Asymptotically safe f(R)-gravity coupled to matter

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N. Alkofer and F. Saueressig, arXiv:1802.00498

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Outline

- introduction
- $f(R)$-gravity minimally coupled to matter
    - setup
    - coarse-graining operators
    - operator traces as spectral sums
- solutions I: polynomial expansion
- solutions II: complete solutions
- summary and open issues

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Related work:

[N. Ohta, R. Percacci, G.-P. Vacca, arXiv:1507.00968]

[N. Ohta, R. Percacci, G.-P. Vacca, arXiv:1511.09393]

introduction

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- controls the UV-behavior of the RG-trajectory
- ensures the absence of UV-divergences

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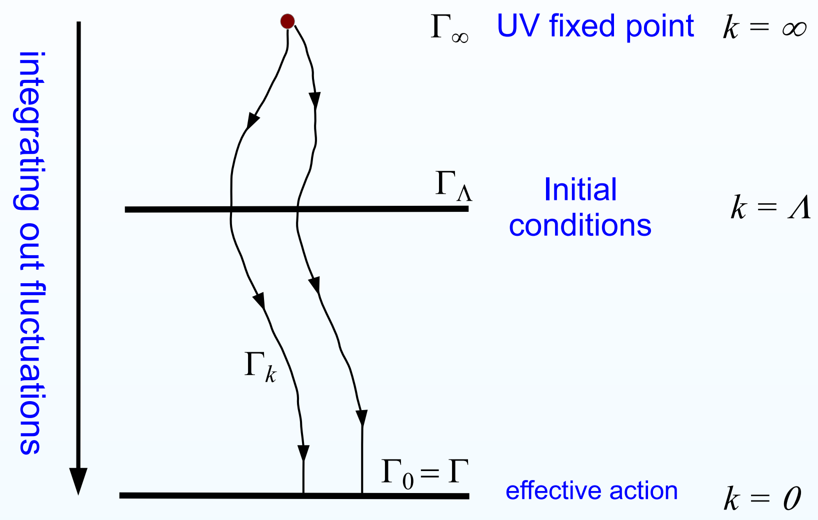
d) structural demands
   - resolution of singularities: (black holes, Landau poles, . . .)

Functional renormalization group equation (FRGE) for gravity

M. Reuter, Phys. Rev. D **57** (1998) 971

central idea: integrate out quantum fluctuations shell-by-shell in momentum-space



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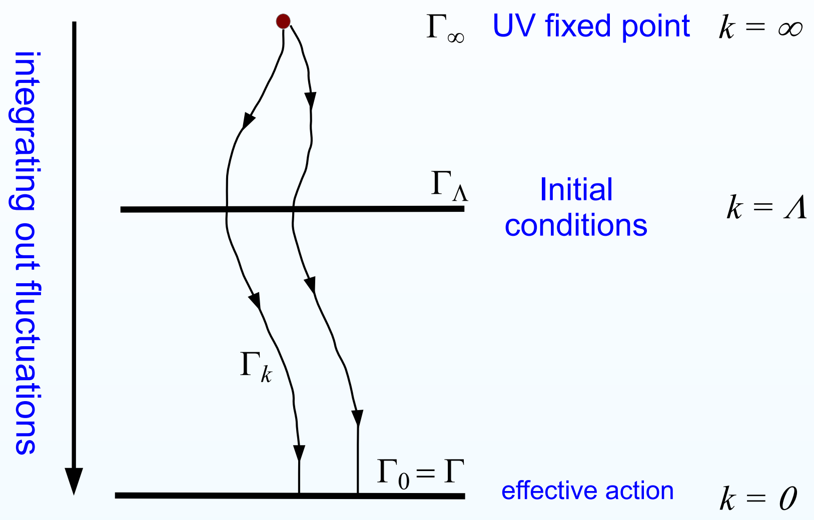

- scale-dependence governed by functional renormalization group equation

$$k\partial_k \Gamma_k[h_{\mu\nu}; \bar{g}_{\mu\nu}] = \frac{1}{2}\mathrm{STr}\left[\left(\Gamma_k^{(2)} + \mathcal{R}_k\right)^{-1} k\partial_k \mathcal{R}_k\right]$$

  - uses background field formalism
  - effective vertices capture quantum-corrections with $p^2 > k^2$

Constructing non-perturbative approximate solutions of the FRGE

ansatz for Γ_k restricting to a subset of all monomials

$$\Gamma_k[h_{\mu\nu};\bar{g}_{\mu\nu}] = \sum_{i=1}^{N} \bar{u}_i(k)\, \mathcal{O}_i[h_{\mu\nu};\bar{g}_{\mu\nu}]$$

$\Longrightarrow$ substitute ansatz into FRGE

$\Longrightarrow$ projection of flow onto ansatz gives β-functions for $\bar{u}_i(k)$

$$k\partial_k \bar{u}_i(k) = \beta_i(\bar{u}_i;k)$$

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classes of approximations:

- "single-metric" trunctions (level 0):

$$\partial_t \Gamma_k[h_{\mu\nu}; \bar{g}_{\mu\nu}]|_{h_{\mu\nu}=0} = \ldots$$

- "bi-metric" truncations (level n) retain information about fluctuation fields

$$\partial_t \Gamma_k^{(n,0)}[h_{\mu\nu}; \bar{g}_{\mu\nu}]\Big|_{h_{\mu\nu}=0} = \ldots$$

selected asymptotic safety highlights

pure gravity:

- non-Gaussian fixed point established in a wide range of approximations
  - also including the Goroff-Sagnotti counterterm

[H. Gies, B. Knorr, S. Lippoldt and F. Saueressig, arXiv:1601.01800]

- low number of relevant parameters ($\simeq$ 3):

[R. Percacci and A. Codello, arXiv:0705.1769]
[P.F. Machado and F. Saueressig, arXiv:0712.0445]
[T. Denz, J. Pawlowski, M. Reichert, arXiv:1612.07315]
[K. Falls, C. S. King, D. F. Litim, K. Nikolakopoulos and C. Rahmede, arXiv:1801.00162]

- non-Gaussian fixed point is connected to a classical regime by crossover

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gravity coupled to matter:

[R. Percacci and D. Perini, hep-th/0207033]
[M. Shaposhnikov and C. Wetterich, arXiv:0912.0208]
[P. Dona, A. Eichhorn and R. Percacci, arXiv:1311.2898]
[J. Meibohm, J. M. Pawlowski and M. Reichert, arXiv:1510.07018]
[J. Biemans, A. Platania and F. Saueressig, arXiv:1702.06539]
[N. Christiansen, D. Litim, J. Pawlowski, M. Reichert, arXiv:1710.04665]
[A. Eichhorn, Y. Hamada, J. Lumma and M. Yamada, arXiv:1712.00319]
[A. Eichhorn and A. Held, arXiv:1803.04027]

⋮

$f(R)$-gravity coupled to matter

Exploring the single-metric theory space spanned by $\Gamma_k^{\mathrm{grav}}[g]$

Einstein-Hilbert truncation

polynomial $f(R)$-truncation

$\vdots$			
R^8	$\ldots$		
R^7	$\ldots$		
R^6	$\ldots$		
R^5	$\ldots$		
R^4	$\ldots$		
R^3	$C_{\mu\nu}{}^{\rho\sigma} C_{\rho\sigma}{}^{\kappa\lambda} C_{\kappa\lambda}{}^{\mu\nu}$	$R \Box R$	+ 5 more
R^2	$C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma}$	$R_{\mu\nu} R^{\mu\nu}$	
R			
$\mathbb{1}$			

finite-dimensional truncations

polynomial expansions of $f(R)$-gravity

[A. Codello, R. Percacci, C. Rahmede, ’07]
[P. Machado, F. Saueressig, ’07]
[A. Codello, R. Percacci, C. Rahmede, ’09]
[A. Bonanno, A. Contillo, R. Percacci, ’11]
[K. Falls, D. F. Litim, K. Nikolakopoulos, C. Rahmede, ’13]
[K. Falls, D. F. Litim, K. Nikolakopoulos, C. Rahmede, ’14]

[A. Eichhorn, ’15]

Polynomial expansion of $f(R)$-gravity

[A. Codello, R. Percacci, C. Rahmede, '07]
[P. Machado, F. Saueressig, '07]

flow equation for $f(R)$-gravity:

$$\Gamma_k^{\text{grav}}[g] \simeq \int d^4x \sqrt{g}\, f_k(R)$$

- supplemented by geometric gauge

  FRGE $\Rightarrow$ partial differential equation governing k-dependence of $f_k(R)$

UV properties of RG flow:

- polynomial expansion: $f_k(R) = \sum_{n=0}^{N} \bar{u}_n\, R^n$
- boundary conditions: $\bar{u}_{N+1} = \bar{u}_{N+2} = 0$
- expand flow equation $\Longrightarrow$ β-functions for $g_n = \bar{u}_n k^{2n-4}$

$$k\partial_k g_n = \beta_{g_n}(g_0, g_1, \ldots)\,, \quad n = 0, \ldots, N$$

- reduces search for NGFP to algebraic problem

Renormalization group flow of $f(R)$-gravity

- Polynomial expansion: $f_k(R) = \sum_{n=0}^{N} g_n \, (R/k^2)^n \, k^4 + \ldots$

$$k\partial_k g_i = \beta_{g_i}(g_0, g_1, \ldots)\,, \quad i = 0, \ldots, N$$

- NGFP can be traced through extensions of truncation subspace

N	g_0^*	g_1^*	g_2^*	g_3^*	g_4^*	g_5^*	g_6^*
1	0.00523	-0.0202					
2	0.00333	-0.0125	0.00149				
3	0.00518	-0.0196	0.00070	-0.0104			
4	0.00505	-0.0206	0.00026	-0.0120	-0.0101		
5	0.00506	-0.0206	0.00023	-0.0105	-0.0096	-0.00455	
6	0.00504	-0.0208	0.00012	-0.0110	-0.0109	-0.00473	0.00238

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NGFP is stable under extension of truncation subspace

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- linearized RG flow at NGFP $\Longrightarrow$ three UV relevant directions

N	Re $\theta_{0,1}$	Im $\theta_{0,1}$	θ_2	θ_3	θ_4	θ_5	θ_6
1	2.38	2.17					
2	1.26	2.44	27.0				
3	2.67	2.26	2.07	-4.42			
4	2.83	2.42	1.54	-4.28	-5.09		
5	2.57	2.67	1.73	-4.40	-3.97 + $4.57i$	-3.97 - $4.57i$	
6	2.39	2.38	1.51	-4.16	-4.67 + $6.08i$	-4.67 - $6.08i$	-8.67

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NGFP is stable under extension of truncation subspace

good evidence: fundamental theory has finite number of relevant parameters

$f(R)$-gravity

minimally coupled to matter

ansatz: gravity supplemented by minimally coupled matter fields

$f(R)$-ansatz in gravitational sector

$$\Gamma_k^{\mathrm{grav}}[g] \simeq \frac{1}{(4\pi)^2} \int d^4x \sqrt{g}\, f_k(R)$$

matter sector:

N_S scalar fields: $S_S = \frac{N_S}{2} \int d^4x \sqrt{g}\, g^{\mu\nu}\, (\partial_\mu \phi)(\partial_\nu \phi)$

N_D Dirac spinors: $S_D = i\, N_D \int d^4x \sqrt{g}\, \bar{\psi} \not{\nabla} \psi$

N_V gauge fields: $S_V = \frac{N_V}{4} \int d^4x \sqrt{g}\, g^{\mu\alpha} g^{\nu\beta} F_{\mu\nu}\, F_{\alpha\beta} + S^{\mathrm{gf}} + S^{\mathrm{gh}}$

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- gauge-fixing: physical gauge
- parameterization of metric fluctuations: $g_{\mu\nu} = \bar{g}_{\mu\rho} \left[e^{\bar{g}^{-1} h} \right]^{\rho}{}_{\nu}$
- background geometry: maximally symmetric d-sphere

technicalities I

coarse graining operators

construction of flow equation

uses transverse-traceless decomposition:

- metric fluctuations:

$$h_{\mu\nu} = h^{\mathrm{TT}}_{\mu\nu} + \bar{D}_\mu \xi^{\mathrm{T}}_\nu + \bar{D}_\nu \xi^{\mathrm{T}}_\mu + (\bar{D}_\mu \bar{D}_\nu - \frac{1}{d}\bar{g}_{\mu\nu}\bar{D}^2)\sigma + \frac{1}{4}\bar{g}_{\mu\nu} h,$$

- vector fluctuations:

$$A_\mu = A^{\mathrm{T}}_\mu + \bar{D}_\mu\, a\,, \qquad \bar{D}^\mu A^{\mathrm{T}}_\mu = 0\,.$$

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for physical gauge:

- 3 traces in gravitational sector

$$h^{\mathrm{TT}}_{\mu\nu}\,,\ \xi^{\mathrm{T}}_\mu\,,\ s$$

- 4 traces in matter sector

$$\underbrace{\phi}_{\text{scalars}}\,, \qquad \underbrace{\bar{\psi}\psi}_{\text{Dirac fermions}}\,, \qquad \underbrace{A^{\mathrm{T}}_\mu\,,\ a}_{\text{vector fields}}$$

coarse graining operator

mass-type regulator $\mathcal{R}_k(\Box)$:

- suppresses fluctuations with eigenvalue $\lambda_\Box < k^2$ by mass-term

structure of the coarse graining operator:

$$\Box = \Delta - \alpha \bar{R}$$

- $\Delta = -\bar{D}^2$: Laplacian on d-sphere
- α: endomorphism parameter

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setup contains 7 endomorphism parameters

- gravity: $\alpha_T^G,\ \alpha_V^G,\ \alpha_S^G$
- matter: $\alpha_S^M,\ \alpha_D^M,\ \alpha_{V_1}^M,\ \alpha_{V_2}^M$

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allow relative shifts among modes being integrated out at scale k

relevant coarse graining operators

- Type I: no shifts in the spectra

$$\alpha = 0$$

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- Type ID: uses $\Box = -\not{D}^2$ in the fermionic sector

$$\alpha_D^M = -\frac{1}{4}\,, \qquad \alpha = 0 \qquad \text{all other fields}$$

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- Type II: removes curvature terms from propagators

$$\alpha_T^G = -\tfrac{1}{6}\,,\; \alpha_S^G = \tfrac{1}{3}\,,\; \alpha_V^G = \tfrac{1}{4}\,,\; \alpha_D^M = -\tfrac{1}{4}\,,\; \alpha_{V_1}^M = -\tfrac{1}{4}\,,\; \alpha_{V_2}^M = \alpha_S^M = 0$$

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- Type i: interpolates smoothly between Type I ($c = 0$) and Type II ($c = 1$)

$$\alpha_T^G = -\tfrac{c}{6}\,,\ \alpha_S^G = \tfrac{c}{3}\,,\ \alpha_V^G = \tfrac{c}{4}\,,\ \alpha_D^M = -\tfrac{c}{4}\,,\ \alpha_{V_1}^M = -\tfrac{c}{4}\,,\ \alpha_{V_2}^M = \alpha_S^M = 0$$

technicalities II

evaluating traces through spectral sums

spectrum of Laplacian on d-sphere

Laplacian Δ on d-sphere with radius a:

- eigenvalues: $\lambda_\ell^{(s)}$
- degeneracies: $M_\ell^{(s)}$

spin s	$\lambda_\ell^{(s)}$	$M_\ell^{(s)}$	
0	$\frac{1}{a^2}\ell(\ell+d-1)$	$\frac{(\ell+d-2)!}{(d-1)!\,\ell!}(2\ell+d-1)$	$\ell=0,1,\ldots$
$\frac{1}{2}$	$\frac{1}{a^2}(\ell^2+d\ell+\frac{d}{4})$	$2^{\lfloor d/2+1\rfloor}\frac{(\ell+d-1)!}{(d-1)!\,\ell!}$	$\ell=0,1,\ldots$
1	$\frac{1}{a^2}\left(\ell(\ell+d-1)-1\right)$	$\frac{(\ell+d-3)!}{(d-2)!(\ell+1)!}(2\ell+d-1)(\ell+d-1)\ell$	$\ell=1,2,\ldots$
2	$\frac{1}{a^2}\left(\ell(\ell+d-1)-2\right)$	$\frac{(d+1)(d-2)(l+d)(l-1)(2l+d-1)(l+d-3)!}{2(d-1)!(l+1)!}$	$\ell=2,3,\ldots$

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fermions obey Lichnerowicz formula:

$$\not\nabla^2=\Delta+\frac{1}{4}\bar{R}$$

spectral sums

Litim regulator

$$R_k(z) = (k^2 - z)\theta(k^2 - z)$$

traces become (finite) sums over degeneracies

$$S_d^{(s)}(N) \equiv \sum_{\ell=\ell_{\min}}^{N} M_\ell^{(s)} \,,$$

$$\widetilde{S}_d^{(s)}(N) \equiv \sum_{\ell=\ell_{\min}}^{N} \lambda_\ell^{(s)} \, M_\ell^{(s)}$$

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- sums can be done analytically
- convergence guaranteed

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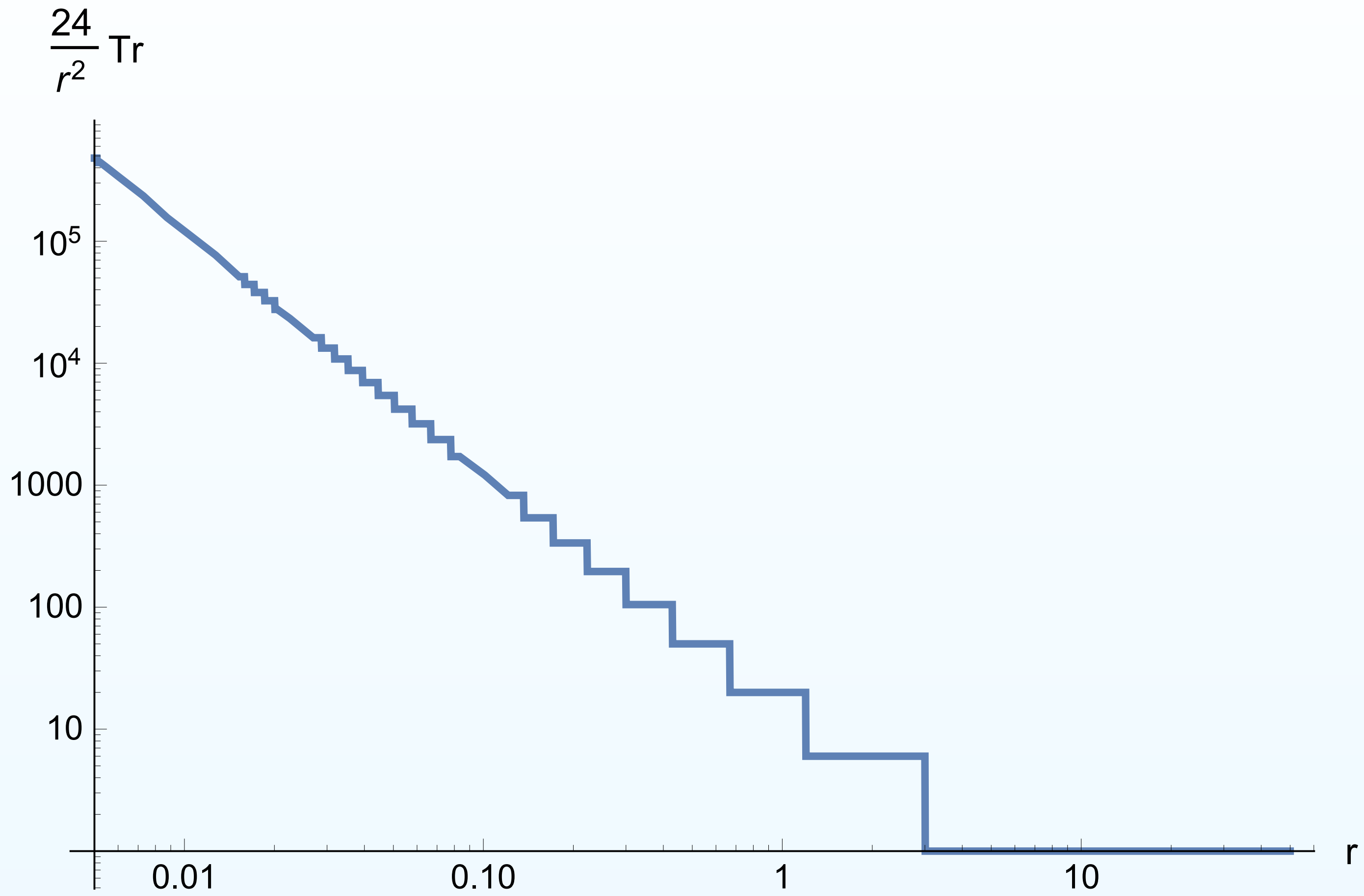
- discrete eigenvalue spectrum $\Rightarrow$ discontinuities in traces

spectral sums: discontinuities in scalar trace



- dimensionless curvature $r \equiv Rk^{-2}$

spectral sums: interpolation schemes

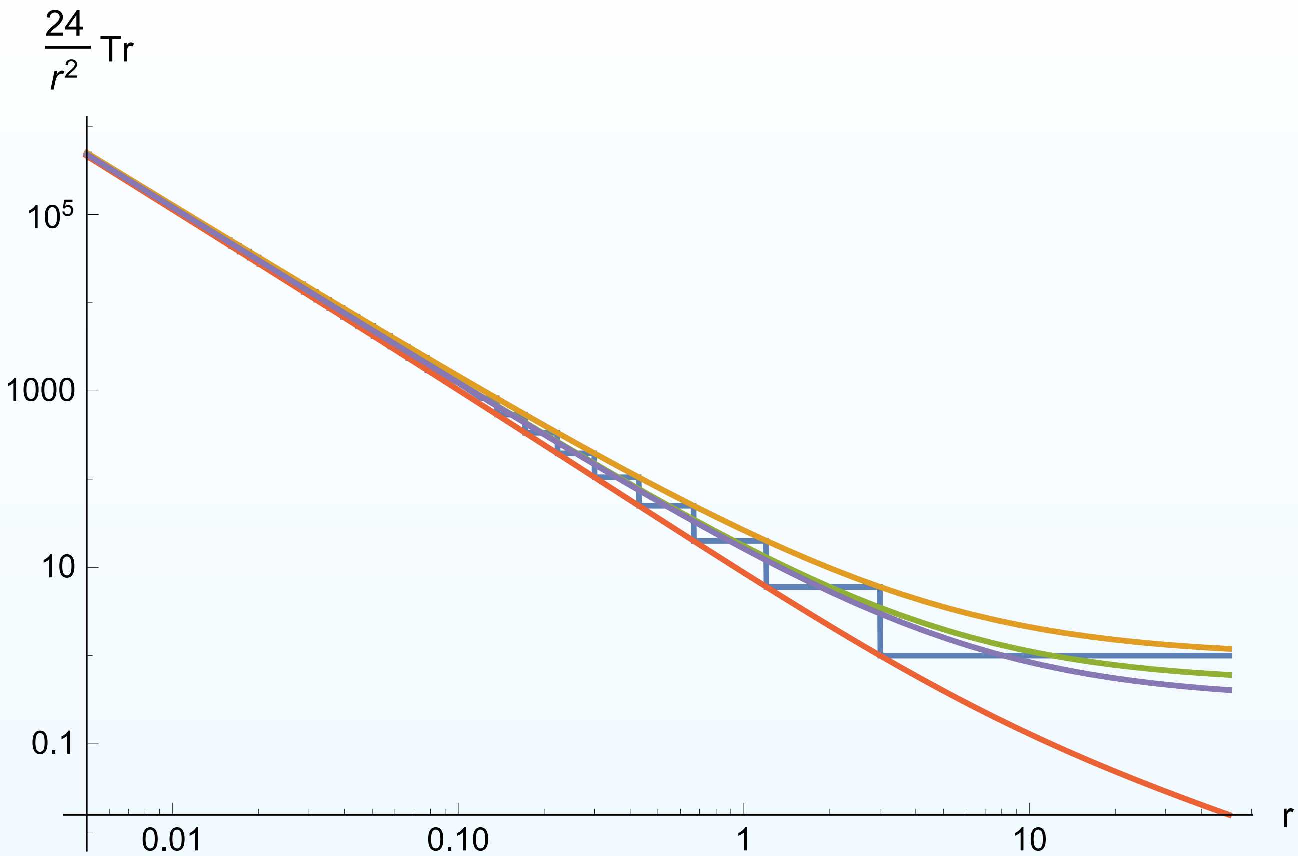
continuous flow equation $\Rightarrow$ treat N as continuous in k

interpolation schemes

- upper staircase
  - connects upper points of staircase
- lower staircase
  - connects lower points of staircase
- averaged interpolation
  - averages the upper and lower staircase
- optimized averaged interpolation
  - tailored to reproduce the early-time expansion (heat-kernel)

$\vdots$

spectral sums: interpolation schemes



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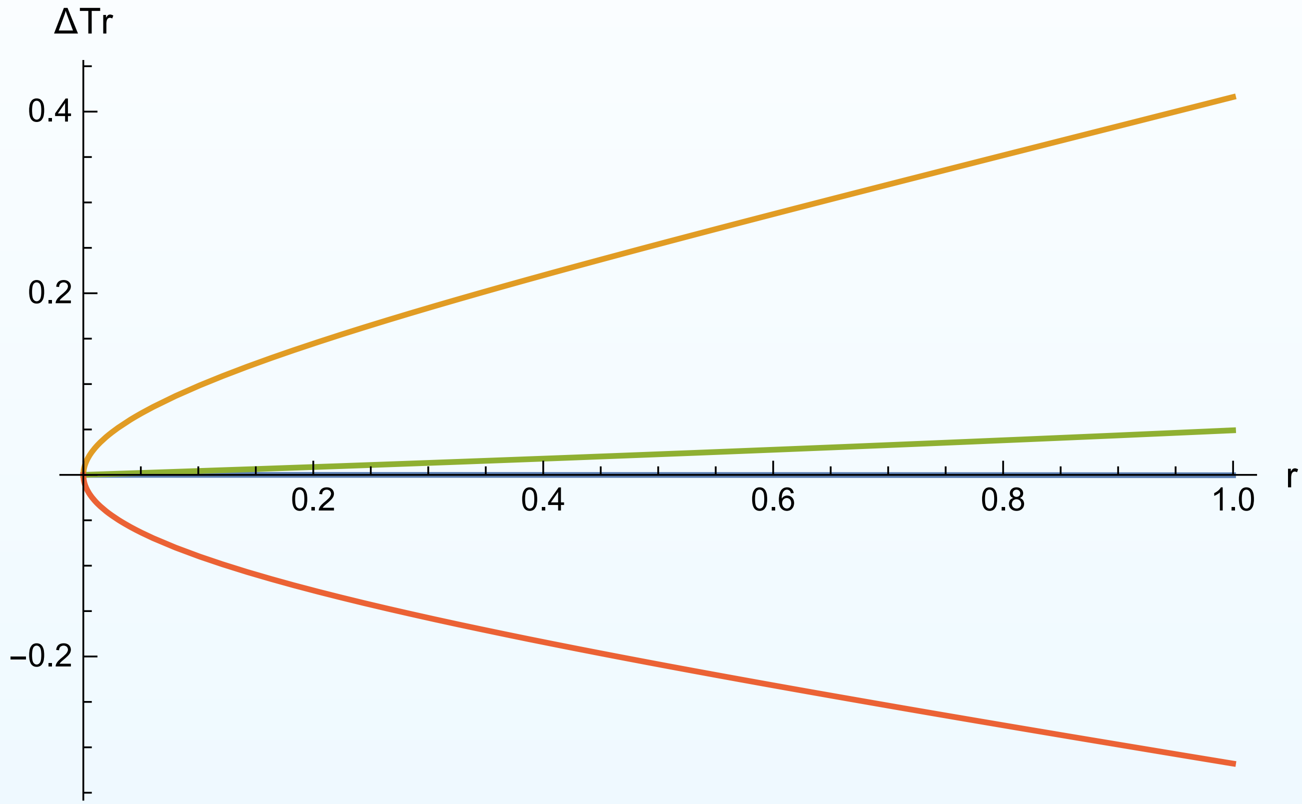
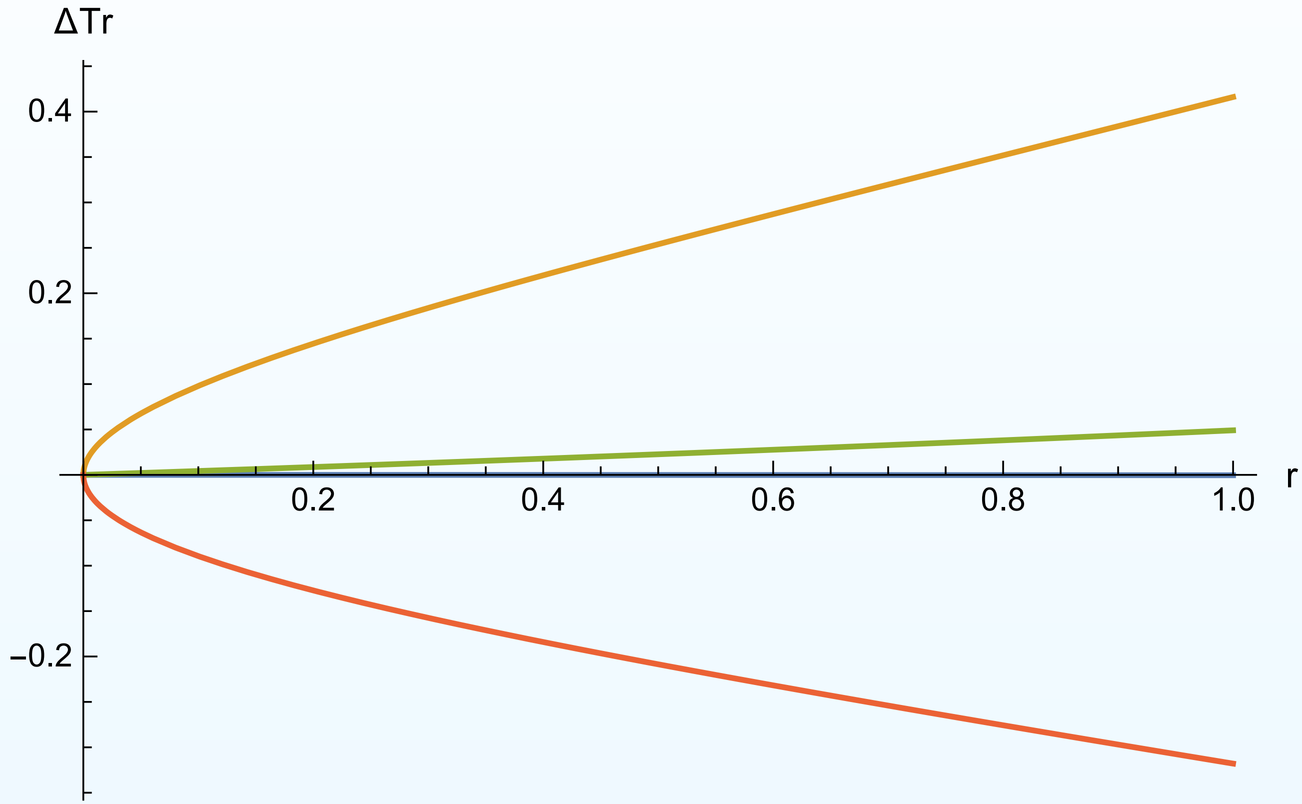
spectral sums: interpolation schemes comparison

deviation from the heat-kernel (Euler-MacLaurin summation):



spectral sums: interpolation schemes comparison

deviation from the heat-kernel (Euler-MacLaurin summation):



! averaged interpolation $\neq$ early-time expansion heat-kernel !

$f(R)$-gravity matter: partial differential equation

$$\dot\varphi + 4\varphi - 2r\varphi' = \mathcal{T}^{\rm TT} + \mathcal{T}^{\rm ghost} + \mathcal{T}^{\rm sinv} + \mathcal{T}^{\rm scalar} + \mathcal{T}^{\rm Dirac} + \mathcal{T}^{\rm vector}$$

gravitational sector:

[N. Ohta, R. Percacci, G.-P. Vacca, arXiv:1507.00968]

[N. Ohta, R. Percacci, G.-P. Vacca, arXiv:1511.09393]

$$\mathcal{T}^{\rm TT} = \frac{5}{2(4\pi)^2}\,\frac{1}{1+\left(\alpha_T^G+\frac{1}{6}\right)r}\left(1+\left(\alpha_T^G-\frac{1}{6}\right)r\right)\left(1+\left(\alpha_T^G-\frac{1}{12}\right)r\right)$$

$$+\frac{5}{12(4\pi)^2}\,\frac{\dot\varphi'+2\varphi'-2r\varphi''}{\varphi'}\left(1+\left(\alpha_T^G-\frac{2}{3}\right)r\right)\left(1+\left(\alpha_T^G-\frac{1}{6}\right)r\right)\;,$$

$$\mathcal{T}^{\rm sinv} = \frac{1}{2(4\pi)^2}\,\frac{\varphi''}{\left(1+\left(\alpha_S^G-\frac{1}{3}\right)r\right)\varphi''+\frac{1}{3}\varphi'}\left(1+\left(\alpha_S^G-\frac{1}{2}\right)r\right)\left(1+\left(\alpha_s^G+\frac{11}{12}\right)r\right)$$

$$+\frac{1}{12(4\pi)^2}\,\frac{\dot\varphi''-2r\varphi'''}{\left(1+\left(\alpha_S^G-\frac{1}{3}\right)r\right)\varphi''+\frac{1}{3}\varphi'}\left(1+\left(\alpha_S^G+\frac{3}{2}\right)r\right)\left(1+\left(\alpha_s^G-\frac{1}{3}\right)r\right)\left(1+\left(\alpha_S^G-\frac{5}{6}\right)r\right)$$

$$\mathcal{T}^{\rm ghost} = -\frac{1}{48(4\pi)^2}\,\frac{1}{1+(\alpha_V^G-\frac{1}{4})r}\left(72+18r(1+8\alpha_V^G)-r^2(19-18\alpha_V^G-72(\alpha_V^G)^2)\right)$$

$f(R)$-gravity matter: partial differential equation

$$\dot{\varphi} + 4\varphi - 2r\varphi' = \mathcal{T}^{\rm TT} + \mathcal{T}^{\rm ghost} + \mathcal{T}^{\rm sinv} + \mathcal{T}^{\rm scalar} + \mathcal{T}^{\rm Dirac} + \mathcal{T}^{\rm vector}$$

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$$\mathcal{T}^{\rm TT} = \frac{5}{2(4\pi)^2} \frac{1}{1+\left(\alpha_T^G + \frac{1}{6}\right) r} \left(1 + \left(\alpha_T^G - \frac{1}{6}\right) r\right) \left(1 + \left(\alpha_T^G - \frac{1}{12}\right) r\right)$$

$$+ \frac{5}{12(4\pi)^2} \frac{\dot{\varphi}' + 2\varphi' - 2r\varphi''}{\varphi'} \left(1 + \left(\alpha_T^G - \frac{2}{3}\right) r\right) \left(1 + \left(\alpha_T^G - \frac{1}{6}\right) r\right) ,$$

$$\mathcal{T}^{\rm sinv} = \frac{1}{2(4\pi)^2} \frac{\varphi''}{\left(1+\left(\alpha_S^G - \frac{1}{3}\right) r\right)\varphi'' + \frac{1}{3}\varphi'} \left(1 + \left(\alpha_S^G - \frac{1}{2}\right) r\right) \left(1 + \left(\alpha_s^G + \frac{11}{12}\right) r\right)$$

$$+ \frac{1}{12(4\pi)^2} \frac{\dot{\varphi}'' - 2r\varphi'''}{\left(1+\left(\alpha_S^G - \frac{1}{3}\right) r\right)\varphi'' + \frac{1}{3}\varphi'} \left(1 + \left(\alpha_S^G + \frac{3}{2}\right) r\right) \left(1 + \left(\alpha_s^G - \frac{1}{3}\right) r\right) \left(1 + \left(\alpha_S^G - \frac{5}{6}\right) r\right)$$

$$\mathcal{T}^{\rm ghost} = -\frac{1}{48(4\pi)^2} \frac{1}{1+(\alpha_V^G - \frac{1}{4})r} \left(72 + 18r(1 + 8\alpha_V^G) - r^2(19 - 18\alpha_V^G - 72(\alpha_V^G)^2)\right)$$

- partial differential equation of third order in r

$f(R)$-gravity matter: partial differential equation

$$\dot\varphi + 4\varphi - 2r\varphi' = \mathcal{T}^{\rm TT} + \mathcal{T}^{\rm ghost} + \mathcal{T}^{\rm sinv} + \mathcal{T}^{\rm scalar} + \mathcal{T}^{\rm Dirac} + \mathcal{T}^{\rm vector}$$

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[N. Ohta, R. Percacci, G.-P. Vacca, arXiv:1507.00968]

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$$+\frac{5}{12(4\pi)^2}\frac{\dot\varphi'+2\varphi'-2r\varphi''}{\varphi'}\left(1+\left(\alpha_T^G-\tfrac{2}{3}\right)r\right)\left(1+\left(\alpha_T^G-\tfrac{1}{6}\right)r\right),$$

$$\mathcal{T}^{\rm sinv} = \frac{1}{2(4\pi)^2}\frac{\varphi''}{\left(1+\left(\alpha_S^G-\frac{1}{3}\right)r\right)\varphi''+\frac{1}{3}\varphi'}\left(1+\left(\alpha_S^G-\tfrac{1}{2}\right)r\right)\left(1+\left(\alpha_s^G+\tfrac{11}{12}\right)r\right)$$

$$+\frac{1}{12(4\pi)^2}\frac{\dot\varphi''-2r\varphi'''}{\left(1+\left(\alpha_S^G-\frac{1}{3}\right)r\right)\varphi''+\frac{1}{3}\varphi'}\left(1+\left(\alpha_S^G+\tfrac{3}{2}\right)r\right)\left(1+\left(\alpha_s^G-\tfrac{1}{3}\right)r\right)\left(1+\left(\alpha_S^G-\tfrac{5}{6}\right)r\right)$$

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- partial differential equation of third order in r
- only derivatives of $\varphi(r) \Longrightarrow$ rhs independent of cosmological constant

$f(R)$-gravity matter: partial differential equation

$$\dot{\varphi} + 4\varphi - 2r\varphi' = \mathcal{T}^{\mathrm{TT}} + \mathcal{T}^{\mathrm{ghost}} + \mathcal{T}^{\mathrm{sinv}} + \mathcal{T}^{\mathrm{scalar}} + \mathcal{T}^{\mathrm{Dirac}} + \mathcal{T}^{\mathrm{vector}}$$

gravitational sector:

[N. Ohta, R. Percacci, G.-P. Vacca, arXiv:1507.00968]

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$$\mathcal{T}^{\mathrm{TT}} = \frac{5}{2(4\pi)^2} \frac{1}{1+\left(\alpha_T^G + \frac{1}{6}\right)r} \left(1 + \left(\alpha_T^G - \frac{1}{6}\right) r\right) \left(1 + \left(\alpha_T^G - \frac{1}{12}\right) r\right)$$
$$+ \frac{5}{12(4\pi)^2} \frac{\dot{\varphi}' + 2\varphi' - 2r\varphi''}{\varphi'} \left(1 + \left(\alpha_T^G - \frac{2}{3}\right) r\right) \left(1 + \left(\alpha_T^G - \frac{1}{6}\right) r\right) \,,$$

$$\mathcal{T}^{\mathrm{sinv}} = \frac{1}{2(4\pi)^2} \frac{\varphi''}{\left(1+\left(\alpha_S^G - \frac{1}{3}\right)r\right)\varphi'' + \frac{1}{3}\varphi'} \left(1 + \left(\alpha_S^G - \frac{1}{2}\right) r\right) \left(1 + \left(\alpha_S^G + \frac{11}{12}\right) r\right)$$
$$+ \frac{1}{12(4\pi)^2} \frac{\dot{\varphi}'' - 2r\varphi'''}{\left(1+\left(\alpha_S^G - \frac{1}{3}\right)r\right)\varphi'' + \frac{1}{3}\varphi'} \left(1 + \left(\alpha_S^G + \frac{3}{2}\right) r\right) \left(1 + \left(\alpha_S^G - \frac{1}{3}\right) r\right) \left(1 + \left(\alpha_S^G - \frac{5}{6}\right) r\right)$$

$$\mathcal{T}^{\mathrm{ghost}} = -\frac{1}{48(4\pi)^2} \frac{1}{1+(\alpha_V^G - \frac{1}{4})r} \left(72 + 18r(1 + 8\alpha_V^G) - r^2(19 - 18\alpha_V^G - 72(\alpha_V^G)^2)\right)$$

- partial differential equation of third order in r
- only derivatives of $\varphi(r) \Longrightarrow$ rhs independent of cosmological constant
- moving singularity at $\varphi'(r) = 0$

$f(R)$-gravity matter: partial differential equation

$$\dot{\varphi} + 4\varphi - 2r\varphi' = \mathcal{T}^{\rm TT} + \mathcal{T}^{\rm ghost} + \mathcal{T}^{\rm sinv} + \mathcal{T}^{\rm scalar} + \mathcal{T}^{\rm Dirac} + \mathcal{T}^{\rm vector}$$

gravitational sector:

[N. Ohta, R. Percacci, G.-P. Vacca, arXiv:1507.00968]

[N. Ohta, R. Percacci, G.-P. Vacca, arXiv:1511.09393]

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$$+\frac{5}{12(4\pi)^2}\,\frac{\dot{\varphi}'+2\varphi'-2r\varphi''}{\varphi'}\left(1+\left(\alpha_T^G-\frac{2}{3}\right)r\right)\left(1+\left(\alpha_T^G-\frac{1}{6}\right)r\right)\,,$$

$$\mathcal{T}^{\rm sinv} = \frac{1}{2(4\pi)^2}\,\frac{\varphi''}{\left(1+\left(\alpha_S^G-\frac{1}{3}\right)r\right)\varphi''+\frac{1}{3}\varphi'}\left(1+\left(\alpha_S^G-\frac{1}{2}\right)r\right)\left(1+\left(\alpha_S^G+\frac{11}{12}\right)r\right)$$

$$+\frac{1}{12(4\pi)^2}\,\frac{\dot{\varphi}''-2r\varphi'''}{\left(1+\left(\alpha_S^G-\frac{1}{3}\right)r\right)\varphi''+\frac{1}{3}\varphi'}\left(1+\left(\alpha_S^G+\frac{3}{2}\right)r\right)\left(1+\left(\alpha_S^G-\frac{1}{3}\right)r\right)\left(1+\left(\alpha_S^G-\frac{5}{6}\right)r\right)$$

$$\mathcal{T}^{\rm ghost} = -\frac{1}{48(4\pi)^2}\,\frac{1}{1+(\alpha_V^G-\frac{1}{4})r}\left(72+18r(1+8\alpha_V^G)-r^2(19-18\alpha_V^G-72(\alpha_V^G)^2)\right)$$

- partial differential equation of third order in r
- only derivatives of $\varphi(r) \Longrightarrow$ rhs independent of cosmological constant
- moving singularity at $\varphi'(r)=0$
- 3 endomorphism parameters

$f(R)$-gravity matter: partial differential equation

$$\dot{\varphi} + 4\varphi - 2r\varphi' = \mathcal{T}^{\rm TT} + \mathcal{T}^{\rm ghost} + \mathcal{T}^{\rm sinv} + \mathcal{T}^{\rm scalar} + \mathcal{T}^{\rm Dirac} + \mathcal{T}^{\rm vector}$$

matter sector

$$\mathcal{T}^{\rm scalar} = \frac{N_S}{2(4\pi)^2}\,\frac{1}{1+\alpha_S^M r}\,\left(1+\left(\alpha_S^M+\tfrac{1}{4}\right)r\right)\left(1+\left(\alpha_S^M+\tfrac{1}{6}\right)r\right),$$

$$\mathcal{T}^{\rm Dirac} = -\frac{2N_D}{(4\pi)^2}\,\left(1+\left(\alpha_D^M+\tfrac{1}{6}\right)r\right),$$

$$\mathcal{T}^{\rm vector} = \frac{N_V}{2(4\pi)^2}\left(\frac{3}{1+\left(\alpha_{V_1}^M+\frac{1}{4}\right)r}\left(1+\left(\alpha_{V_1}^M+\tfrac{1}{6}\right)r\right)\left(1+\left(\alpha_{V_1}^M+\tfrac{1}{12}\right)r\right)\right.$$
$$\left.-\frac{1}{1+\alpha_{V_2}^M r}\left(1+(\alpha_{V_2}^M+\tfrac{1}{2})r\right)\left(1+(\alpha_{V_2}^M-\tfrac{1}{12})r\right)\right)$$

$f(R)$-gravity matter: partial differential equation

$$\dot{\varphi} + 4\varphi - 2r\varphi' = \mathcal{T}^{\mathrm{TT}} + \mathcal{T}^{\mathrm{ghost}} + \mathcal{T}^{\mathrm{sinv}} + \mathcal{T}^{\mathrm{scalar}} + \mathcal{T}^{\mathrm{Dirac}} + \mathcal{T}^{\mathrm{vector}}$$

matter sector

$$\mathcal{T}^{\mathrm{scalar}} = \frac{N_S}{2(4\pi)^2} \frac{1}{1+\alpha_S^M r} \left(1 + \left(\alpha_S^M + \tfrac{1}{4}\right) r\right) \left(1 + \left(\alpha_S^M + \tfrac{1}{6}\right) r\right) ,$$

$$\mathcal{T}^{\mathrm{Dirac}} = -\frac{2N_D}{(4\pi)^2} \left(1 + \left(\alpha_D^M + \tfrac{1}{6}\right) r\right) ,$$

$$\mathcal{T}^{\mathrm{vector}} = \frac{N_V}{2(4\pi)^2} \Bigg(\frac{3}{1+\left(\alpha_{V_1}^M + \frac{1}{4}\right) r} \left(1 + \left(\alpha_{V_1}^M + \tfrac{1}{6}\right) r\right) \left(1 + \left(\alpha_{V_1}^M + \tfrac{1}{12}\right) r\right) - \frac{1}{1+\alpha_{V_2}^M r} \left(1 + (\alpha_{V_2}^M + \tfrac{1}{2}) r\right) \left(1 + (\alpha_{V_2}^M - \tfrac{1}{12}) r\right) \Bigg)$$

- independent of $\varphi(r)$

$f(R)$-gravity matter: partial differential equation

$$\dot{\varphi} + 4\varphi - 2r\varphi' = \mathcal{T}^{\mathrm{TT}} + \mathcal{T}^{\mathrm{ghost}} + \mathcal{T}^{\mathrm{sinv}} + \mathcal{T}^{\mathrm{scalar}} + \mathcal{T}^{\mathrm{Dirac}} + \mathcal{T}^{\mathrm{vector}}$$

matter sector:

$$\mathcal{T}^{\mathrm{scalar}} = \frac{N_S}{2(4\pi)^2} \frac{1}{1+\alpha_S^M r} \left(1 + \left(\alpha_S^M + \tfrac{1}{4}\right) r\right) \left(1 + \left(\alpha_S^M + \tfrac{1}{6}\right) r\right) ,$$

$$\mathcal{T}^{\mathrm{Dirac}} = -\frac{2N_D}{(4\pi)^2} \left(1 + \left(\alpha_D^M + \tfrac{1}{6}\right) r\right) ,$$

$$\mathcal{T}^{\mathrm{vector}} = \frac{N_V}{2(4\pi)^2} \left(\tfrac{3}{1+\left(\alpha_{V_1}^M + \frac{1}{4}\right) r} \left(1 + \left(\alpha_{V_1}^M + \tfrac{1}{6}\right) r\right) \left(1 + \left(\alpha_{V_1}^M + \tfrac{1}{12}\right) r\right) - \tfrac{1}{1+\alpha_{V_2}^M r} \left(1 + (\alpha_{V_2}^M + \tfrac{1}{2}) r\right) \left(1 + (\alpha_{V_2}^M - \tfrac{1}{12}) r\right) \right)$$

- independent of $\varphi(r)$
- 4 endomorphism parameters

$f(R)$-gravity matter: partial differential equation

$$\dot{\varphi} + 4\varphi - 2r\varphi' = \mathcal{T}^{\mathrm{TT}} + \mathcal{T}^{\mathrm{ghost}} + \mathcal{T}^{\mathrm{sinv}} + \mathcal{T}^{\mathrm{scalar}} + \mathcal{T}^{\mathrm{Dirac}} + \mathcal{T}^{\mathrm{vector}}$$

matter sector

$$\mathcal{T}^{\mathrm{scalar}} = \frac{N_S}{2(4\pi)^2} \frac{1}{1+\alpha_S^M r} \left(1 + \left(\alpha_S^M + \tfrac{1}{4}\right) r\right) \left(1 + \left(\alpha_S^M + \tfrac{1}{6}\right) r\right) ,$$

$$\mathcal{T}^{\mathrm{Dirac}} = -\frac{2N_D}{(4\pi)^2} \left(1 + \left(\alpha_D^M + \tfrac{1}{6}\right) r\right) ,$$

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- independent of $\varphi(r)$
- 4 endomorphism parameters
- fermions contribute with a first order polynomial in r

$f(R)$-gravity matter: partial differential equation

$$\dot{\varphi} + 4\varphi - 2r\varphi' = \mathcal{T}^{\mathrm{TT}} + \mathcal{T}^{\mathrm{ghost}} + \mathcal{T}^{\mathrm{sinv}} + \mathcal{T}^{\mathrm{scalar}} + \mathcal{T}^{\mathrm{Dirac}} + \mathcal{T}^{\mathrm{vector}}$$

matter sector

$$\mathcal{T}^{\mathrm{scalar}} = \frac{N_S}{2(4\pi)^2} \frac{1}{1+\alpha_S^M r} \left(1 + \left(\alpha_S^M + \tfrac{1}{4}\right) r\right) \left(1 + \left(\alpha_S^M + \tfrac{1}{6}\right) r\right) \,,$$

$$\mathcal{T}^{\mathrm{Dirac}} = -\frac{2N_D}{(4\pi)^2} \left(1 + \left(\alpha_D^M + \tfrac{1}{6}\right) r\right) ,$$

$$\mathcal{T}^{\mathrm{vector}} = \frac{N_V}{2(4\pi)^2} \Bigg(\frac{3}{1+\left(\alpha_{V_1}^M + \frac{1}{4}\right) r} \left(1 + \left(\alpha_{V_1}^M + \tfrac{1}{6}\right) r\right) \left(1 + \left(\alpha_{V_1}^M + \tfrac{1}{12}\right) r\right) - \frac{1}{1+\alpha_{V_2}^M r} \left(1 + (\alpha_{V_2}^M + \tfrac{1}{2}) r\right) \left(1 + (\alpha_{V_2}^M - \tfrac{1}{12}) r\right) \Bigg)$$

- independent of $\varphi(r)$
- 4 endomorphism parameters
- fermions contribute with a first order polynomial in r

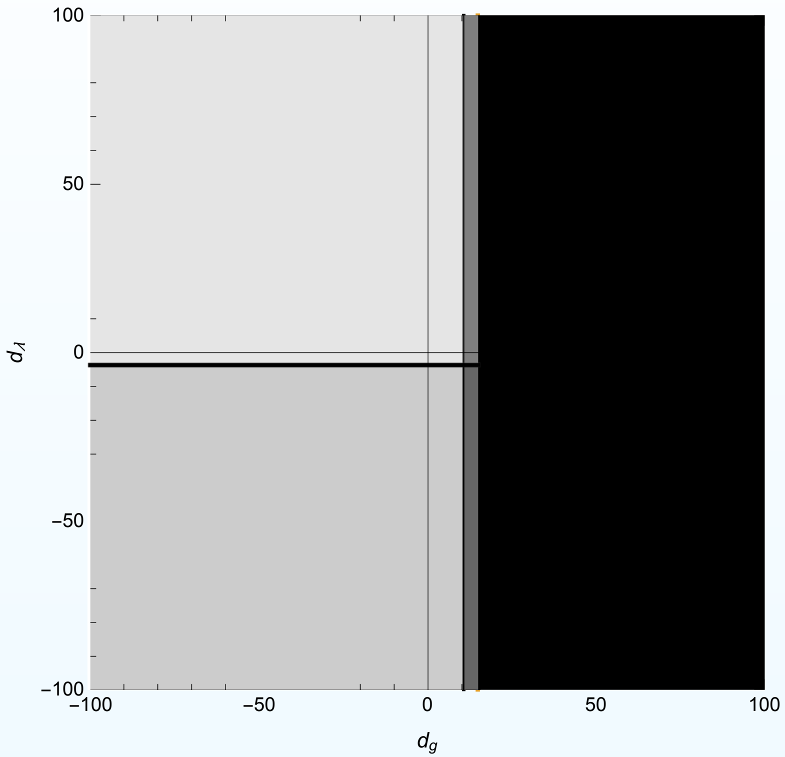
fixed functions: stationary (r-independent) solutions of partial differential equation

solutions I

polynomial expansion

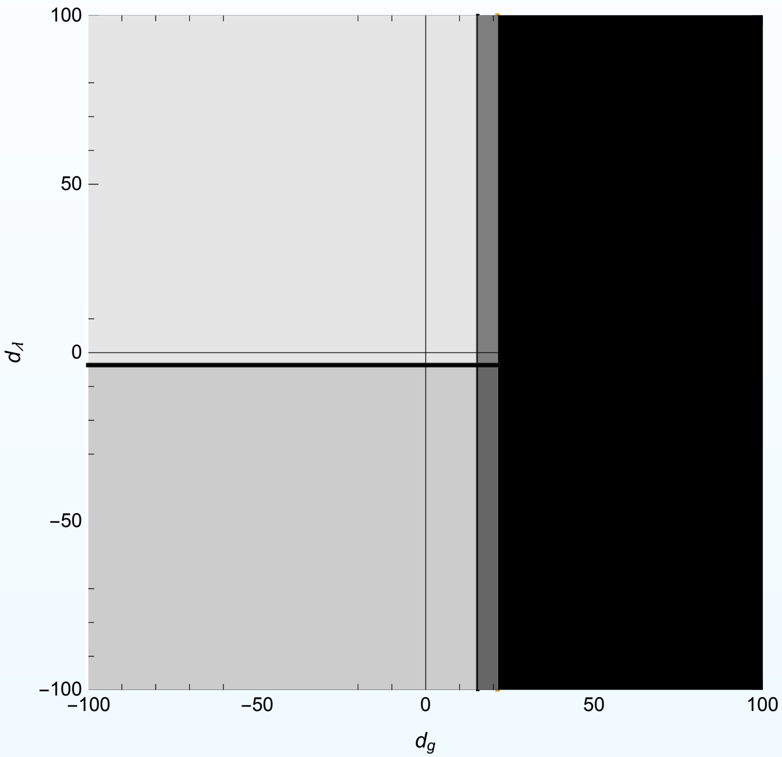
scanning for NGFPs: $N = 1$

$$d_\lambda = N_S + 2N_V - 4N_D$$



Type I

$$d_g = \frac{5}{4}N_S - \frac{5}{4}N_V - 2N_D$$

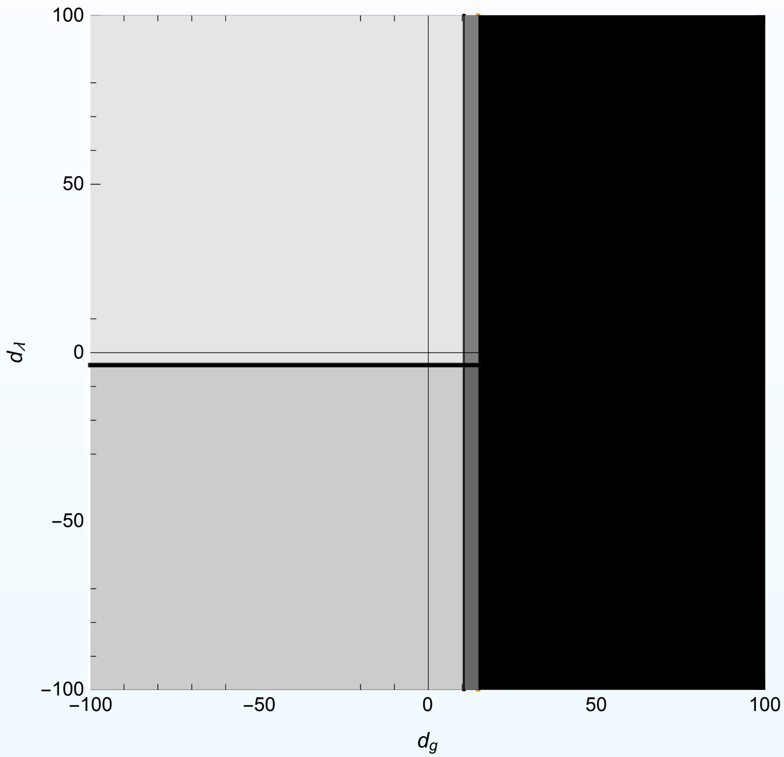


Type II

$$d_g = \frac{5}{4}N_S - \frac{7}{2}N_V + N_D$$

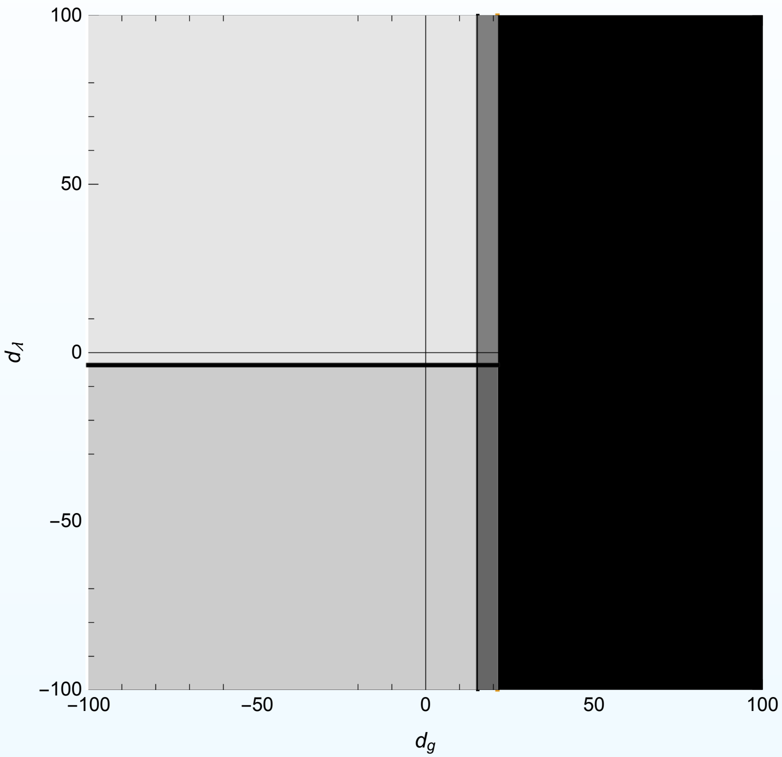
scanning for NGFPs: $N = 1$

$$d_\lambda = N_S + 2N_V - 4N_D$$



Type I

$$d_g = \frac{5}{4}N_S - \frac{5}{4}N_V - 2N_D$$



Type II

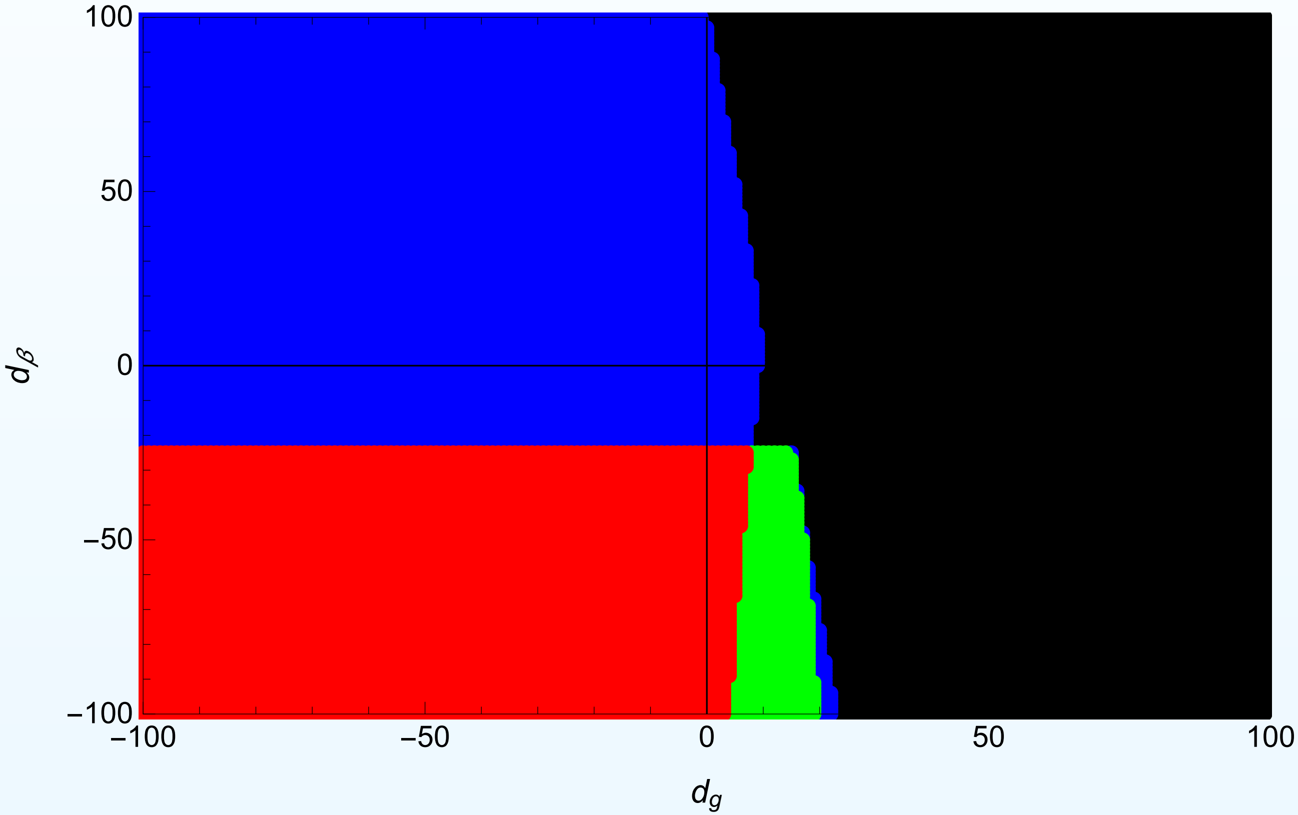
$$d_g = \frac{5}{4}N_S - \frac{7}{2}N_V + N_D$$

!!! upper bound on the number of scalars !!!

scanning for NGFPs: $N=2$, Type I

deformation parameters in beta functions for g^1, g^2:

$$d_g = \frac{5}{4}N_S - \frac{5}{4}N_V - 2N_D\,, \qquad d_\beta = N_S + 2N_V$$



black: 0 NGFP

blue: 1 NGFP

green: 2 NGFPs

red: 3 NGFPs