Asymptotic safety and field parametrization dependence in the $f(R)$ truncation

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Motivation

Asymptotically Safe Quantum Gravity:

- Relies on the existence of nontrivial fixed points in the RG-flow;
- We can control UV divergences with the idea of non-perturbative renormalization;
- Evidences for the existence of FP depends on non-perturbative calculations
  $\rightarrow$ Functional Renormalization Group is the usual framework;
  $\rightarrow$ Some scheme of approximation is still necessary $\sim$ Truncations;

Possible Ambiguities in ASQG:

- Standard QFT quantization of gravity is constructed upon several ambiguities
  $\rightarrow$ Field parametrization, Gauge fixing choice and so on...
- Such ambiguities may affect the behavior of off-shell quantities (*e.g.* beta functions);
- Important question: How the structure of FP can be affected?

Field Parametrization in Quantum Gravity

Background Field Method × Field Parametrization

- The background field method is very a useful tool in order to track diffeomorphism invariance in our calculations;
  → It requires the introduction of a non-dynamical background metric $\bar{g}_{\mu\nu}$;
  → We perform the quantization of metric fluctuations $h_{\mu\nu}$;

- How should we decompose the physical metric in terms of $\bar{g}_{\mu\nu}$ and $h_{\mu\nu}$?

  ◆ Linear parametrization: $g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$;

  ◆ Exponential parametrization: $g_{\mu\nu} = \bar{g}_{\mu\alpha}[e^{h}]^{\alpha}{}_{\nu}$;

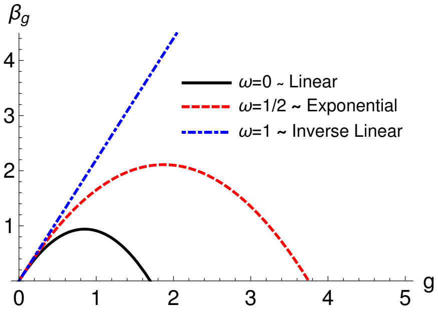
  ◆ Inverse linear parametrization $g^{\mu\nu} = \bar{g}^{\mu\nu} - h^{\mu\nu}$;

- How different choices of parametrization can affect the structure of FP?
  [Gies, Knorr and Lippoldt, PRD 92 (2015) 084020]

A simple example of parametrization dependence:

One-loop beta-function in EH quantum gravity (without C.C.) with the so-called interpolating parametrization

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu} + \omega\, h_{\mu\alpha} h^{\alpha}{}_{\nu} \tag{1}$$



→ No interacting fixed point for $\omega = 1$.

Field Parametrization in $f(R)$-Truncation

Our goal:

Investigate how different choices of field parametrization can affect the structure of fixed points in Asymptotically Safe Quantum Gravity.

Choice of truncation:

$$\Gamma_k = \int_x \sqrt{g}\, f_k(R) + \Gamma_{gf} + \Gamma_{FP}. \tag{2}$$

- Gauge fixing contribution:

$$\Gamma_{gf} = \frac{Z_\alpha}{2\,\alpha} \int_x \sqrt{\bar{g}} \left[\bar{\nabla}_\mu h^{\mu\nu} - \frac{1 + (1 + d\, m)\beta}{d} \bar{\nabla}^\nu h \right]^2. \tag{3}$$

- Faddeev-Popov sector:

$$\Gamma_{FP} = \int_x \sqrt{\bar{g}}\, \bar{C}^\mu \left[\bar{g}_{\mu\nu} \bar{\nabla}^2 + \left(1 - 2\frac{1+\beta}{d} \right) \bar{\nabla}_\mu \bar{\nabla}_\nu + \bar{R}_{\mu\nu} \right] C^\nu. \tag{4}$$

Choice of field parametrization:

$$g_{\mu\nu} =(\bar{g}_{\mu\nu} + h_{\mu\nu})(1 + m\, h) + \omega\, h_{\mu\alpha} h^{\alpha}{}_{\nu} + \\ + \frac{1}{2}\bar{g}_{\mu\nu}\left[m\,(2\omega - 1) h_{\alpha\beta} h^{\alpha\beta} + m^2 h^2\right]; \tag{5}$$

- This kind of field parametrization was recently employed in the investigation of 1-loop divergences in the context of Einstein-Hilbert, Higher-Derivative and $f(R, R^2_{\mu\nu})$ theories; [Ohta, Percacci and Pereira, JHEP 06 (2016) 115; EPJC 77 (2017) 611; PRD 97 (2018) 104039]

- For $m \neq 0$, the actual dynamical variable correspond to a tensorial density constructed with the full metric;

- ω works as an interpolating parameter:

  - ◆ $\omega = 0$ and $m = 0 \rightarrow$ Linear parametrization;

  - ◆ $\omega = 1/2$ and $m = 0 \rightarrow$ Exponential parametrization;

  - ◆ $\omega = 1$ and $m = 0 \rightarrow$ Inverse linear parametrization;

Some techinical details:

- Gauge fixing contribution:
  $\rightarrow$ All calculations were performed with Landau gauge ($\alpha \rightarrow 0$);
  $\rightarrow$ Computations were done with $\beta = 0$ and $\beta \rightarrow -\infty$;

- York decomposition both for the gravitational and Faddeev-Popov sectors:
  $\rightarrow$ No field redefinition $\Rightarrow$ We have to consider Jacobians $\Rightarrow$ Auxiliary fields;
  $\rightarrow$ Spurious modes should be removed from the computation of traces;

- Background approximation has been employed;

- Computations were performed with a maximally symmetric background (d-sphere);

- Calculations were done with type-I cutoff and optimized (Litim's) regulator function;

Results for general $f(R)$

- The FRG-flow equation becomes independent of the parameters ω and m if we set the background to be on-shell;

- In general, the RG-flow depends on the four parameters introduced before:
  $\rightarrow$ Field parameters $\Rightarrow \omega$ and m;
  $\rightarrow$ Gauge fixing parameters α and β;

- Certain choices of some of the parameter minimizes the dependence on the others:
  $\rightarrow m = -1/d$, $\beta = 0$ and $\alpha \rightarrow 0 \Rightarrow$ the RG-flow becomes independent of ω;
  $\rightarrow \omega = 1/2$ and $\beta \rightarrow -\infty \Rightarrow$ the RG-flow does not depend on α and m;

- "Duality" – The FRG-flow equation is invariant under the following transformation:
  [Ohta, Percacci and Pereira, JHEP 06 (2016) 115; EPJC 77 (2017) 611; PRD 97 (2018) 104039]

$$(\omega, m) \mapsto \left(1 - \omega, -m - \frac{2}{d}\right). \tag{6}$$

Results for polynomial $f(R)$: Fixed Point Structure

From now on let us concentrate our attention to the case of polynomial $f(R)$-functions:

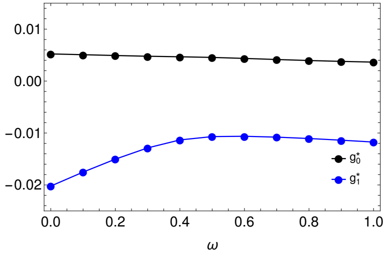
$$f_k(R) = \sum_{n=0}^{N} k^{d-2n} g_n(k) R^n. \tag{7}$$

- We computed the beta functions for the dimensionless couplings up to the case $N = 6$ (we also set $d = 4$);
- The FP structure was investigated for several choices of the parameters ($\omega \in [0, 1]$):
  - $m = 0$, $\beta \to 0$ and $\alpha \to 0$
  - $m = -1/4$, $\beta \to 0$ and $\alpha \to 0$
  - $m = -1/2$, $\beta \to 0$ and $\alpha \to 0$
  - $m = 0$, $\beta \to -\infty$ and $\alpha \to 0$
  - $m = -1/4$, $\beta \to -\infty$ and $\alpha \to 0$
  - $m = -1/2$, $\beta \to -\infty$ and $\alpha \to 0$

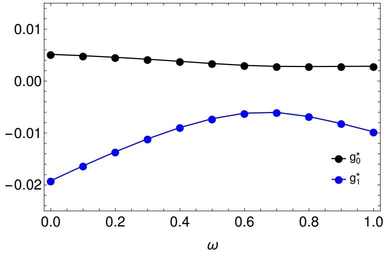
Result: In all cases we found suitable FP's, however, the number of relevant directions depends on the choice of parametrization.

The EH truncation ($N = 1$)

Gauge parameter: $\beta = 0$



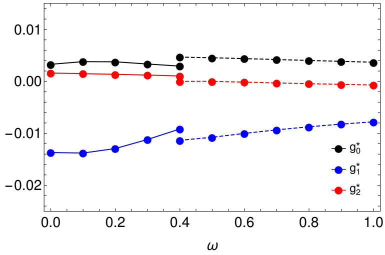
Gauge parameter: $\beta \to -\infty$



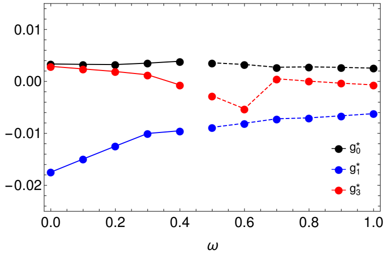
- In all cases we found FP's with 2 relevant directions;

The R^2 truncation ($N = 2$)

Gauge parameter: $\beta = 0$

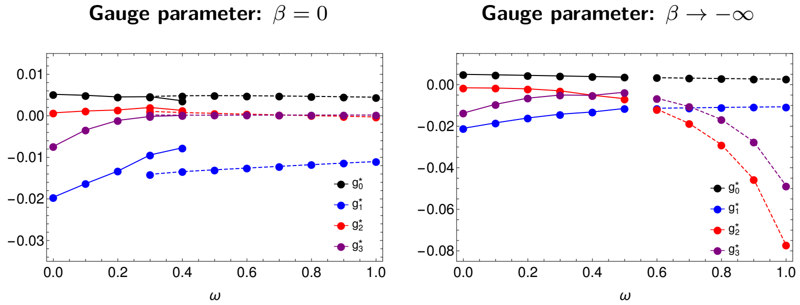


Gauge parameter: $\beta \to -\infty$



- Continuous line $\to$ 3 relevant directions ;
- Dashed line $\to$ 2 relevant directions ;

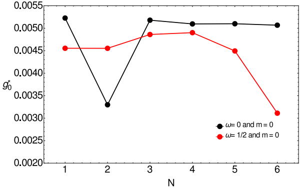
The R^3 truncation ($N = 3$)

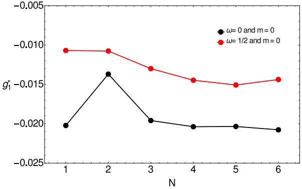


- Continuous line $\to$ 3 relevant directions ;
- Dashed line $\to$ 2 relevant directions ;

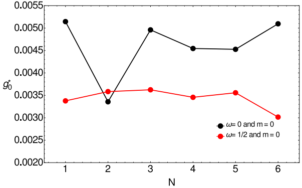
FP's for different truncations

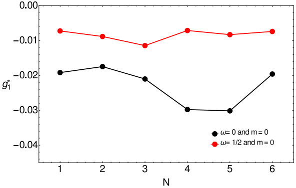
Results for $\beta = 0$:





Results for $\beta \to -\infty$:





Discussion

Summing up our results regarding the ω-dependence for polynomial truncations:

- In all cases we found suitable fixed points;
- The number of relevant directions depends on the parameter ω;
- Results obtained with the linear parametrization remains "stable" up to $N = 6$;
- For the exponential parametrization, the number of relevant directions depends on the choice of the gauge parameter
  $\rightarrow \beta = 0 \Rightarrow$ 2 relevant directions;
  $\rightarrow \beta \rightarrow -\infty \Rightarrow$ 3 relevant directions;
- In order to investigate the numerical convergence of the FP's we should go to higher order in our truncation with different choices of parametrization;
  [Falls, Litim, Nikolakopoulos and Rahmede, PRD 93 (2016) 104022;
  Alkofer and Saueressig, AOP 396 (2018) 173.]

A controversial result for the exponential parametrization?

Discussion on the results for the exponential parametrization with the so-called physical gauge ($\alpha \to 0$ and $\beta \to -\infty$):

- Our result $\to$ 3 relevant directions (slide 13);
- Results reported in the literature $\to$ 2 relevant directions;
  [Ohta, Percacci and Vacca, EPJC 76 (2016) 46;
  Alkofer and Saueressig, AOP 396 (2018) 173.]

What is the source of such ambiguity?

- Field redefinition in the spin-0 sector $(\sigma, h) \mapsto (s, \chi)$;
- Different treatment for the Jacobians $\to$ leading to different flow equations;

Remark: It was not a controversial result, however, we should be very careful with all schemes employed during our computations.

Final Remarks

We discussed how different choices of field parametrization for the quantum fluctuations affect the RG-flow in $f(R)$-truncations.

How different field parametrization may affect the structure of FP's?

- For all combinations of parameters considered we found interacting fixed points;
- Different choices of field parametrization may lead to different numbers of relevant directions;
- The exponential parametrization turns out to be sensitive on the change of gauge fixing parameter
  $\rightarrow$ Disagreement with the literature? $\Rightarrow$ Scheme dependent results!

Our finding reflects the difficulties of the background approximation for the FRGE:
[Litim and Pawlowski, PLB 546 (2002) 279; Bridle, Dietz and Morris, JHEP 03 (2014) 093.]

- Results obtained with background approximation may carry ambiguities;
- In our perspective, this is a limitation of this approximation and further constraints should be imposed on the calculations;

Thank you for your attention!