Consistent early and late time cosmology from the RG flow of gravity

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Based on arXiv:1806.10147
(with Robin Ooijer and Frank Saueressig)

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PART I: COSMOLOGICAL OBSERVATIONS - TIME EVOLUTION AS AN ENERGY DIAL

The concordance model of (late-time) cosmology

- assuming validity of General Relativity plus homogeneity and isotropy, the background evolution is governed by the Friedmann equations

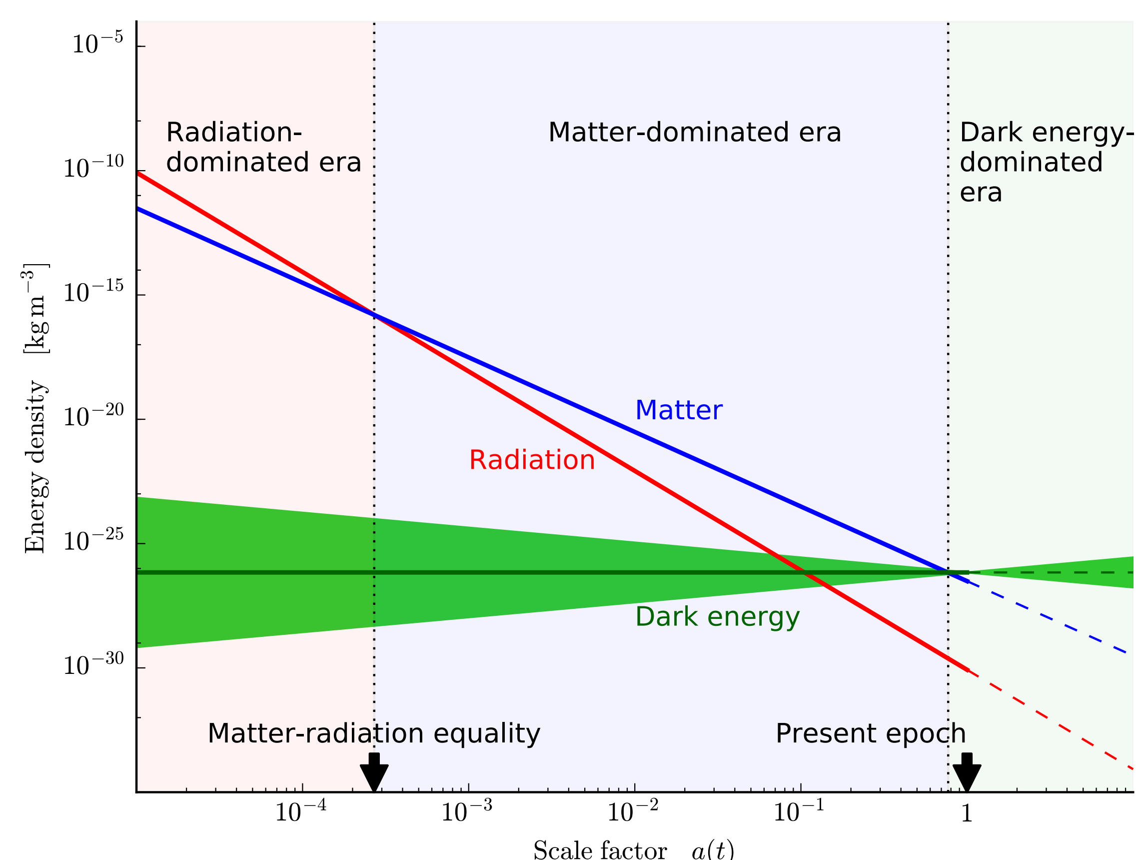
$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{kc^2}{a^2} = \frac{8\pi G}{3}\rho$$

$$2\left(\frac{\ddot{a}}{a}\right) + \left(\frac{\dot{a}}{a}\right) + \frac{kc^2}{a^2} = -\frac{8\pi G}{c^2}p$$

these can be solved once the energy content is defined: $\rho = \rho_{\mathrm{m}} + \rho_{\mathrm{rad}} + \rho_{\Lambda}$

$$\dot{\rho} + 3\frac{\dot{a}}{a}\left(\rho + \frac{p}{c^2}\right) = 0$$

$$\longrightarrow \begin{cases} \rho_{\mathrm{m}}(a) \sim a^{-3} \\ \rho_{\mathrm{rad}}(a) \sim a^{-4} \\ \rho_{\Lambda}(a) = \mathrm{const} \end{cases}$$



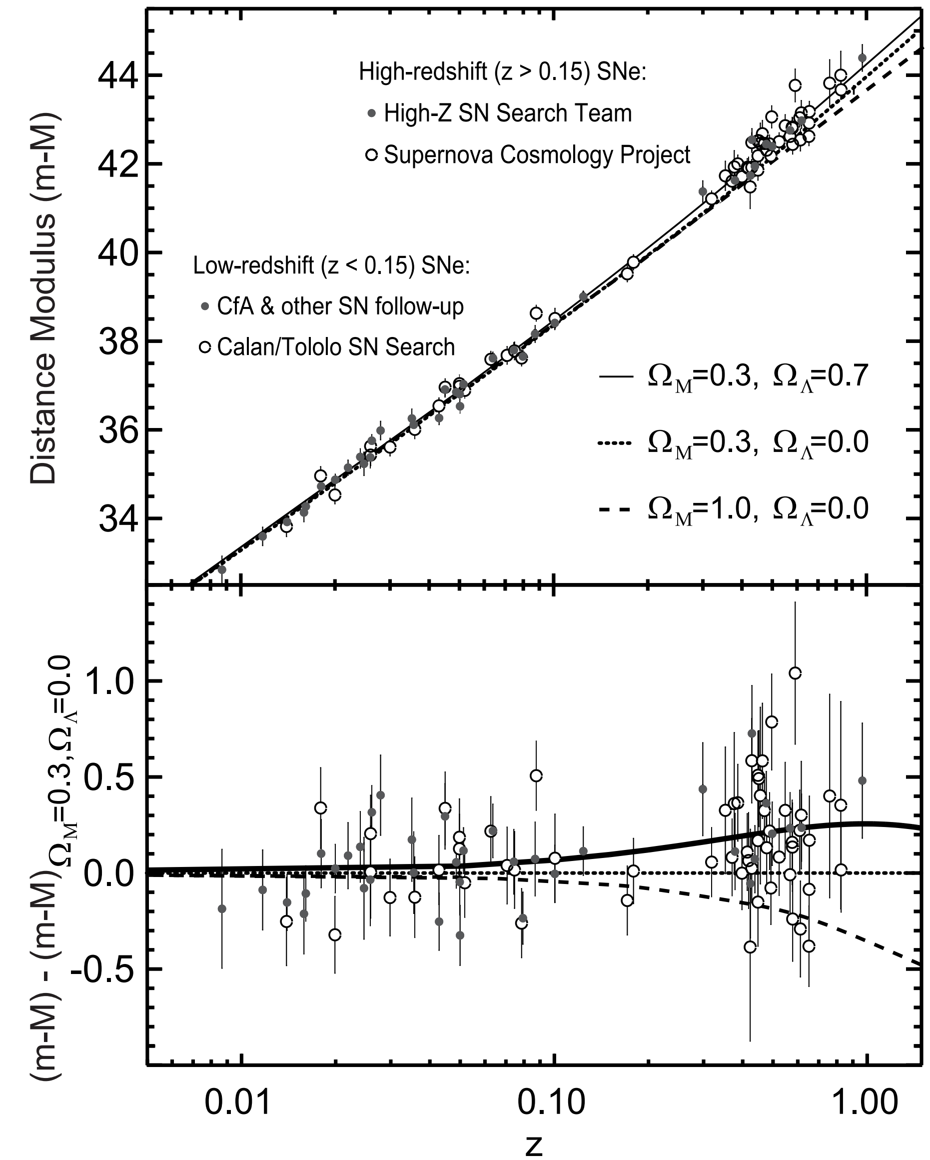
•*Debono, Smoot - Universe 2016*

Observational evidence for the vacuum energy density

- the amount of vacuum energy density was originally inferred from the distance-luminosity relation of supernovae

$$\mu \equiv m_i - M_j = 2.5\log(L_j/f_i) = 5\log[H_0 d_L(z;\Omega_m,\Omega_{DE},w(z))] - 5\log H_0 + K_{ij}(z)$$

$$d_L(z;\Omega_m,\Omega_\Lambda) = \frac{c(1+z)}{H_0}|\Omega_k|^{-1/2} S_k\left(\int_{1/(1+z)}^{1} |\Omega_k|^{1/2} \frac{da}{a^2[\Omega_m a^{-3} + \Omega_\Lambda + \Omega_k a^{-2}]^{1/2}}\right)$$



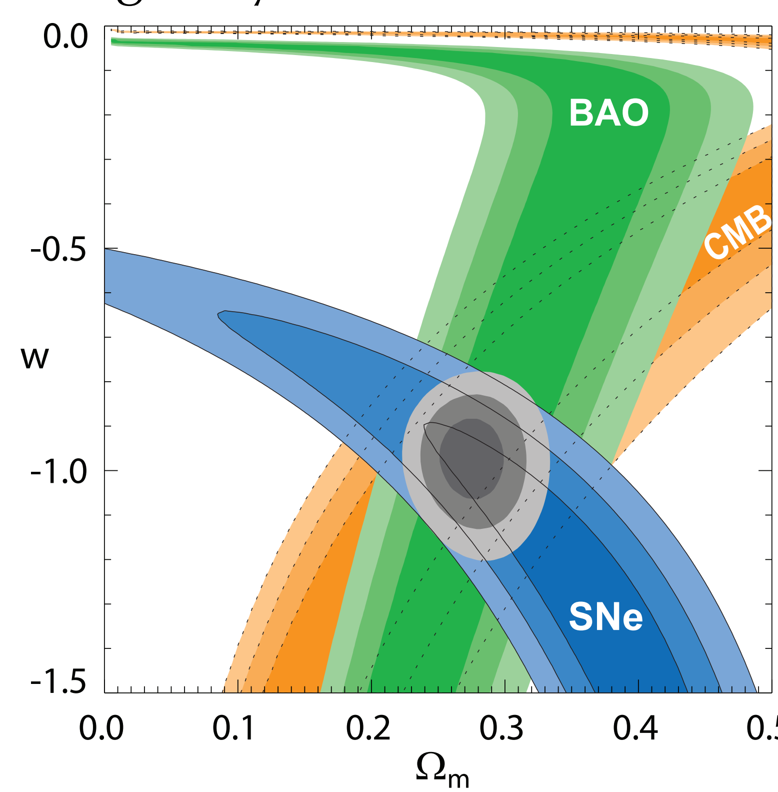
picture from the 1998 discovery of accelerated expansion (Nobel prize 2011)
The Supernova Cosmology Project and The High-z Supernova Search Team

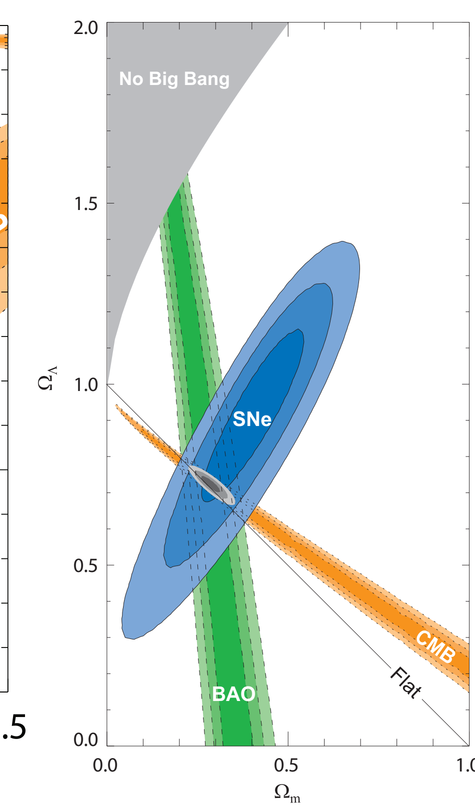
- nowadays we combine also measurements of the cosmic microwave background and large scale galaxy correlations (baryon acoustic oscillations)

$$\Omega_i \equiv \frac{\rho_i}{\rho_c} = \frac{8\pi G\rho_i}{3H_0^2}$$

$$w = p/\rho$$

in flat universe: $\sum_i \Omega_i = 1$





•*see e.g. review by Frieman - arXiv:0904.1833*

Gravitational action of very-late-time cosmology

- observations are compatible with a universe whose expansion was accelerating during the last 5 Gyrs (i.e. since redshift z~0.5)

- the recent history of the universe can thus be encoded in the gravitational action including a cosmological constant (matter is sub-dominant)

$$S_{\text{late times}} = \frac{1}{16\pi G} \int d^4x \sqrt{-g}\,(2\Lambda - R)$$

- the parameter Λ is deduced from the observational constraints:

$$\Omega_\Lambda \equiv \frac{\rho_\Lambda}{\rho_c} = \frac{\Lambda}{8\pi G}\frac{8\pi G}{3H_0^2} \longrightarrow \Lambda = \Omega_\Lambda \cdot 3H_0^2$$

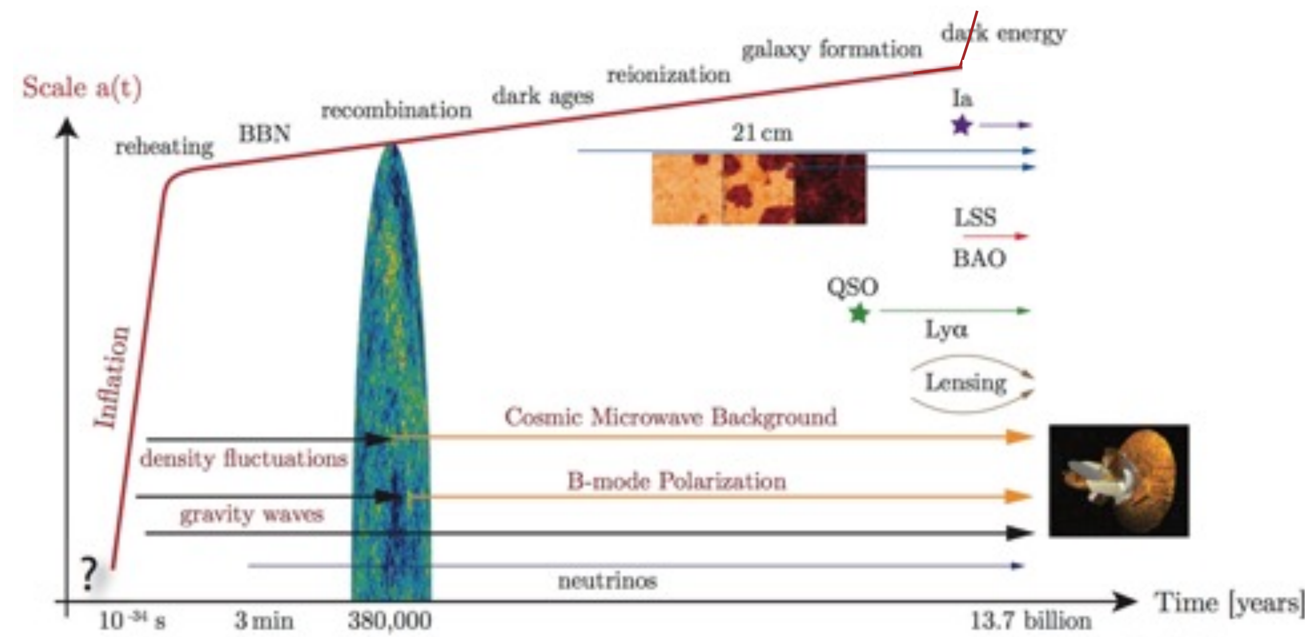
$$\Omega_\Lambda \simeq 0.7$$

$$H_0 \simeq 70\,\mathrm{km\,s^{-1}\,Mpc^{-1}} \simeq 10^{-33}\mathrm{eV} \longrightarrow \Lambda \simeq 4 \times 10^{-66}\mathrm{eV}^2$$

Cosmology of very early times - inflation

- observations of the Cosmic Microwave Background are consistent* with another epoch of accelerated expansion at early times (inflation)

to have a mechanisms that drives this acceleration and then switches off to leave room for standard cosmological evolution requires to go beyond GR



- the simplest (and oldest) model adds higher-order curvature terms to the Einstein-Hilbert action (Starobinsky inflation - 1980)

$$S_{\text{inflation}} = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\,(-R + B\,R^2)$$

this can also be written as the Higgs inflation model

$$S_{\text{Higgs}} = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\left(-R + \frac{1}{2}(\nabla\varphi)^2 + V(\varphi)\right)$$

$$V(\varphi) \equiv -\frac{M_P^2}{8B}\left[1 - e^{-\sqrt{2/3}\varphi/M_P}\right]^2$$

$$M_P = (8\pi G)^{-1/2}$$

*fine print warning: consistency does not mean that inflation has been proven to be correct

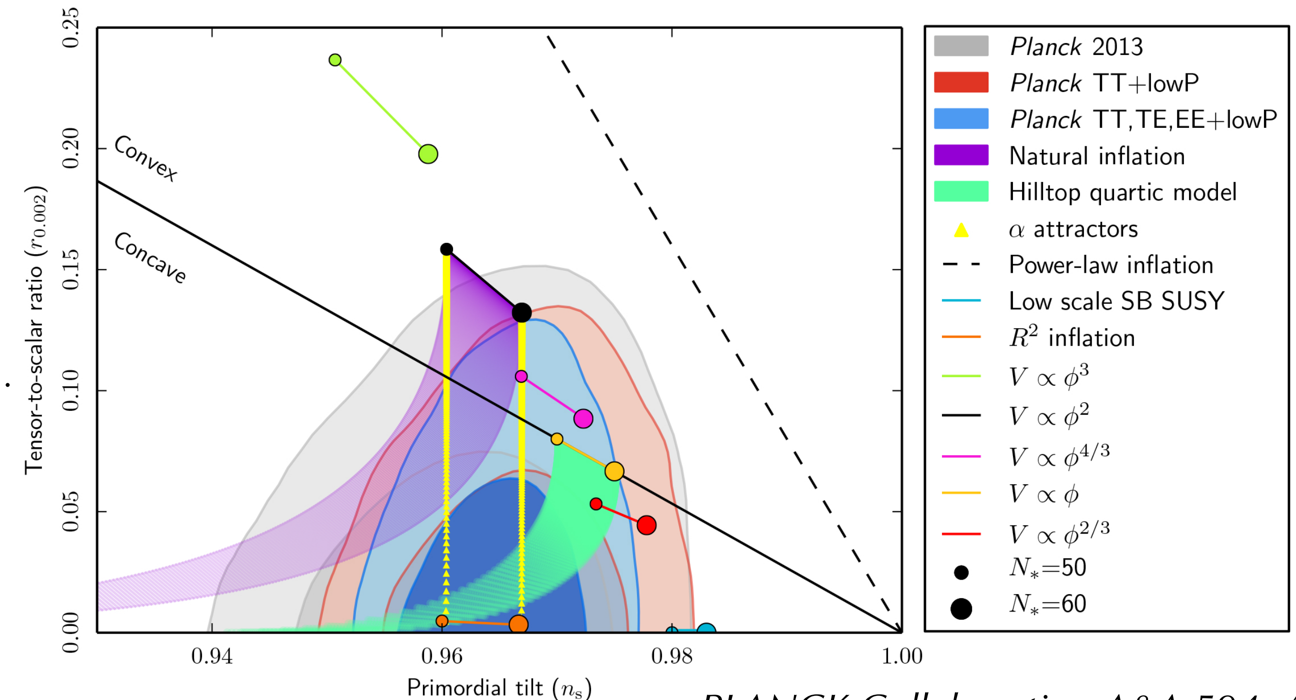
Inflation - observational constraints

✦ Starobinsky inflation is currently one of the best-fitting inflationary models when compared to observations of the CMB

$$\mathcal{P}_{\mathcal{R}}(k) = \frac{k^3}{2\pi^2}|\mathcal{R}_k|^2 = A_s\left(\frac{k}{k_*}\right)^{n_s-1+\frac{1}{2}\frac{dn_s}{d\ln k}\ln(k/k_*)+\ldots}$$

$$\mathcal{P}_t(k) = \frac{k^3}{2\pi^2}\left(|h_k^+|^2+|h_k^\times|^2\right) = A_t\left(\frac{k}{k_*}\right)^{n_t+\frac{1}{2}\frac{dn_t}{d\ln k}\ln(k/k_*)+\ldots}$$

$$\frac{A_t}{A_s} \equiv r$$



• *PLANCK Collaboration A&A 594, A20 (2016)*

✦ this is quite remarkable given that this is a 'zero parameter' model:

the only free parameter B is determined by the amplitude of the primordial power spectrum

$$V(\varphi) \equiv M^4 v(\varphi) \longrightarrow \left(\frac{M}{M_P}\right)^4 = 24\pi^2 A_s \frac{\epsilon_{1*}}{v(\varphi_*)}$$

$$M_P^2\, B_k \simeq -1\times 10^9$$

the other observables are only linked to the properties of the subsequent reheating phase

$$n_s - 1 \approx -8(4N_* + 9)/(4N_* + 3)^2$$

$$r \approx 192/(4N_* + 3)^2$$

Towards a unified framework for late time and early time cosmology

- two different gravitational theories seem to govern late time and early time cosmology

$$S_{\text{inflation}} = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\left(-R + B\,R^2\right)$$

$$S_{\text{late times}} = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\,(2\Lambda - R)$$

- these two actions actually correspond to two very different regimes of gravity

$$k_{\text{late times}} = H_0 \simeq 10^{-33}\text{eV}$$

$$k_{\text{infl}} = H_{\text{infl}} \simeq 10^{22}\,\text{eV}$$

- we can posit that the complete cosmic history is indeed described by the full action

$$S[g] = \frac{1}{16\pi G}\int \mathrm{d}^4\,x\sqrt{-g}\left(2\Lambda - R + BR^2\right)$$

with the cosmological constant term only being relevant on very small energy scales and the higher curvature term being dominant on large energy scales

- shall we regard this just as a phenomenological model or does it have a deeper origin?

Towards a unified framework for late time and early time cosmology

- this motivates us to search for a underlying mechanism that can produce the full action describing the complete cosmic history, interpolate between the two limiting regimes and explain the observed values of the coupling constants at the relevant scales

Energy scale (eV)	Constraint
$k_{\text{infl}} = 10^{22}$	$B = -1.7 \times 10^{-46}\ \text{eV}^{-2}$
$k_{\text{lab}} = 10^{-5}$	$G = 6.7 \times 10^{-57}\ \text{eV}^{-2}$
$k_{\text{Hub}} = 10^{-33}$	$\Lambda = 4 \times 10^{-66}\ \text{eV}^{2}$

In this context, in principle the Newton's constant is also one of the free parameters of the action, whose value we measure at 'human' (laboratory) scales

PART II: COSMOLOGY FROM RG FLOWS

Lightning review of Asymptotic Safety

- Idea: integrate out quantum fluctuations shell-by-shell in path integral
- RG flow governed by FRGE

$$\partial_t \Gamma_k\left[h_{\mu\nu}; \bar{g}_{\mu\nu}\right] = \frac{1}{2}\mathrm{Tr}\left[\left(\Gamma^{(2)} + R_k\right)^{-1} \partial_t R_k\right]$$

- Γ_k describes effective dynamics with quantum corrections at scale k

RG flow of Quantum Gravity

- Basis in curvature invariants:
- $f(R)$: $1, R, R^2, R^3, R^4, \ldots$
- Curvature²: $R_{\mu\nu}^2, C^2$
- Curvature³
- $\vdots$
- Cosmology constrains these operators!

$f(R)$-gravity

- RG equations (polynomial expansion)
  - Machado, Saueressig [2008]
- Truncation:

$$f_k(R) = \frac{1}{16\pi\, G_k}\left(2\Lambda_k - R + B_k R^2\right)$$

- Dimensionless couplings:

$$G_k = k^{-2} g_k; \quad \Lambda_k = k^2 \lambda_k; \quad B_k = k^2 b_k$$

- Fixed point:

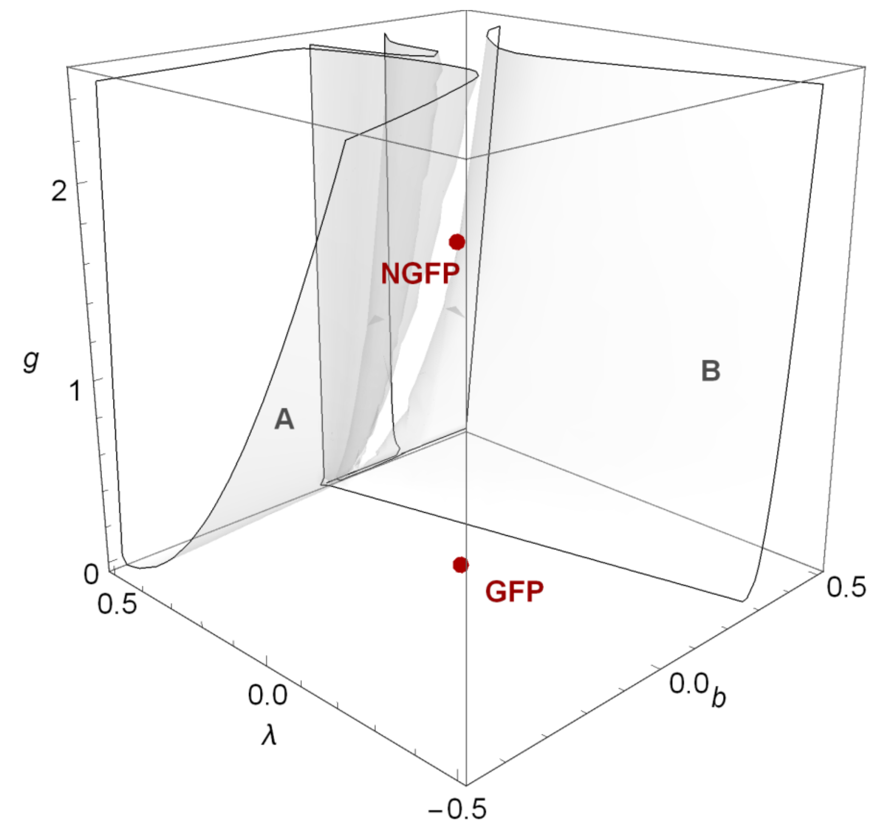
$\lambda_* = 0.133$ $\qquad g_* = 1.59$ $\qquad b_* = 0.119$

- Critical exponents:

$\theta_{1,2} = 1.26 \pm 2.45\mathrm{i}$ $\qquad \theta_3 = 27.0$

Phase diagram: global structures

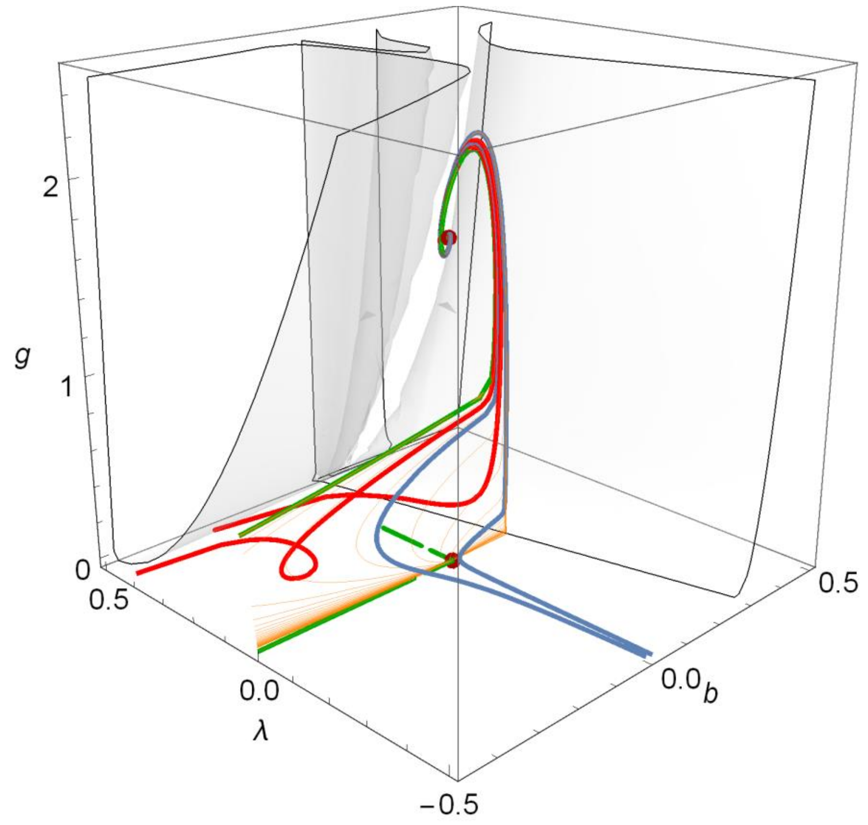
- Singularities in β-functions:
  - Singularity in η_N (A)
  - Singularity in R^2 (B)
- GFP: $(0,0,0)$
- NGFP



Phase diagram: trajectories

- Classification:
  - Type Ia
  - Type IIa
  - Type IIIa

- …but which trajectory satisfies Planck constraints?

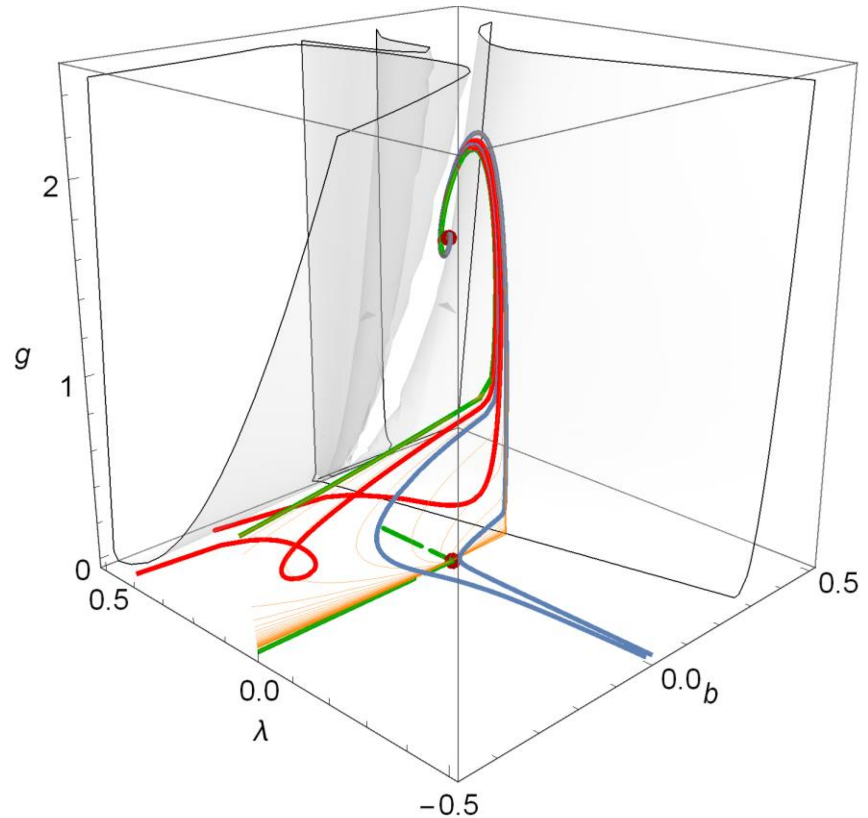


Physical trajectory

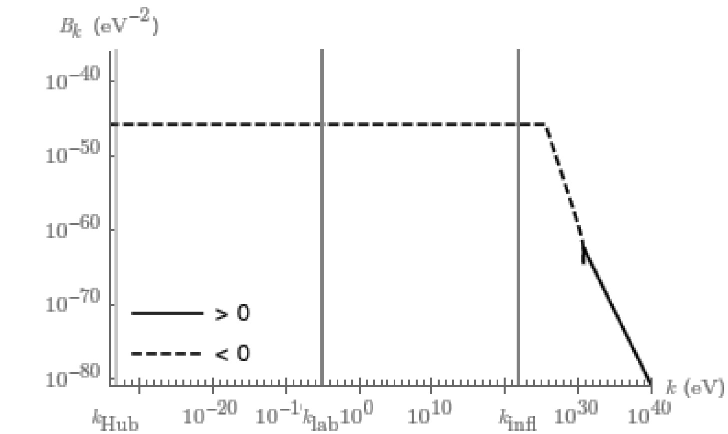
Energy	Constraint
$k \simeq k_{\text{infl}}$	$B_k \simeq -1.7 \times 10^{-46}\ \text{eV}^{-2}$
$k \simeq k_{\text{lab}}$	$G_k \simeq 6.7 \times 10^{-57}\ \text{eV}^{-2}$
$k \simeq k_{\text{Hub}}$	$\Lambda_k \simeq 4 \times 10^{-66}\ \text{eV}^{-2}$

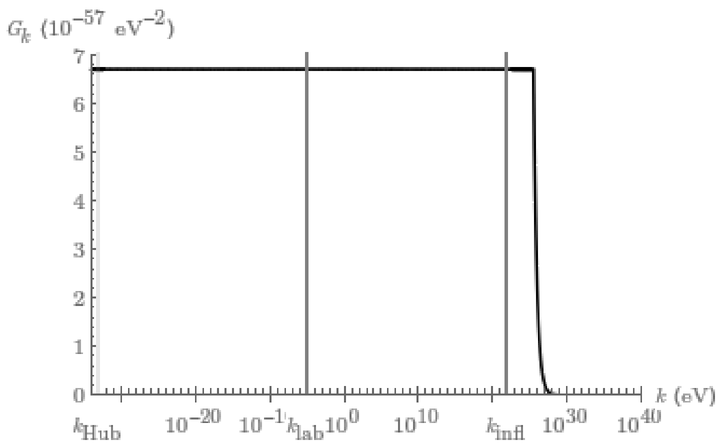
- Take $k_0 = k_{\text{lab}} \Rightarrow g_{k_0} \ll 1$: classical regime
- Initial conditions:

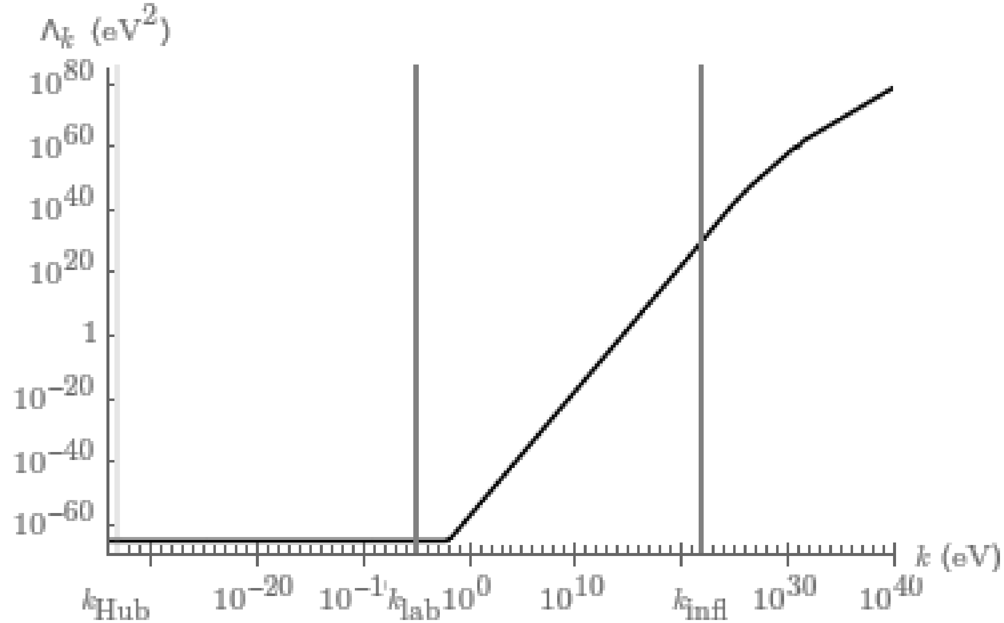
$$g_{k_0} = 6.71 \times 10^{-67}$$
$$\lambda_{k_0} = 3.99 \times 10^{-56}$$
$$b_{k_0} = -1.7 \times 10^{-56}$$



Physical trajectory







Λ starts to run at $k \simeq 10^{-2}$ eV $\simeq 10^{-4}$ m
⇒ observable?

Conclusions and outlook

- Inflation: testing ground for QG
- EH+R^2-gravity:
  - Asymptotically safe
  - Consistent with inflation/late time cosmology

- Outlook:
  - Proper predictions from irrelevant operators: e.g. R^n, C^2
  - Cosmological constant problem: matter effects?