

How perturbative is quantum gravity?

Asymptotic Safety Online Seminar

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Technical implications

- Guideline for future truncations
- A posteriori justification of previous truncations
- Application of perturbative methods in transplanckian regime
→ provide indications for existence of fixed point

[M. R. Niedermaier, 2009]

Physical implications

- Asymptotically safe Standard Model favors perturbative nature of fixed point
- Appealing scenario: interactions at high energies
→ strong enough to induce asymptotic safety
→ near canonical scaling for higher order operators

[Falls, Litim, Nikolakopoulos and Rahmede, 2013]

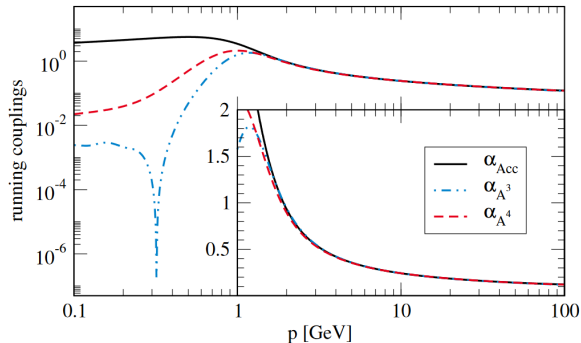
[Falls, Litim, Nikolakopoulos and Rahmede, 2014]

[Falls, Litim and Schröder, 2018]

Avatars of a gauge coupling

Non-abelian gauge theories: different incarnations of the gauge coupling!

QCD



[A. K. Cyrol, L. Fister, M. Mitter, J.M. Pawłowski and N. Strodthoff, 2016]

Gravity + Matter

?



**Effective Universality at UV
fixed point**

Avatars of the Newton coupling (I)

Classical level:

$$S = \int d^4x \left[\frac{\sqrt{g}}{G} R + \sqrt{g} \bar{\psi} \not{\nabla} \psi \right] \rightarrow \Gamma \sim \int d^4x \left[\left(h \partial^2 h + \sqrt{G} h (\partial h) (\partial h) + \dots \right) + \left(\bar{\psi} \partial \psi + \sqrt{G} h \bar{\psi} \partial \psi + \dots \right) \right]$$

- Linear split: $g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$
 - $\rightarrow \sqrt{g} R \approx \partial^2 h + h \partial^2 h + h (\partial h) (\partial h) + \mathcal{O}(h^4)$
 - $\rightarrow \sqrt{g} \bar{\psi} \not{\nabla} \psi \approx \bar{\psi} \not{\partial} \psi + h \bar{\psi} \partial \gamma \psi + \mathcal{O}(h^2)$
- Rescale: $h_{\mu\nu} \rightarrow \sqrt{G} h_{\mu\nu}$

Quantum level:

$$S = \int d^4x \left[\frac{\sqrt{g}}{G} R + \sqrt{g} \bar{\psi} \not{\nabla} \psi \right] \rightarrow \Gamma \sim \int d^4x \left[\left(h \partial^2 h + \sqrt{G_h} h (\partial h) (\partial h) + \dots \right) + S_{\text{gf}}(\bar{g}, h) + \left(\bar{\psi} \partial \psi + \sqrt{G_\psi} h \bar{\psi} \partial \psi + \dots \right) \right]$$

- Linear split: $g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$
 $\rightarrow \sqrt{g} R \approx \partial^2 h + h \partial^2 h + h (\partial h) (\partial h) + \mathcal{O}(h^4)$
 $\rightarrow \sqrt{g} \bar{\psi} \not{\nabla} \psi \approx \bar{\psi} \not{\partial} \psi + h \bar{\psi} \partial \gamma \psi + \mathcal{O}(h^2)$
- Rescale: $h_{\mu\nu} \rightarrow \sqrt{G} h_{\mu\nu}$
- Quantum level: Introduce gauge fixing $S_{\text{gf}}(\bar{g}, h)$ and Regularization

- analogously for
ghosts $\rightarrow G_c$
scalars $\rightarrow G_\varphi$
vectors $\rightarrow G_A$

(Effective) Universality

- QCD:
 $[\alpha_s] = 0$
→ 2-loop universality
→ perturbative regime:
one gauge coupling
- Gravity:
 $[G_N] = -2$
→ no universality
→ gauge symmetry encoded in
relations for avatars

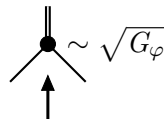
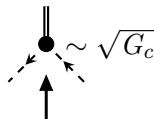
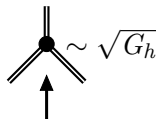
- **Effective Universality:** semi-quantitative agreement of β -functions

[\[A. Eichhorn, P. Labus, J. M. Pawłowski and M. Reichert, 2018\]](#)

- Quantify deviation from effective universality:

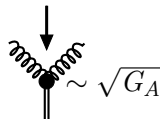
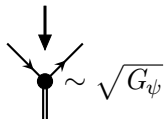
$$\varepsilon_{ij}(G, \mu, \lambda_3) \sim |\beta_{G_i} - \beta_{G_j}|_{G_i=G_j=G}$$

- Computation of β -functions: Functional Renormalization Group



$$S = \frac{1}{16\pi G_N} \int d^4x \sqrt{g} (2\Lambda - R) + S_{\text{gf,grav}} + S_{\text{gh,grav}} + \frac{1}{2} \int d^4x \sqrt{g} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi$$

$$+ \int d^4x \sqrt{g} \bar{\psi} \not{\nabla} \psi + \frac{1}{2} \int d^4x \sqrt{g} g^{\mu\nu} g^{\rho\sigma} \text{tr} F_{\mu\rho} F_{\nu\sigma} + S_{\text{gf,gauge}} + S_{\text{gh,gauge}}$$



- Regularization and gauge fixing:

$$\Gamma_k = \Gamma_k[\bar{g}, h, \Phi]$$

- Vertices depend on momentum configuration

$$\Gamma_k^{(n,l)} = \frac{\delta^{n+l} \Gamma_k[\bar{g}, h, \Phi]}{\delta h^n(p_{h_1}, \dots, p_{h_n}) \delta \Phi^l(p_{\Phi_1}, \dots, p_{\Phi_l})}$$

- Perform Vertex expansion of $\Gamma_k[\bar{g}, h, \Phi]$ up to three-point level

Diagrammatic representation of vertex flows

$$\begin{aligned}
 \partial_t \Gamma_k^{(2h)} &= \frac{1}{2} \tilde{\partial}_t \text{[Diagram 1]} - N_f \tilde{\partial}_t \text{[Diagram 2]} - \frac{1}{2} \tilde{\partial}_t \text{[Diagram 3]} + \tilde{\partial}_t \text{[Diagram 4]} \\
 &\quad + N_f \tilde{\partial}_t \text{[Diagram 5]} \\
 \partial_t \Gamma_k^{(\bar{\psi}, \psi)} &= \frac{1}{2} \tilde{\partial}_t \text{[Diagram 6]} - \tilde{\partial}_t \text{[Diagram 7]} \\
 \partial_t \Gamma_k^{(3h)} &= \frac{1}{2} \tilde{\partial}_t \text{[Diagram 8]} - N_f \tilde{\partial}_t \text{[Diagram 9]} - \frac{3}{2} \tilde{\partial}_t \text{[Diagram 10]} + 3 N_f \tilde{\partial}_t \text{[Diagram 11]} \\
 &\quad + \tilde{\partial}_t \text{[Diagram 12]} - 2 \tilde{\partial}_t \text{[Diagram 13]} - 2 N_f \tilde{\partial}_t \text{[Diagram 14]} \\
 \partial_t \Gamma_k^{(h, \bar{\psi}, \psi)} &= \frac{1}{2} \tilde{\partial}_t \text{[Diagram 15]} - \frac{1}{2} \tilde{\partial}_t \text{[Diagram 16]} - 2 \tilde{\partial}_t \text{[Diagram 17]} + \tilde{\partial}_t \text{[Diagram 18]} \\
 &\quad + \tilde{\partial}_t \text{[Diagram 19]}
 \end{aligned}$$

- Flow of graviton-fermion three-point vertex
(at momentum symmetric point)

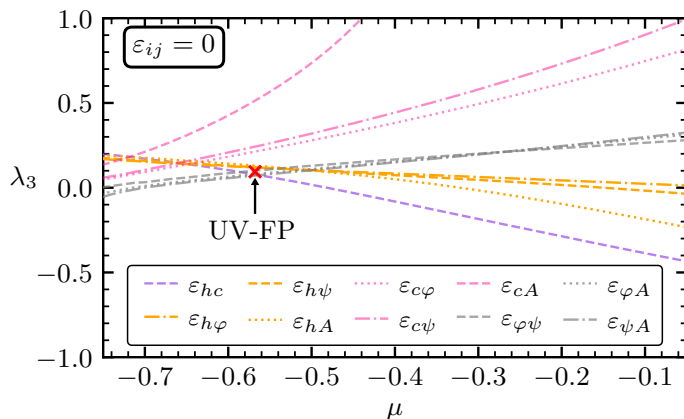
$$\partial_t \left[\sqrt{Z_h(p^2)} Z_\psi(p^2) \sqrt{G_\psi(p^2) k^{-2} p^2} \right] = \sqrt{Z_h(p^2)} Z_\psi(p^2) \underbrace{\partial_t \Gamma_k^{(h, \bar{\psi}, \psi)}(p^2)}_{\text{Flow}^{(h, \bar{\psi}, \psi)}(p^2)} \Big|_{G_\psi}$$

- Evaluate vertex flows at $p^2 = 0$ and $p^2 = k^2$
- Bilocal flow equation:

$$\beta_{G_\psi} = \left(2 + \frac{1}{2} \eta_h(k^2) + \eta_\psi(k^2) \right) G_\psi + 2 \sqrt{G_\psi} \text{Flow}^{(h, \bar{\psi}, \psi)}(k^2)$$

- Analogously for G_h , G_c , G_φ , G_A

Effective Universality



- Pairwise comparison of β -functions
- $\varepsilon_{ij} \sim |\beta_{G_i} - \beta_{G_j}|$
- $\varepsilon_{ij} = 0$ lines
- Red cross: UV-FP

Key Result

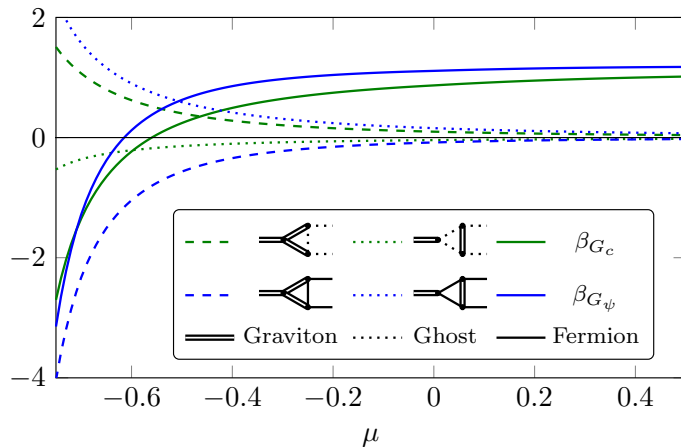
UV-FP lies in symmetry preferred region.

Key Result

$\varepsilon_{ij} = 0$ lines exist for all avatars of the Newton coupling.

Non-trivial cancellations

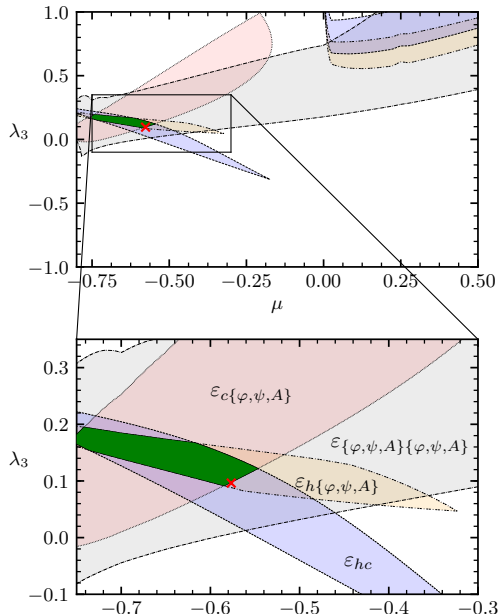
- Compare e.g. β_{G_c} and β_{G_ψ} and β_{G_ψ}
- Specific diagrams do not agree
- Full β -functions are similar



Key Result

Only non-trivial cancellations between diagrams lead to effective universality.

Effective Universality at UV Fixed Point



- At UV-FP present:

- ▶ Higher curvature couplings

- [Benedetti, Machado and Saueressig, 2009]
 - [Falls, Litim, Nikolakopolous and Rahmede, 2013]
 - [Falls, Litim, Nikolakopolous and Rahmede, 2014]
 - [Gies, Knorr, Lippoldt and Saueressig, 2016]
 - [Christiansen, Falls, Pawłowski, Reichert, 2017]
 - [De Brito, Otha, Pereira, Tomaz, Yamada, 2018]
 - [Falls, Litim and Schröder, 2018]

- ▶ Non-minimal matter gravity couplings

- [Narain and Percacci, 2009]
 - [Percacci and Vacca, 2015]
 - [Eichhorn, Lippoldt and Skrinjar, 2017]

- ▶ Matter self-interactions

- [Eichhorn and Gies, 2011], [Eichhorn, 2012]
 - [Meibohm and Pawłowski, 2015]
 - [Eichhorn, Held and Pawłowski, 2016]
 - [Eichhorn and Held, 2017]
 - [Christiansen and Eichhorn, 2017]

→ systematic error $\delta\mathcal{E}$

- Lower bound: $\delta\mathcal{E} \approx 0.2$

- [Meibohm, Pawłowski and Reichert, 2015]
 - [Denz, Pawłowski and Reichert, 2016]

- Gauge-fixing and regularization:
fluctuation couplings are related by Slavnov Taylor identities (STIs)
- STIs: sourced by quantum effects
perturbative limit \rightarrow trivial STIs
non-perturbative limit \rightarrow non-trivial STIs, e.g. QCD
[\[A. K. Cyrol, L. Fister, M. Mitter, J.M. Pawłowski and N. Strodthoff, 2016\]](#)

- $\varepsilon_{ij} \approx 0.2 \rightarrow \frac{G_i^*}{G_j^*} \approx 0.7$

$$(G_h^*, G_c^*, G_\varphi^*, G_\psi^*, G_A^*) = (0.58, 0.55, 0.74, 0.75, 0.84)$$

especially: $\frac{G_{\min}^*}{G_{\max}^*} \approx 0.65$

Key Result

Hint towards near-perturbative nature of asymptotically safe fixed point.

Supression of higher-order operators

- Right-hand side of flow equation: overlap with all operators allowed by symmetry
- E.g. on level of three-graviton coupling: $\sqrt{g}R$ and $\sqrt{g}R_{\mu\nu}R^{\mu\nu}$
- Breaking of diffeomorphism-invariance: difficult to disentangle different contributions
- Assume contamination $\delta\beta_{\text{Ric}}$ to β_{G_h}
 $\Rightarrow \varepsilon_{hi} \gtrsim 0.3$

Key Result

$\varepsilon_{ij} \lesssim 0.2$ provides hint for subleading nature of higher order operators.

- Effective universality:
 - ▶ hint for physical nature of asymptotically safe fixed point
 - ▶ hint for geometric origin of $h_{\mu\nu}$
 - ▶ indication for small impact of higher-order operators
 - ▶ indication for near-perturbative nature of quantum gravity
- Extend analysis to phenomenologically relevant numbers of scalars, fermions and vector fields

