Controllable Instantons

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outline

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1. Nano-review of instantons
2. Micro-review of QCD conformal window
3. Micro-review of LISA (Litim-Sannino model, 1406.2337)
4. Instantons $\subset$ Banks-Zaks and LISA

Reference: 1802.10372

Nano-review of Instantons

- $Z[\mathcal{J}] = \int \mathcal{D}\phi \, e^{\mathrm{i}S[\phi;\lambda] + \mathcal{J}\phi}$
- $Z[\mathcal{J}] = \int \mathcal{D}\phi \; e^{\mathrm{i}\left[S[\bar{\phi}] + \frac{1}{2}\phi S^{(2)}[\bar{\phi}]\phi + \mathcal{O}(\phi^3)\right] + \mathcal{J}\phi}$
- Euclideanize $\implies$ if $\exists > 1$ degenerate vacuum then topologically stable solutions to Euclidean EOM may $\exists$
- These solutions are called pseudoparticles or instantons and should be included in the steepest descent method!

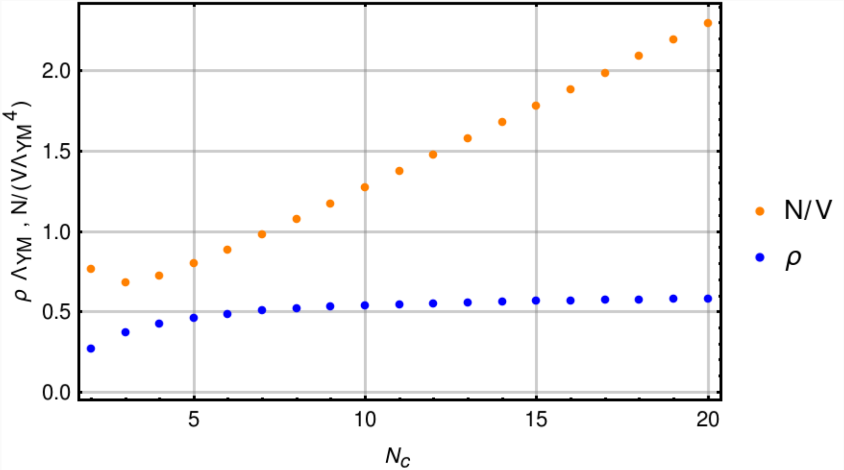
- from the POV of homotopy th. instantons are non-trivial maps labelled by an integer (topological charge)
- $n = \frac{g^2}{32\pi^2} \int_x G_a^{\mu\nu} \tilde{G}_{a\mu\nu}, \qquad n \in \mathbb{Z}$
- $A_\mu^a = \frac{2}{g} \eta_{a\mu\nu} \frac{(x-x_0)^\nu}{(x-x_0)^2+\rho^2}$, BPST instanton in regular gauge

- $\langle 0|0\rangle$ in presence of a single instanton is given by the following 1-loop instanton calculus result for an SU(N_c) pure YM:
- $W^{(1)} = C_c \int d^4x d\rho \rho^{-5} \left(\frac{8\pi^2}{g_0^2}\right)^{2N_c} \exp\left(-\frac{8\pi^2}{g_{1L}^2}\right)$
- this requires: IR regularization and renormalization
- including fermions this becomes:
- $W^{(1)} = C_{cf}\ m^{N_f} \int d^4x d\rho \rho^{-5+N_f} (\frac{8\pi^2}{g_0^2})^{2N_c} \exp(-\frac{8\pi^2}{g_{1L}^2})$
- caveat: this appears to vanish for $m \to 0$

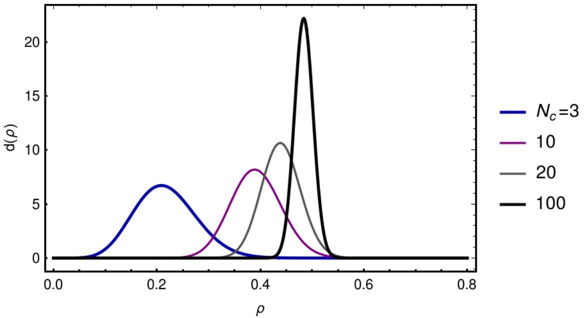
- $m = 0$ could be bypassed in real world through condensates:
- $m_{eff} = -\frac{2\pi^2\rho^2}{N_c}\langle\bar{q}(1+\gamma_5)q\rangle$, solves problem if $m \to m_{eff} \neq 0$
- IR div. could be bypassed through instanton interactions
- originally done empirically, later systematically in MFT
- description in terms of an instanton fluid, characterized by $\overline{\rho}$ and $\langle N/V\rangle$
- $\langle N/V\rangle$ directly related to pheno. observables, e.g.:
- gluon condensate, vacuum energy, χ_{top}, U(1) axial anomaly,..

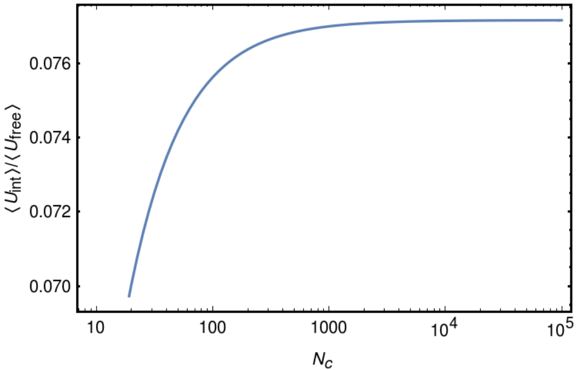
- MFT starts from the following ansatz:
- $\frac{Z}{Z_{ptb}}\Big|_{reg,1L} \geq \frac{1}{N_+!N_-!} \int \prod_i^{N_+ + N_-} d\gamma_i \; d(\rho_i) e^{-\beta(\bar{\rho}) U_{int}(\gamma_i)}$
- notation: $\beta(\rho) \equiv 8\pi^2/g^2(\rho)$
- only 2-body interaction: $U_{int}^{2-body}(\rho_1, \rho_2) = \gamma^2 \rho_1^2 \rho_2^2$
- notation: $\gamma^2 = \frac{27\pi^2}{4} \frac{N_c}{N_c^2 - 1}$
- variational principle gives us optimal instanton density profile:
- $\mu(\rho) = d(\rho) \exp\left(-\frac{\beta\gamma^2 N}{V} \overline{\rho^2} \rho^2\right)$

- from $\overline{\rho^2} = \frac{1}{\mu^0} \int d\rho \; \rho^2 \mu_(\rho)$ and the above $\mu(\rho)$ we get:
- $(\overline{\rho^2})^2 = \nu/(\beta\gamma^2 N/V)$, where $\nu = \frac{b-4}{2}$
- maximizing $\left.\frac{Z}{Z_{ptb}}\right|_{reg,1L}$ wrt N we also get:
- $\langle N \rangle = V\Lambda^4_{YM} \left(\Gamma(\nu) C_{cf} \tilde{\beta}^{2N_c} (\beta\gamma^2\nu)^{-\nu/2}\right)^{\frac{2}{\nu+2}}$
- we have to solve consistently for N/V and ρ!



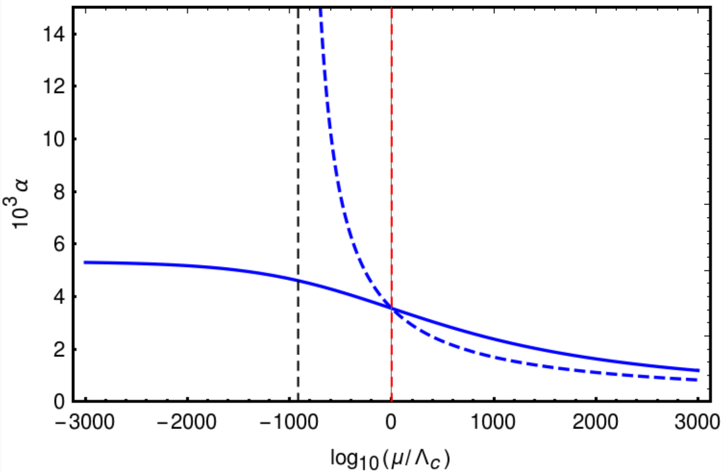
2.0
1.5
1.0
0.5
0.0
5
10
15
20
N_c
$\rho\ \Lambda_{YM}$, $N/(V/\Lambda_{YM}{}^4)$
N/V
ρ


d(ρ)
20
15
10
5
0
0.0
0.2
0.4
0.6
0.8
ρ
N_c=3
10
20
100


⟨$U_{\rm int}$⟩/⟨$U_{\rm free}$⟩
0.076
0.074
0.072
0.070
10
100
1000
10^4
10^5
N_c

Micro-review of QCD Conformal Window

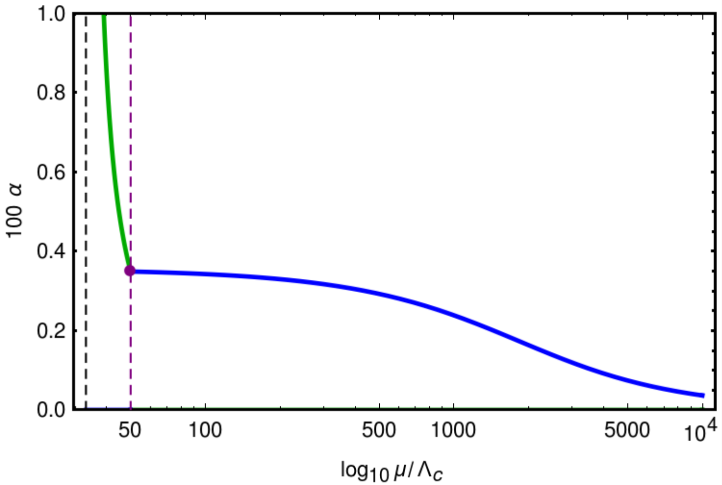
- consider QCD with N_c colors and N_f massless fermions
- let $N_f \lesssim \frac{11}{2} N_c$ so that asymptotic freedom is preserved
- let also $N_c, N_f \to \infty$ so that we go to Veneziano limit:
- expansion parameter: $\epsilon = \frac{N_f}{N_c} - \frac{11}{2} < 0$
- Banks and Zaks observed that beta functions admit an interacting IR fixed point (starting from 2 loops)!


14
12
10
8
6
4
2
0
$10^3\alpha$
−3000
−2000
−1000
0
1000
2000
3000
$\log_{10}(\mu/\Lambda_c)$

- "exact" 2-loop running (no deep-UV exansion)
- $\alpha(\mu) = \alpha_*/(1 + W(z(\mu)))$, where $\alpha = g^2 N_c/(4\pi)^2$
- $\mu\partial_\mu\alpha = -B\alpha^2 + C\alpha^3 \ \rightarrow \alpha_* = B/C$, $B = -\frac{4}{3}\epsilon > 0$, $C \simeq 25$
- $\partial_\alpha\beta_\alpha = 0$ for $\alpha = \frac{2}{3}\alpha_* \equiv \alpha_c \quad \Longrightarrow \ \mu(\alpha_c) \equiv \Lambda_c$
- $z(\rho) = \#(\rho\Lambda_c)^{-\theta_*}$; $\theta_* =$ e'value of the stab. matrix @ BZ FP

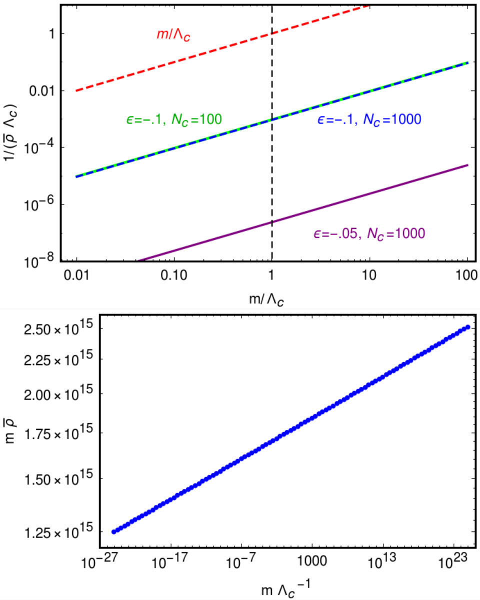
Instantons in Banks-Zaks model

- FP arbitrarily weakly interacting $\rightarrow$ controllable ϵ-expansion
- conformal IR dynamics; distinct from the χSB QCD scenario
- ptb control $\rightarrow$ include fermion effects @1L by including their contribution to the beta function of the gauge coupling
- we investigate mass-deformed perturbative CFT (massless fermions $\implies$ trivial vacuum-to-vaccum transition amplitude)


1.0
0.8
0.6
0.4
0.2
0.0
100 α
50
100
500
1000
5000
10^4
$\log_{10} \mu/\Lambda_c$

- our first task is to characterize the instanton fluid!
- we use "exact" 2-loop RG running in the master equation
- this provides us with RG-improved instanton density $d_{2L}(\rho)$
- $d_{2L}(\rho) = f_{c,f}(\bar{\rho}) m^{N_f} \rho^{N_f-5} (\rho\Lambda_c)^{\frac{1}{2}BN_c}$, where
- $f_{c,f}(\bar{\rho}) \sim log(M^2\overline{\rho^2})^{2N_c} W(\overline{\rho^2})^{N_c/(2\alpha_*)}$

- we now use variational principle to find $\overline{\rho^2}$, and arrive at
- $(\overline{\rho^2})^2 = \nu/(\beta\gamma^2 N/V)$, where $\nu = \frac{1}{2}(\frac{1}{2}BN_c + N_f - 4)$
- also need N/V, which we get by minimizing partition func.:
- $\langle N\rangle/(V\Lambda_c^4) = (\Gamma(\nu)(\frac{m}{\Lambda_c})^{N_f} f(\overline{\rho})(\beta\gamma^2\nu)^{-\nu/2})^{2/(2+\nu)}$
- these equations need to be solved consistently!
- we are looking for a simultaneous solution for N/V and $\overline{\rho}$



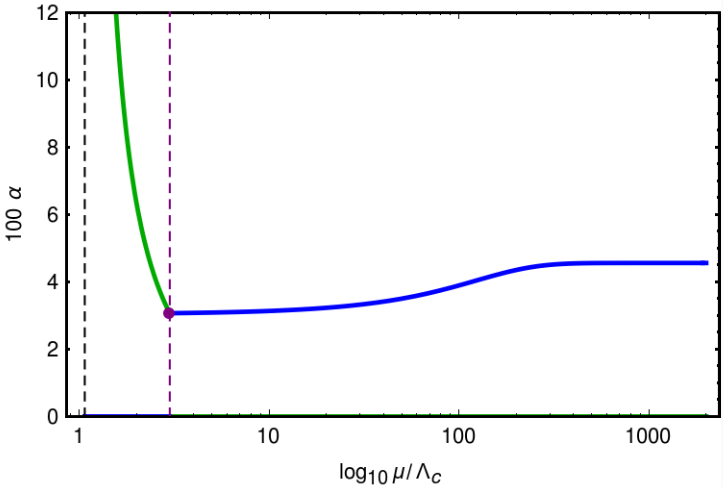
m/Λc
ϵ=-.1, Nc=100
ϵ=-.1, Nc=1000
ϵ=-.05, Nc=1000
1/(ρ̄ Λc)
1
0.01
10^-4
10^-6
10^-8
0.01
0.10
1
10
100
m/Λc
m ρ̄
2.50 × 10^15
2.25 × 10^15
2.00 × 10^15
1.75 × 10^15
1.50 × 10^15
1.25 × 10^15
10^-27
10^-17
10^-7
1000
10^13
10^23
m Λc^-1

- instantons found well within the pure YM regime
- calculation of instanton density and topological susceptibility closely follows pure YM
- we have the following choice of units: Λ_{YM} or Λ_c
- $\rho\Lambda_{YM}$ also ϵ-independent for a, m, N_c all fixed
- then N/V essentially depends only on the explicit $(\overline{\rho^2})^2$
- since $\chi_{top} \sim \langle N \rangle$ in units of Λ_{YM} $\chi_{top}(m, \epsilon)$ is constant

Micro-review of Litim-Sannino (LISA) Model

- $\mathcal{L}_{LISA} = \mathcal{L}_{YM} + \mathcal{L}_{\psi} + \mathcal{L}_{Yuk} + \mathcal{L}_{\phi}$, see: 1406.2337
- LO: $\mu\partial_{\mu}\alpha_g = -B\alpha_g^2$, where $B = -4\epsilon/3$
- opposite from BZ, $\epsilon = N_f/N_c - 11/2 > 0$, and $|\epsilon| \ll 1$
- @1L loss of asymptotic freedom, triviality problem
- NLO: $\mu\partial_{\mu}\alpha_g = -B\alpha_g^2 + C\alpha_g^3 \rightarrow \alpha_g^* = B/C$
- UV NGFP unphysical without Yukawa! (B flips sgn wrt BZ)
- including β_{yuk}@1L and β_{ϕ}@0L gives physical UV NGFP
- dubbed 2-1-0 scheme. FP confirmed to $\exists$ in 3-2-1 scheme!

- eff. flow of α_g along the separatrix is of the NLO form with:
- $C = -2/3(57 - 46\epsilon - 8\epsilon^2)/(13 + \epsilon)$, see 1501.03061
- analogous to the BZ running $\rightarrow \alpha_g(\mu) = \alpha_*/(1 + W(z(\mu)))$
- from the POV of α_g high and low energies "exchange" roles


12
10
8
6
4
2
0
100 α
1
10
100
1000
log10 μ/Λc

Instantons in LISA

- work in 2-1-0 scheme so $\mathcal{L}_{Yuk}$ and $\mathcal{L}_\phi$ drop out
- the remaining computation closely follows the BZ computation
- deep in perturbative regime so there are no condensates
- introduce mass operator for fermions
- obtain $\mu(\rho)$, $\overline{\rho}$, and $\langle N/V \rangle$ by analogy
- parameters $f(\overline{\rho})$, ν and β have to be appropriately modified

- using LISA beta functions instantons found at $\overline{\rho}^{-1} < m$
- next we look for instantons below m
- we find them and confirm that they're consistent
- in this respect situation with LISA is equivalent to Banks-Zaks
- instantons are always deep-IR objects
- (deep-IR is $\simeq$Gaussian for LISA but opposite for BZ)

recap & outlook

- we've investigated instanton dynamics for fundamental QFTs featuring either asymptotically safe or free dynamics
- we've determined the number of instantons per unit volume and instanton density as function of a common fermion mass
- its importance comes from direct connection with observables
- ambitious goal is to determine to which extent the instanton dynamics is responsible for the loss of conformality

Thank you!