Effective universality in quantum gravity

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Based on

Eichhorn, Labus, Pawlowski, MR: arXiv:1804.00012

Asymptotic Safety Seminar

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Key question 1: Slavnov-Taylor identities

- Gauge theories feature different *avatars* of the gauge coupling
- Marginal couplings exhibit two-loop universality
  (in gravity the R^2 and $R_{\mu\nu}^2$ couplings)
- Newton's coupling related to R is not marginal.
  The dynamical multi-graviton Newton's couplings
  - agree on a classical level
  - are related by Slavnov-Taylor identities in quantum gravity

Can these identities simplify?

And allow for approximate identifications of the avatars of a coupling?

Key question 2: Nielsen identities

- Background couplings are related by background diffeomorphism invariance
- Flow of background couplings is solely driven by dynamical couplings
- Background and dynamical couplings are related by Nielsen identities
- Previous works showed differences between background and dynamical couplings

What is the important information carried by the Nielsen identity?

Can we construct a truncation that captures the behaviour of the dynamical couplings but is similar to the background-field approximation?

Relations between the avatars of Newton's coupling

	Background diffeomorphism invariance		Background diffeomorphism invariance	
$\bar{G}$	$=$	$\bar{G}_{\bar{g}^n}$	$=$	$\bar{G}_{\bar{g}^n\varphi\varphi}$
$\bar{G} \sim \Gamma\big\vert_R$		$\bar{G}_{\bar{g}^n} \sim \frac{\delta^n}{\delta\bar{g}^n}\Gamma\big\vert_R$		$\bar{G}_{\bar{g}^n\varphi\varphi} \sim \frac{\delta^{n+2}}{\delta\bar{g}^n\delta\varphi^2}\Gamma\big\vert_{\varphi\Delta\varphi}$
		Nielsen identities $\updownarrow$		Nielsen identities $\updownarrow$
G_{h^m}	$\longleftrightarrow$	G_{h^n}	$\longleftrightarrow$	$G_{h^n\varphi\varphi}$
$G_{h^m} \sim \frac{\delta^m}{\delta h^m}\Gamma\big\vert_R$	Slavnov Taylor identities	$G_{h^n} \sim \frac{\delta^n}{\delta h^n}\Gamma\big\vert_R$	Slavnov Taylor identities	$G_{h^n\varphi\varphi} \sim \frac{\delta^{n+2}}{\delta h^n\delta\varphi^2}\Gamma\big\vert_{\varphi\Delta\varphi}$

Relations between the avatars of Newton's coupling

$\bar{G}$ $\quad$ Background diffeomorphism invariance $=$ $\quad$ $\bar{G}_{\bar{g}^n}$ $\quad$ Background diffeomorphism invariance $=$ $\quad$ $\bar{G}_{\bar{g}^n\varphi\varphi}$

$\bar{G} \sim \Gamma\big|_R$ $\qquad$ $\bar{G}_{\bar{g}^n} \sim \frac{\delta^n}{\delta \bar{g}^n}\Gamma\big|_R$ $\qquad$ $\bar{G}_{\bar{g}^n\varphi\varphi} \sim \frac{\delta^{n+2}}{\delta \bar{g}^n \delta\varphi^2}\Gamma\big|_{\varphi\Delta\varphi}$

Nielsen identities $\updownarrow$ $\quad$ Key question 2 $\quad$ Nielsen identities $\updownarrow$

G_{h^m} $\longleftrightarrow$ G_{h^n} $\longleftrightarrow$ $G_{h^n\varphi\varphi}$

$G_{h^m} \sim \frac{\delta^m}{\delta h^m}\Gamma\big|_R$ $\quad$ Slavnov Taylor identities $\quad$ $G_{h^n} \sim \frac{\delta^n}{\delta h^n}\Gamma\big|_R$ $\quad$ Slavnov Taylor identities $\quad$ $G_{h^n\varphi\varphi} \sim \frac{\delta^{n+2}}{\delta h^n \delta\varphi^2}\Gamma\big|_{\varphi\Delta\varphi}$

Key question 1

Avatars of Newton's coupling

- Gauge fixing and regularisation necessitate split of metric

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \sqrt{Z_h G_N}\, h_{\mu\nu} \tag{1}$$

- Effective action depends separately on $\bar{g}_{\mu\nu}$ and $h_{\mu\nu}$

$$\Gamma_k = \Gamma_k[\bar{g}_{\mu\nu}, h_{\mu\nu}, \varphi] \tag{2}$$

- Vertices depend on momentum configuration

$$\Gamma_k^{(n,m,l)} = \frac{\delta^{n+m+l}\Gamma_k[\bar{g}, h, \varphi]}{\delta\bar{g}^n(p_1,\dots)\delta h^m(\dots,p_i,\dots)\delta\varphi^l(\dots,p_{n+m+l})} \tag{3}$$

- Define dimensionless Newton's couplings $G_{\vec{n}} = G_{(n_{\bar{g}}, n_h, n_\varphi, \dots)}$ at a momentum symmetric point with projection on R or $\varphi\Delta\varphi$

Here: comparison between $G_{(n_h=3, n_\varphi=0)}$ and $G_{(n_h=1, n_\varphi=2)}$

Effective universality

What is effective universality:

- The mSTIs simplify such that there is an approximate equality between the avatars of Newton's coupling

It would provide:

- Consistent closure of the flow
- Justification of previous truncations

Where can we hope for effective universality:

- At $k \to 0$ for the avatars of Newton's coupling
- At $k \to \infty$ due to the scaling regime of the UV fixed point
- The λ_n and μ are special due to the convexity of the effective action

Setup: a scalar-gravity system

Einstein-Hilbert with N_s minimally coupled scalars:

$$S = \frac{1}{16\pi G_N} \int \mathrm{d}^4 x \sqrt{g}\,(2\Lambda - R) + S_{\mathrm{gf}} + S_{\mathrm{gh}} \\ + \frac{1}{2} \sum_{i=1}^{N_s} \int \mathrm{d}^4 x \sqrt{g} g^{\mu\nu} \partial_\mu \varphi^i \partial_\nu \varphi^i \tag{4}$$

- Flat background $\bar{g}_{\mu\nu} = \delta_{\mu\nu}$
- Landau gauge $\alpha = 0$ and $\beta = 1$
- Vertex expansion of effective action up to order three with couplings:

$$\underbrace{\bar{\lambda},\, \bar{G}}_{\bar{\Gamma}},\ \underbrace{\mu,\, \eta_h(p^2)}_{\Gamma^{(2h)}},\ \underbrace{\lambda_3,\ G_{(3,0)}}_{\Gamma^{(3h)}},\ \underbrace{G_{(1,2)}}_{\Gamma^{(h\varphi\varphi)}},\ \underbrace{\eta_\varphi(p^2)}_{\Gamma^{(c\bar{c})}},\ \underbrace{\eta_c(p^2)}_{\Gamma^{(2\varphi)}} \tag{5}$$

Bilocal flow equations

Parameterisation of flow of the three-graviton vertex
(transverse traceless & momentum symmetric point):

$$\partial_t \left[Z_h^{3/2}(p^2)\sqrt{\bar{G}_{(3,0)}(p^2)}(c_1 p^2 + c_2 \Lambda_3) \right] = Z_h^{3/2}(p^2)\mathrm{Flow}_{\mathrm{tt}}^{(3,0)}(p^2) \qquad (6)$$

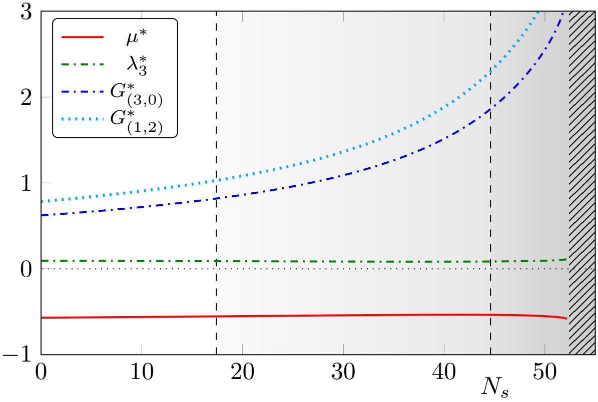
Bilocal flow equation ($\bar{G}_{(3,0)}(p^2) = \bar{G}_{(3,0)}$)

$$\begin{aligned} \beta_{G_{(3,0)}} = \; & \big(2 + 3\eta_h(k^2)\big)\, G_{(3,0)} - \frac{c_2}{c_1}\big(\eta_h(k^2) - \eta_h(0)\big)\,\lambda_3\, G_{(3,0)} \\ & + \frac{2}{c_1}\, G_{(3,0)}^{1/2}\Big(\mathrm{Flow}_{\mathrm{tt}}^{(3,0)}(k^2) - \mathrm{Flow}_{\mathrm{tt}}^{(3,0)}(0)\Big) \end{aligned} \qquad (7)$$

see also Christiansen, Knorr, Meibohm, Pawlowski, MR (2015)

Analogously for $\beta_{G_{(1,2)}}$

Fixed-point values of the fluctuation couplings



Very similar behaviour of $G^*_{(3,0)}$ and $G^*_{(1,2)}$

(Difference of $\sim 20\%$)

Momentum-dependent couplings at the bilocal FP

Three-graviton vertex

$$\partial_t \left[Z_h^{3/2}(p^2)\sqrt{\bar{G}_{(3,0)}(p^2)}(c_1 p^2 + c_2\Lambda_3) \right] = Z_h^{3/2}(p^2)\mathrm{Flow}^{(3,0)}_{\mathrm{tt}}(p^2) \qquad (8)$$

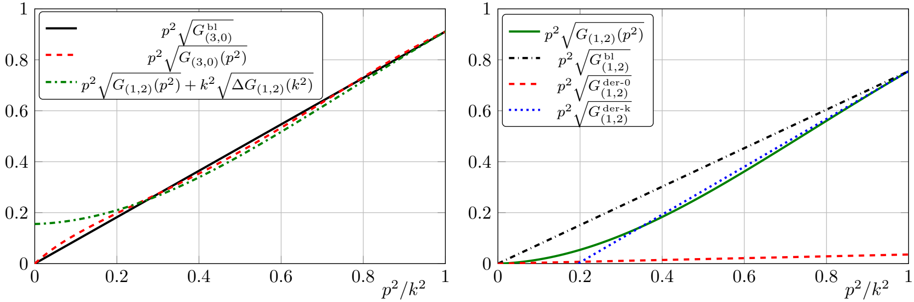
With $\partial_t G_{(3,0)}(p^2) = 0$ and $\lambda_3^* \approx 0$ at the FP

$$p^2 \sqrt{G_{(3,0)}(p^2)} \sim \left. \frac{\mathrm{Flow}^{(3,0)}_{\mathrm{tt}}(p^2) - \mathrm{Flow}^{(3,0)}_{\mathrm{tt}}(0)}{2 + 3\eta_h(p^2)} \right|_{G^*,\mu^*,\lambda_3^*} \qquad (9)$$

Analogously for the scalar-graviton vertex

$$p^2 \sqrt{G_{(1,2)}(p^2)} \sim \left. \frac{\mathrm{Flow}^{(1,2)}(p^2)}{2 + \eta_h(p^2) + 2\eta_\varphi(p^2)} \right|_{G^*,\mu^*,\lambda_3^*} \qquad (10)$$

Momentum dependence at the bilocal fixed point & $N_s = 0$

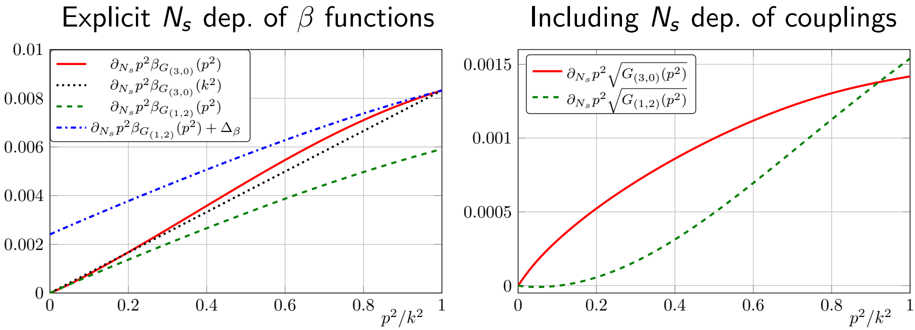


- $G^{\mathrm{bl}}_{(3,0)}$ matches $G_{(3,0)}(p^2)$ well

Christiansen, Knorr, Meibohm, Pawlowski, MR (2015)

- Remarkable agreement of slope of $p^2\sqrt{G_{(3,0)}(p^2)}$ and $p^2\sqrt{G_{(1,2)}(p^2)}$ for $p^2 \gtrsim 0.3\,k^2$
- $G^{\mathrm{der\text{-}0}}_{(1,2)}$ is off by a factor 20 compared to $G_{(1,2)}(p^2)$ at $p^2 = k^2$

N_s derivative of the momentum dependence



- N_s dep. from couplings and explicit N_s dep. almost cancels
- Hint towards N_s-independent FP values?
- $\partial_{N_s} p^2 \beta_{G_{(3,0)}}(p^2)$ and $\partial_{N_s} p^2 \beta_{G_{(1,2)}}(p^2)$ do not agree well
- Hint towards higher-derivative operators?

Effective universality beyond the UV fixed point

Define measure for breaking of effective universality

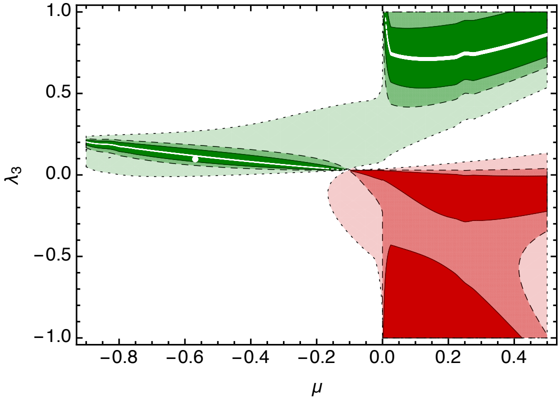
$$\varepsilon(G,\mu,\lambda_3,N_s) = \left|\frac{\Delta\beta_{G_{(3,0)}} - \Delta\beta_{G_{(1,2)}}}{\Delta\beta_{G_{(3,0)}} + \Delta\beta_{G_{(1,2)}}}\right|_{G_{\vec{n}}=G} \tag{11}$$

with

$$\Delta\beta_{G_{(i,j)}} = \beta_{G_{(i,j)}} - 2G_{(i,j)} \tag{12}$$

Properties:

- $\varepsilon = 0$ is effective universality (in the bilocal approximation)
- Compares one-loop coefficients in the limit $G \to 0$
- For $\varepsilon < 1$ ($\varepsilon > 1$) the $\Delta\beta_{G_{(i,j)}}$ have the same (different) sign



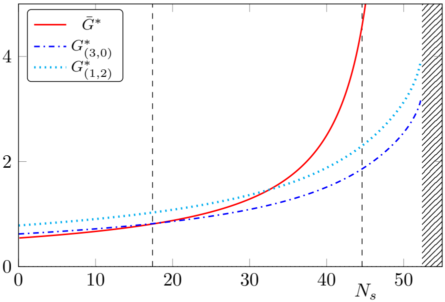
- Input: $\varepsilon(G \to 0, N_s = 1)$
- Dark colours: $\varepsilon < \frac{1}{5}$
- White line: $\varepsilon = 0$
- White dot: bilocal UV-FP
- Green: $\Delta\beta < 0$
- Red: $\Delta\beta > 0$

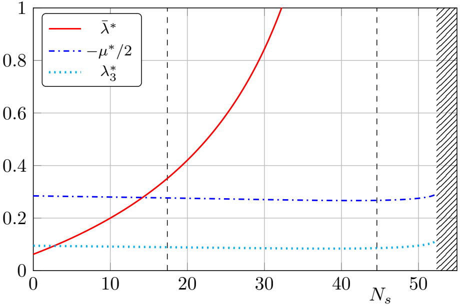
Key result 1

mSTIs between $G_{(3,0)}$ and $G_{(1,2)}$ seem to simplify at the UV-FP

Effective universality for the background couplings

Background couplings $\bar{G}^*$ & $\bar{\lambda}^*$ evaluated on the fluctuation couplings:



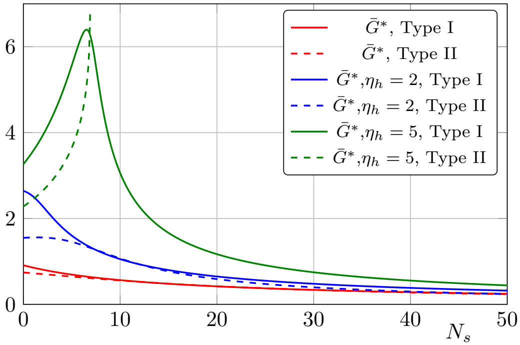


No effective universality with N_s, but qualitative agreement remains

Key result 2

mNIs seem not simplify at the UV fixed point

The background-field approximation and hybrids



- N_s dependence in background-field approximation maximally violates effective universality (due to $\bar{\lambda}$ threshold effects)
- N_s dependence in hybrid computations agrees qualitatively (triggered by large η_h)

Level-one improvement

- Use Nielsen identity to get closer to fluctuation system

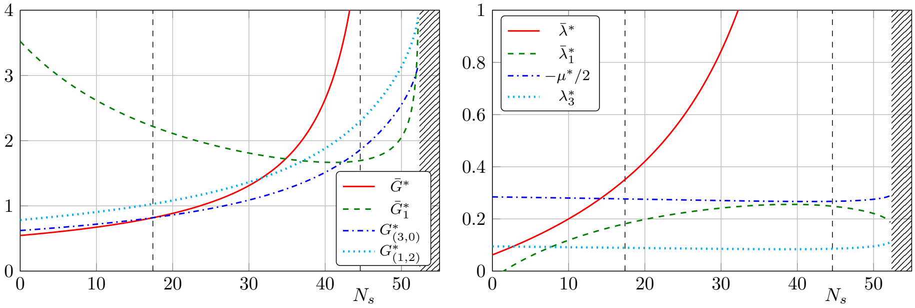
$$\frac{\delta\Gamma_k}{\delta\bar{g}_{\mu\nu}} - \frac{\delta\Gamma_k}{\delta h_{\mu\nu}} = \frac{1}{2}\mathrm{Tr}\left[\frac{1}{\sqrt{\bar{g}}}\frac{\delta\sqrt{\bar{g}}R_k}{\delta\bar{g}_{\mu\nu}}\, G_k\right] + \text{gauge-term} \tag{13}$$

- Approximation to fluctuation two-point function

$$\frac{\delta^2\Gamma_k}{\delta h_{\mu\nu}\delta h_{\rho\sigma}} \approx \frac{\delta^2\Gamma_k}{\delta\bar{g}_{\mu\nu}\delta\bar{g}_{\rho\sigma}} - \frac{1}{2}\frac{\delta}{\delta\bar{g}_{\rho\sigma}}\mathrm{Tr}\left[\frac{1}{\sqrt{\bar{g}}}\frac{\delta\sqrt{\bar{g}}R_k}{\delta\bar{g}_{\mu\nu}}\, G_k\right] \tag{14}$$

- Fully computable with heat-kernel methods
- Leads to flows for the level-one couplings $\bar{G}_1$ and $\bar{\lambda}_1$

Level-one improvement



- $\bar{G}_1^*$ appears even less universal than $\bar{G}^*$
- Improvement for $\bar{\lambda}_1^*$ compared to $\bar{\lambda}^*$
- Level-one improvement or our approximation to the NI is not sufficient

Outlook: more avatars of Newton's coupling

So far comparison between $G_{(3,0)} = G_3$ and $G_{(1,2)} = G^{\varphi}_{(1,2)}$

There are much more avatars of Newton's coupling

- G_4 difficult due to large R^2 contribution

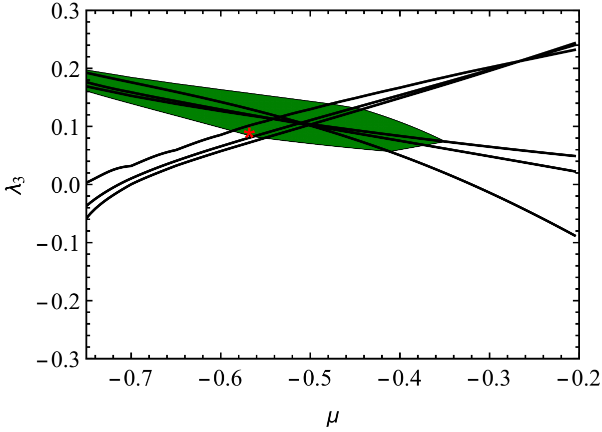
Denz, Pawlowski, MR (2016)

- More avatars from matter-graviton vertices:
  $G^{\psi}_{(1,2)}$, $G^{a}_{(1,2)}$, ...

Eichhorn, Lippoldt, Pawlowski, MR, Schiffer (in preparation)

Outlook: more avatars of Newton's coupling

All $\varepsilon = 0$ lines between G_3, $G^{\varphi}_{(1,2)}$, $G^{\psi}_{(1,2)}$, $G^{a}_{(1,2)}$ & unified $\varepsilon < \frac{1}{5}$ area



All avatars of the dynamical Newton's coupling feature effective universality!

Eichhorn, Lippoldt, Pawlowski, MR, Schiffer (in preparation)

Summary

Results:

- Effective universality is present at the UV fixed point for the avatars of the dynamical Newton's coupling
- N_s dependence weakens effective universality slightly
- No effective universality for background couplings, but qualitative agreement remains

Consequences:

- Justification for previous truncations with equated Newton's couplings
- Guiding principle for future truncations
- Indication for physical nature of UV fixed point