Towards reconstructing the quantum effective action of gravity

Benjamin Knorr and Frank Saueressig, 1804.03846

Radboud University

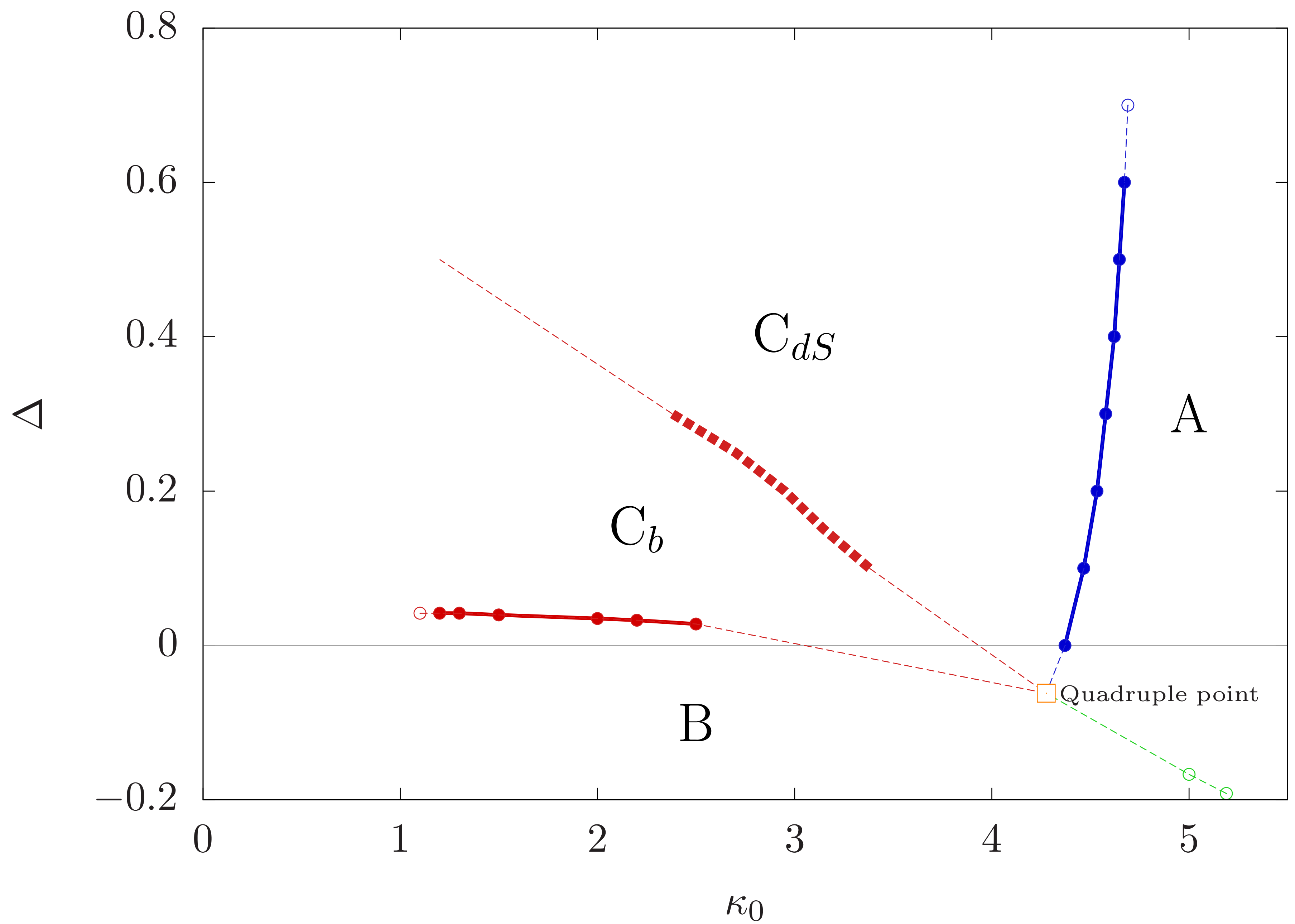
Outline

- CDT 101
- a matching template: reconstructing the effective action
- results and comparison to CDT data
- outlook

CDT 101 - some basics

- central idea in CDT: build spacetime out of fundamental building blocks (simplices)
- implement causal structure by foliation - Wick rotation well-defined
- calculate partition function as usual by Monte Carlo methods
- note: in general, spacelike and timelike links have different lengths

CDT 101 - phase diagram



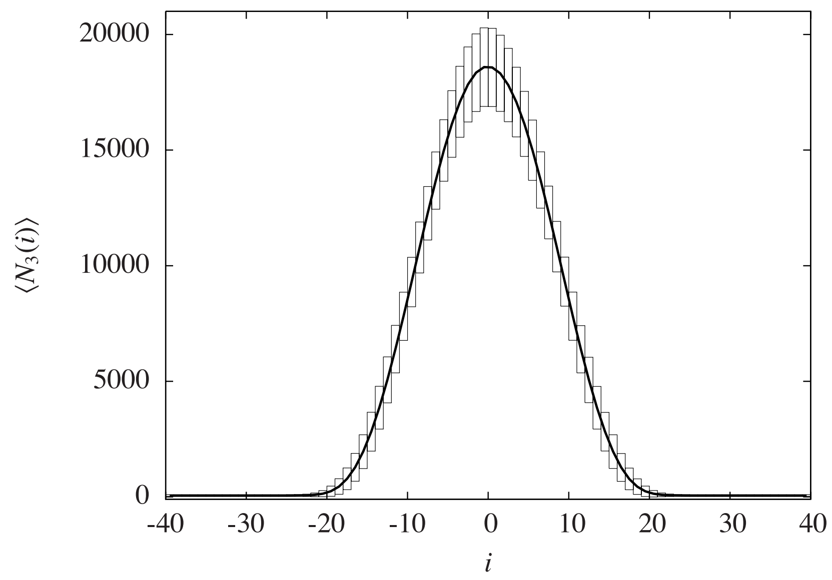
1610.05245

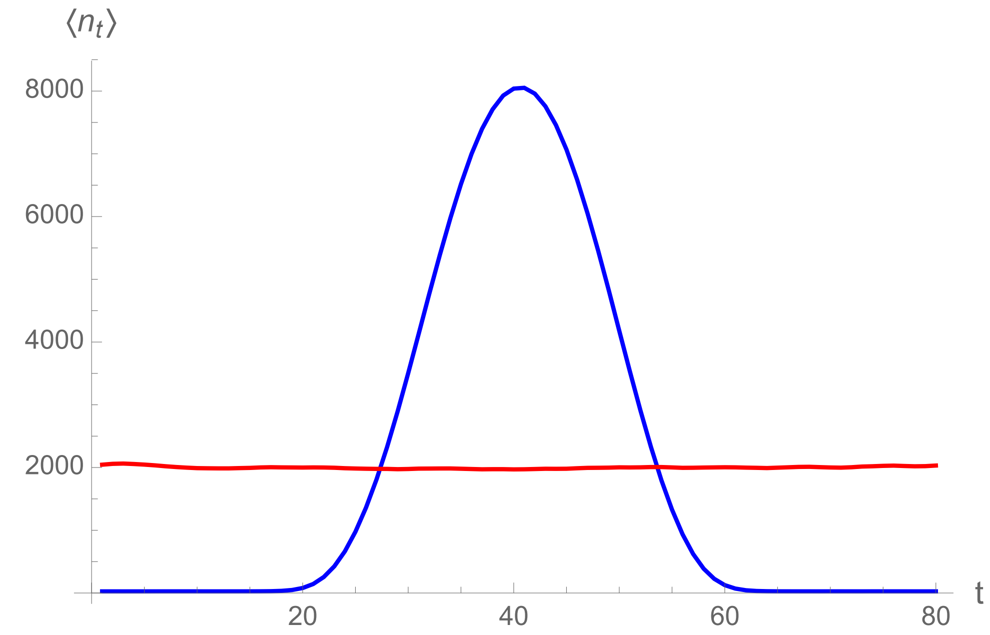
CDT 101 - some results

- in recent years, many results accumulated:
  - phase diagram with 4 different phases
  - study of spectral dimensions
  - autocorrelator of 3-volume fluctuations
  - effect of spatial topology (sphere vs. torus)

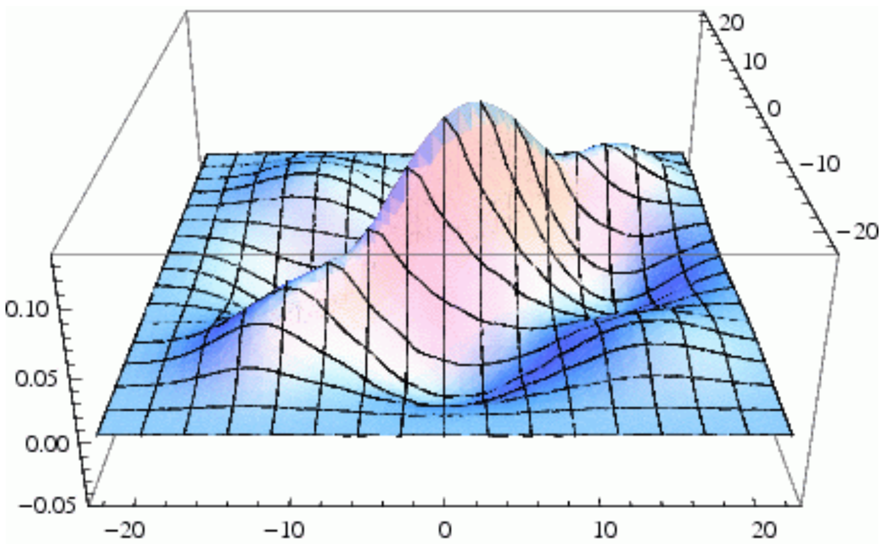
CDT 101 - 3-volume and fluctuations

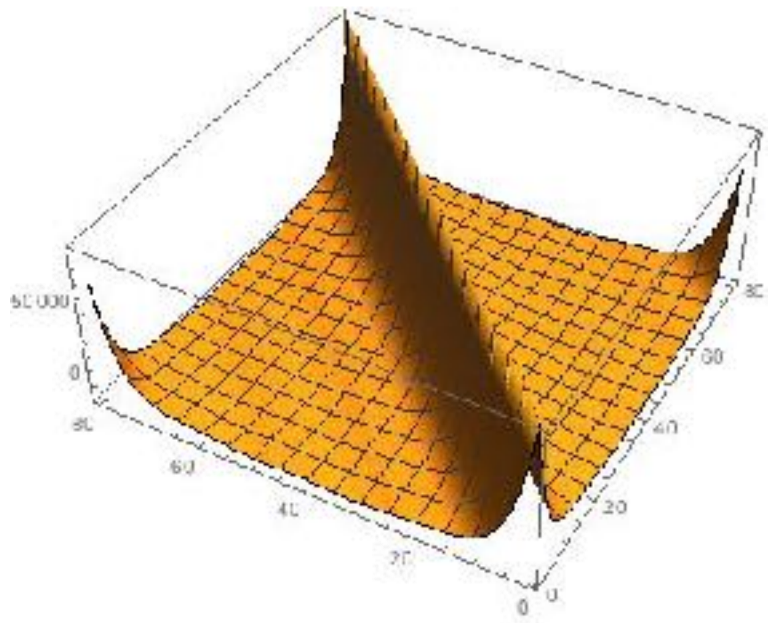
<V3(t)>





$<\delta V_3(t)\delta V_3(t')>$





S^1xS^3 0807.4481

S^1xT^3 1604.08786

Can we extract information on the effective action of gravity from these correlators?

Effective action - a matching template

- idea:
  - start with ansatz for effective action
  - calculate correlators measured in CDT
  - fit to data, extract parameters
  - if fit unsatisfactory: extend ansatz for effective action
- advantages:
  - EFT spirit, independent of UV completion
  - access to IR of quantum gravity

IR effects of quantum gravity

- graviton is supposedly massless
- massless particles are expected to give rise to non-local terms in the effective action
  - 2d QG:

$$\Gamma \propto \int \mathrm{d}^2x \sqrt{-g}\, R \frac{1}{\Box} R$$

  - QCD: non-local interactions correctly describe non-perturbative gluon propagator in IR

$$\Gamma_{\mathrm{nl}} \propto \mathrm{Tr} \int \mathrm{d}^4x\, F_{\mu\nu} \frac{1}{\Box} F^{\mu\nu}$$

- 4d: dynamical explanation for dark energy? (Maggiore-Mancarella model)

Reconstructing the effective action of QG

- inspired by results in Asymptotic Safety and phenomenology, we make the ansatz

$$\Gamma_{\rm QG} \approx \frac{1}{16\pi G_N} \int \mathrm{d}^4x \sqrt{-g} \left[-R + 2\Lambda - \frac{b^2}{6} \mathcal{R} \frac{1}{\Box^2} \mathcal{R} \right]$$

- at present: only one correlator available, cannot fix non-local term uniquely, for definiteness we consider

$$\mathcal{R} = R + \frac{1}{2} {}^{(3)}R$$

- effect of this term: gauge-invariant mass term for two-point function

- in general: inverse operator might contain endomorphism

Calculating the correlator

- CDT data: expectation value of geometry is sphere/torus - use fixed on-shell metric

$$\bar{g}_{\mu\nu} = \mathrm{diag}(1, a(t)^2 \bar{\sigma}_{ij}(x))$$

- use definition of 3-volume:

$$\delta V_3(t) = \frac{1}{2} \int \mathrm{d}^3x \, \sqrt{\bar{\sigma}} \, a(t)^3 \, \bar{\sigma}^{ij} \delta\sigma_{ij}$$

- insert into correlator:

$$\langle \delta V_3(t) \delta V_3(t') \rangle = \frac{1}{4} \int_x \int_{x'} \langle \hat{\sigma}(x,t) \hat{\sigma}(x',t') \rangle$$

$$\hat{\sigma} = a(t)^3 \bar{\sigma}^{ij} \delta\sigma_{ij}$$

Calculating the correlator II

$$\langle \delta V_3(t) \delta V_3(t') \rangle = \frac{1}{4} \int_x \int_{x'} \langle \hat{\sigma}(x,t) \hat{\sigma}(x',t') \rangle$$

- key ideas:
  - propagator admits expansion into product eigenfunctions:

$$\langle \hat{\sigma}(x,t) \hat{\sigma}(x',t') \rangle = \sum_{n,m} \frac{1}{\lambda_n^x + \lambda_m^t} \Phi_n^*(x) \Phi_n(x') \Psi_m^*(t) \Psi_m(t')$$

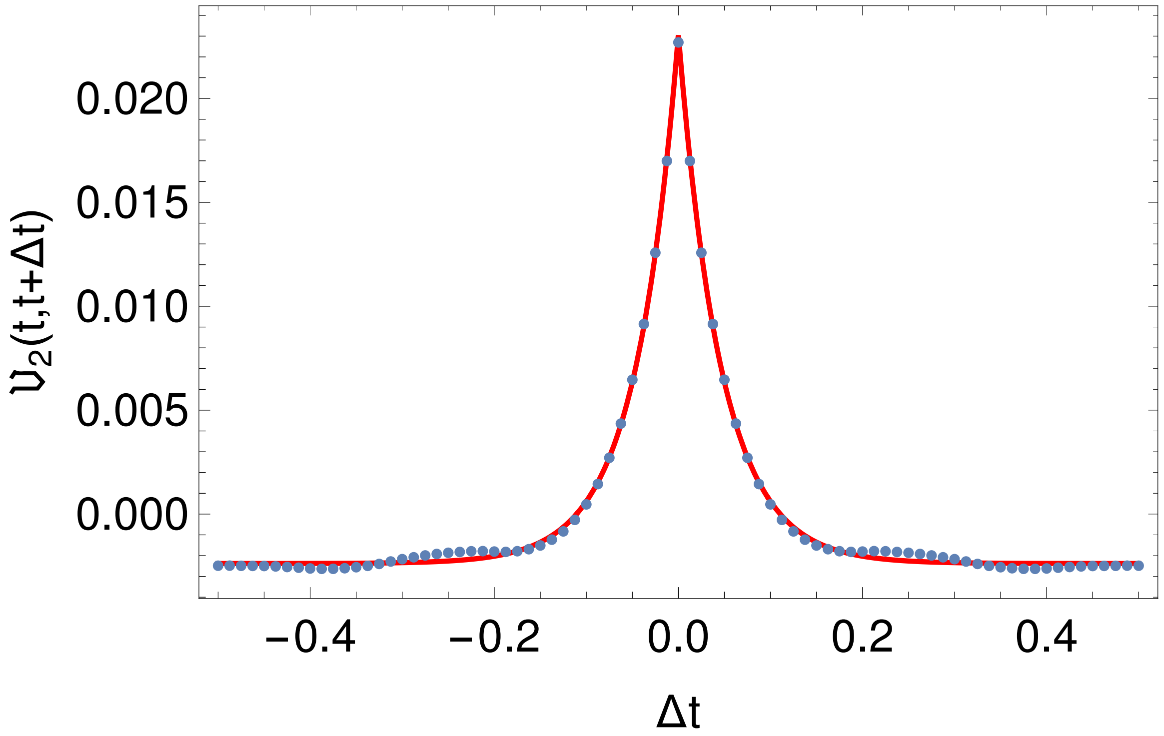
  - use orthogonality of spatial eigenfunctions - only need to know temporal spectrum and eigenfunctions
  - use ansatz for effective action to calculate spectrum and eigenfunctions
  - calculate (temporal) propagator

Calculating the correlator III

- real world check:
  - torus: can do calculation analytically to the end
  - sphere: analytical solution unknown
- general properties:
  - (potentially singular) Sturm-Liouville problem, depending on boundary conditions
  - general expectation: eigenvalues grow quadratically asymptotically, slow convergence of propagator

Comparison to CDT - torus

- due to symmetry, correlator only depends on time difference
- fitting the data:

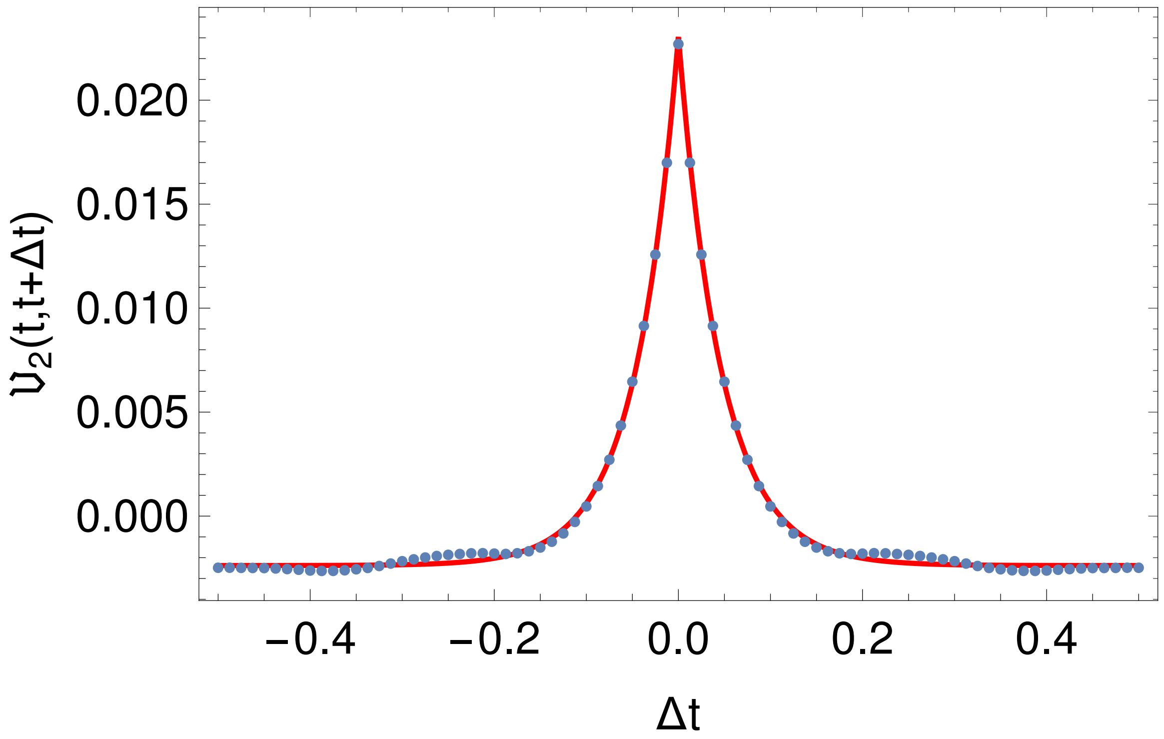


$$G_N = 0.14 a_{CDT}^2$$
$$b = 6.93/a_{CDT}$$

- fitting lowest eigenvalues gives lattice spacing:

$$r = 3.09 a_{CDT}$$

Comparison to CDT - torus - remarks



- non-zero value of b - evidence for non-local interactions from first principle CDT simulations!
- fit with b=0 disagrees *qualitatively*
- checked higher order curvature terms - completely subdominant (ca. 3 orders of magnitude smaller)

Comparison to CDT - sphere

- similar analysis already done in 0807.4481
- eigenproblem agrees (with b=0), thus compatible with data
- can extract Newton's constant by matching lowest eigenvalue:

$$G_N = 0.23 a_{CDT}^2$$

- relation between lattice spacing and physical radius agrees with relation on torus at same bare parameters

Quo vadis background independence?

- different values for coupling constants on different geometries - no background independence?
- three remarks:
  - on sphere, higher order operators contribute to what we called Newton's constant due to non-vanishing curvature
  - endomorphism necessary for well-defined non-local operator, at present not enough information to clearly resolve b on sphere
  - background independence in principle only necessary in continuum limit

Outlook: more correlators to investigate

- the more correlators, the better we can constrain the effective action
- two immediate candidates to extend the present analysis:
  - higher order three-volume autocorrelations:

$$\langle \delta V_3(t_1) \dots \delta V_3(t_n) \rangle$$

  - three-curvature correlations:

$$\langle \delta^{(3)}\mathcal{R}(t)\delta^{(3)}\mathcal{R}(t') \rangle$$

- preliminary insight: non-local operators are nasty…

Summary

- central idea: constrain effective action by correlators that can be calculated in different approaches
- CDT: evidence for non-local interactions which could explain dark energy dynamically
- future work: more correlators

Thank you for your attention!