towards asymptotic freedom

[U. Harst and M. Reuter, JHEP **1105**, 119 (2011), [arXiv:1101.6007]]

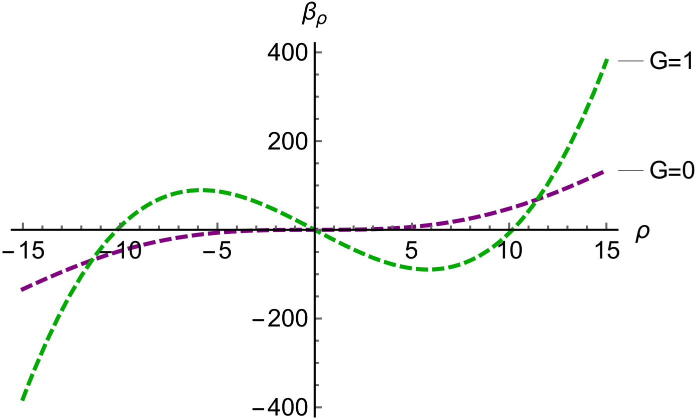
[N. Christiansen and A. Eichhorn, Phys. Lett. B **770**, 154 (2017),[arXiv:1702.07724]]

[J. E. Daum, U. Harst and M. Reuter, JHEP **1001**, 084 (2010),[arXiv:0910.4938]]

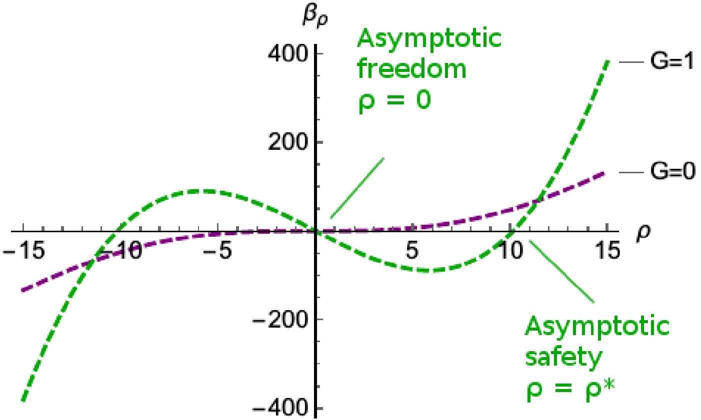
[S. Folkerts, D. F. Litim and J. M. Pawlowski, Phys. Lett. B **709**, 234 (2012), [arXiv:1101.5552]]

UV completion: TT approximation

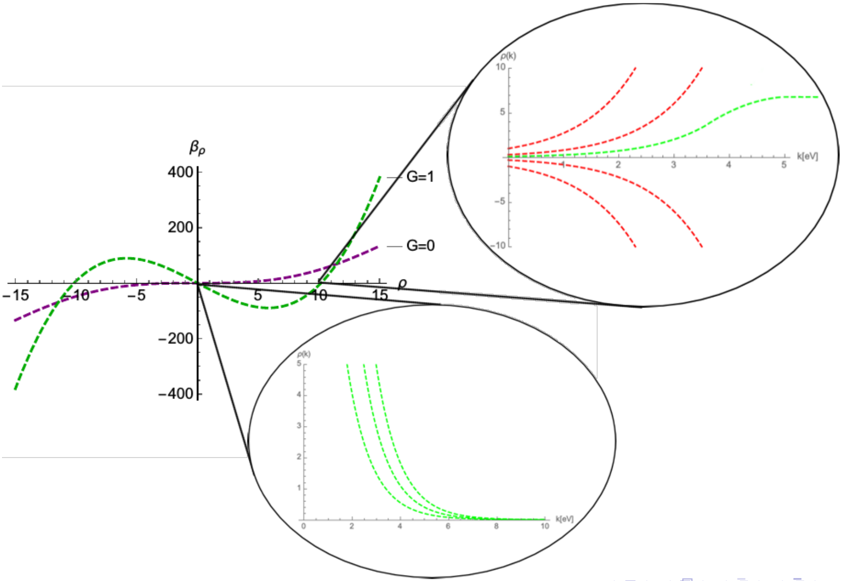
Schematically, what is going on?



UV completion: TT approximation

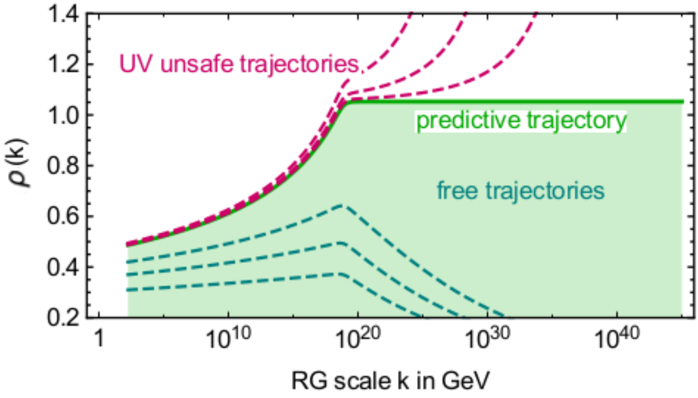


UV completion: TT approximation



UV completion: TT approximation

Upper bound on IR value



UV completion: Gauge parameters

- gravity and matter gauge parameters: α, β, ξ
- Hard implementation of gauge condition: $\alpha = 0$

[H. Gies, B. Knorr and S. Lippoldt, Phys. Rev. D **92**, no. 8, 084020 (2015)

- Demanding that $\beta_{\rho_3} = \beta_{\rho_4} = \frac{\eta_A}{2}$ yields:

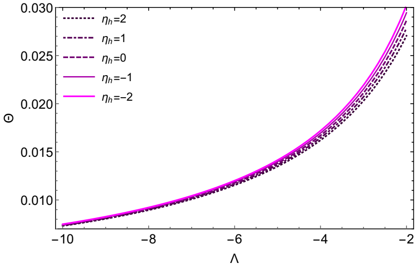
$$\beta = 1, \qquad \xi\text{-independent}$$

- Flow:

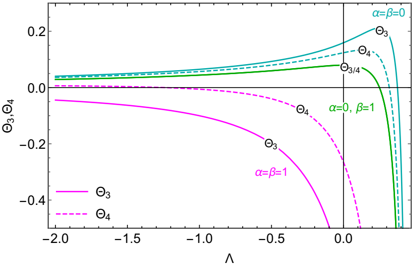
$$\beta_\rho = \frac{1}{48\pi^2}\,\rho^3 - \frac{G\,(4(1-4\Lambda)+\eta_h)}{16\pi(1-2\Lambda)^2}\,\rho.$$

- Critical exponents

$$\Theta_{\text{int}} = -\Theta_{\text{free}} = -\frac{\partial\beta_\rho}{\partial\rho}\Big|_{\rho=\rho^*} = -G\frac{4-16\Lambda+\eta_h}{8\pi(1-2\Lambda)^2}.$$



Robustness under change of η_h.



Preferred gauge $\alpha = 0,\ \beta = 1$.

$\mathrm{U}(1)$ hypercharge

- In the standard model:

$$\beta_{g_Y} = \frac{g_Y^3}{16\pi^2}\frac{41}{6} - g_Y f_g$$
$$g_Y^* = 4\sqrt{\frac{6}{41} f_g \pi}$$

- Where:
  g_Y is the hypercharge coupling
  f_g is the quantum-gravity contribution

- Following the flow to the IR and demanding that the predicted IR value of the $U(1)$ coupling matches the observed value gives constraint:

$$f_g \gtrsim \frac{0.096}{\pi^2}$$

Results

- What we need: $fg \gtrsim \dfrac{0.096}{\pi^2}$

- What we find:

$$\begin{aligned} \beta_{g_Y} &= \frac{g_Y^3}{16\pi^2}\frac{41}{6} - g_Y\, G\frac{1-4\Lambda}{4\pi(1-2\Lambda)^2} \\ \text{and thus} \quad f_g &= G\frac{1-4\Lambda}{4\pi(1-2\Lambda)^2} > 0 \qquad \text{for } G > 0,\ \Lambda < \frac{1}{4} \end{aligned}$$

[S. Folkerts, D. F. Litim and J. M. Pawlowski, Phys. Lett. B **709**, 234 (2012)]

- Interacting fixed point:

$$g_Y^* = \sqrt{\frac{24\pi}{41}}\sqrt{\frac{G(1-4\Lambda)}{(1-2\Lambda)^2}}.$$

Results

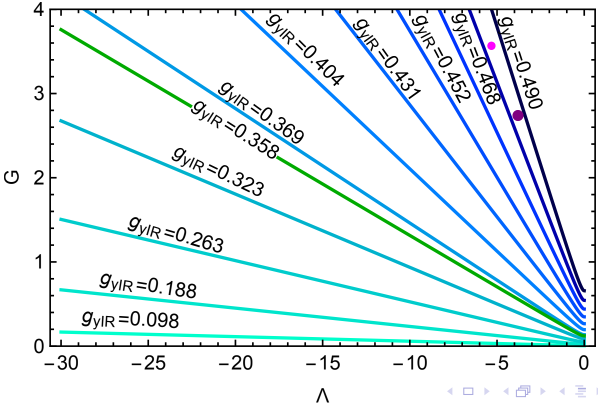
- at $g_Y(k = M_{Pl}) = g_Y^*$ gravity switches off $\rightarrow$ integrate down to IR

- FP values (G, Λ) depend on matter content.

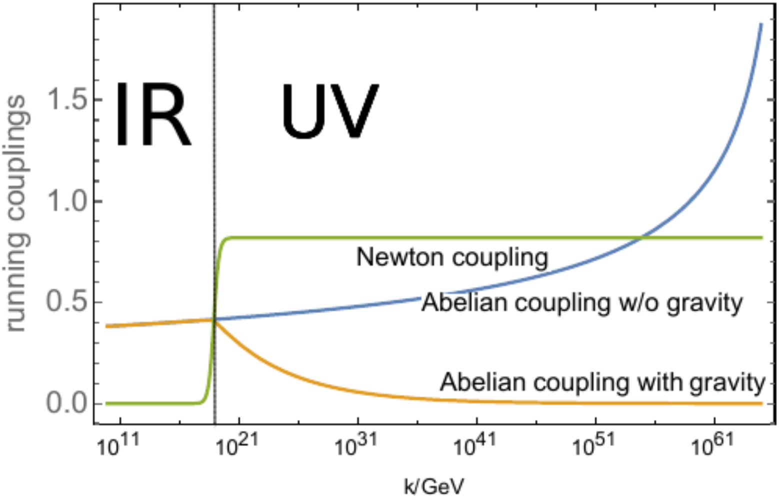
  [S. Lippoldt, to appear], [P. DonÃ , A. Eichhorn and R. Percacci, Phys. Rev. D **89**, no. 8, 084035 (2014)]

  For asymptotically safe EH + minimally coupled SM

  $\Rightarrow$ we find: $g_{Y\,IR} = 0.487$

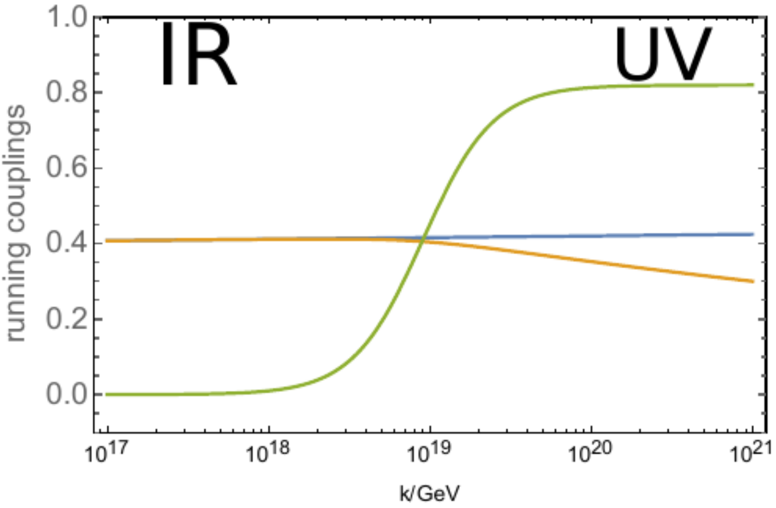


Results



IR
UV
running couplings
1.5
1.0
0.5
0.0
Newton coupling
Abelian coupling w/o gravity
Abelian coupling with gravity
10^{11}
10^{21}
10^{31}
10^{41}
10^{51}
10^{61}
k/GeV

Results



IR
UV
running couplings
1.0
0.8
0.6
0.4
0.2
0.0
10^{17}
10^{18}
10^{19}
10^{20}
10^{21}
k/GeV

Conclusions

- QED breaks down at high energy scales
  $\Rightarrow$ Need new physics

- Adding gravity in truncations yields two potential solutions:

  - Asymptotic freedom (Free FP)
  - Asymptotic safety (Interacting FP)

  $\Rightarrow$ Interacting FP predicts IR value g_Y within 35% range

- Regime where microscopic gravity gives close prediction of g_Y same as regime predicting top/bottom mass

  [A. Eichhorn and A. Held, Phys. Lett. B **777**, 217 (2018)]

Outlook:

Extend truncation to include $(FF)^2$-type terms