Towards Background Independent QG with Tensor models

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Emmy Noether-Programm

DFG Deutsche Forschungsgemeinschaft

Goal: Make sense of

$$\sum_{\text{topologies}} \int [\mathcal{D}g][\mathcal{D}X] e^{-S_{\mathrm{EH}}-S_{\mathrm{SM}}-S_{\mathrm{other}}}$$

- Path integral over geometric d.o.f. & topologies
- Path integral over matter d.o.f

→ ill defined (above Planck scale)

Below Planck scale:
Cosmological perturbation theory &
EFT formulation of QG by Donoghue et al.

A discrete strategy

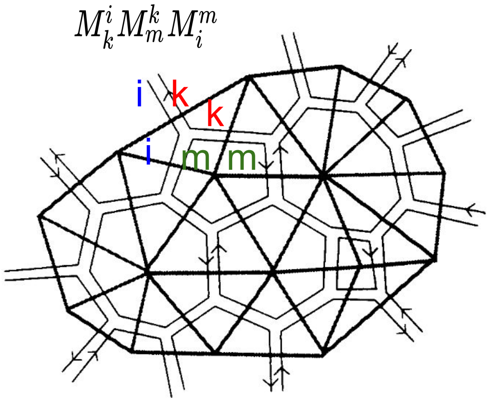
Discretize the continuum Path integral

$$\sum_{\text{topologies}} \int [\mathcal{D}g] \longrightarrow \sum_{\text{triangulations}}$$

How to take the continuum limit ?

Random Matrices

- Random Matrices are dual to triangulations in 2D



Picture from 2D Gravity and Random Matrices, Di Francesco, Ginsparg, Zinn-Justin

2D Euclidean QG

$$\mathcal{Z} = \sum_h \int \mathcal{D}g\, e^{-\beta A + \gamma \chi} \quad (1)$$

Sum over topologies = Sum over handles h

A=Area = $\int \sqrt{g}$

$\chi = 2 - 2h$ is the Euler character

Random Matrices

$$e^{\mathcal{Z}} = \int dM\, e^{-\frac{1}{2} trM^2 + \frac{g}{\sqrt{N}} trM^3} \quad (2)$$

$M \to \frac{M}{\sqrt{N}}$: $N = e^{\gamma}$ and $g = e^{-\beta}$

$\Longrightarrow$ Continuum Limit is 2D Eucl. QG

How to take the continuum limit ?

Double-scaling limit

- Continuum limit from double-scaling limit

$$(g-g_c)^{\frac{1}{\theta}} N = N_0 \qquad (1)$$

- Linearized "RG Flow" in matrix size N [Brezin, Zinn-Justin '92] [Eichhorn, Koslowski '13]

$$g = g_c + \left(\frac{N}{N_0}\right)^{-\theta} \qquad (2)$$

The continuum limit from non-local coarse graining

- RG flows from many to few d.o.f [Eichhorn, Koslowski '13]

$$\partial_t \Gamma_N = \frac{1}{2} \mathrm{Tr} \left(\Gamma_N^{(2)} + R_N \right)^{-1} \partial_t R_N$$

- Background Independent Coarse graining
    - No Notion of Canonical Dimension

$$\Gamma_N^{(2)} = \frac{\delta^2 \Gamma_N}{\delta T_{a_1 a_2 a_3} \delta T_{b_1 b_2 b_3}}$$

R_N IR-Regulator

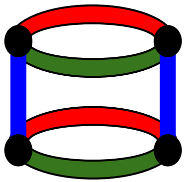
From Matrix to Tensor Models

- Tensor Models = # of indices ≥ 3

- Large-N limit exists for “colored” Tensor Models (indexed by Gurau degree)

[R. Gurau ‘10]

$$T_{a_1 a_2 a_3} T_{a_1 b_2 b_3} T_{b_1 a_2 a_3} T_{b_1 b_2 b_3}$$



Canonical Scaling Dimension in background-independent RG flow

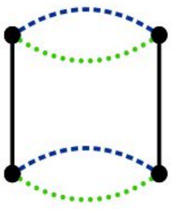
Local Flows

- RG step: local averaging
- Notion of canonical dimensionality defined by mass dimension

Background-independent flows

- RG step: Non-local averaging
- No units, no (a priori) scaling dimension
- Canonical Dimension from autonomous system of beta functions in large-N limit

$\bar{g}_{4,1}^{2,1}$



$$\beta_{\bar{g}_{4,1}^{2,1}} = 2N^2 \left(\bar{g}_{4,1}^{2,1}\right)^2 + \dots \qquad (1)$$

$$g_{4,1}^{2,1} = \bar{g}_{4,1}^{2,1} N^2$$

$$\beta_{g_{4,1}^{2,1}} = 2g_{4,1}^{2,1} + 2\left(g_{4,1}^{2,1}\right)^2 \qquad (2)$$

So Far

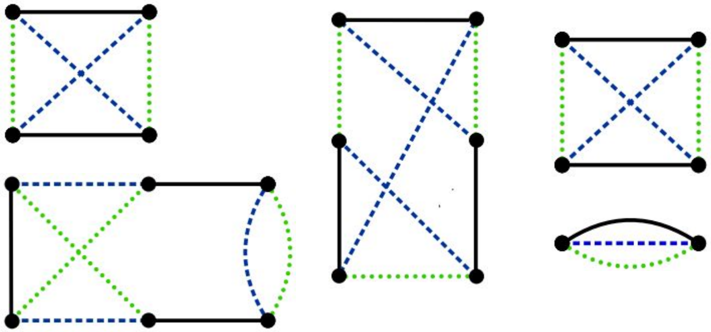
- Matrix Models discretize 2D QG
- Tensor Models generalize Matrix Models to higher D
- Setting-up the FRG
    - No Notion of Scaling Dimension

The Model

- Consider a real rank-3 Tensor model s.t.

$$T_{a_1 a_2 a_3} \to O^{(1)}_{a_1 b_1} O^{(2)}_{a_2 b_2} O^{(3)}_{a_3 b_3} T_{b_1 b_2 b_3}$$

- Enlarged Theory space compared to $U(N)^{\otimes 3}$



etc.

Applying the FRG

- Coarse-graining in *N* [Eichhorn, Koslowski, 2013]

$$\partial_t \Gamma_N = \frac{1}{2} \mathrm{Tr} \left(\Gamma_N^{(2)} + R_N \right)^{-1} \partial_t R_N$$

→ Flow of the Effective Action Γ_N

→ Alternative to path-integral formulation

How to cook with the FRG - A recipe

Ingredients:

- A truncation of the effective action
  - *Amount*: Already small truncations can yield viable results
- A regulator R_N

1) $a_1 + a_2 + a_3 < N : \quad R_N > 0$

2) $N < a_1 + a_2 + a_3 : \quad R_N = 0$

3) $N \to N' \to \infty : \quad R_N \to \infty$

Instructions:

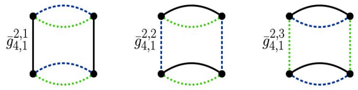
- Compute $\Gamma_N^{(2)}$
- Specify R_N
- Cook β-fcts (or compute) using FRG
- Fix scaling of couplings

Ingredient I: Truncation

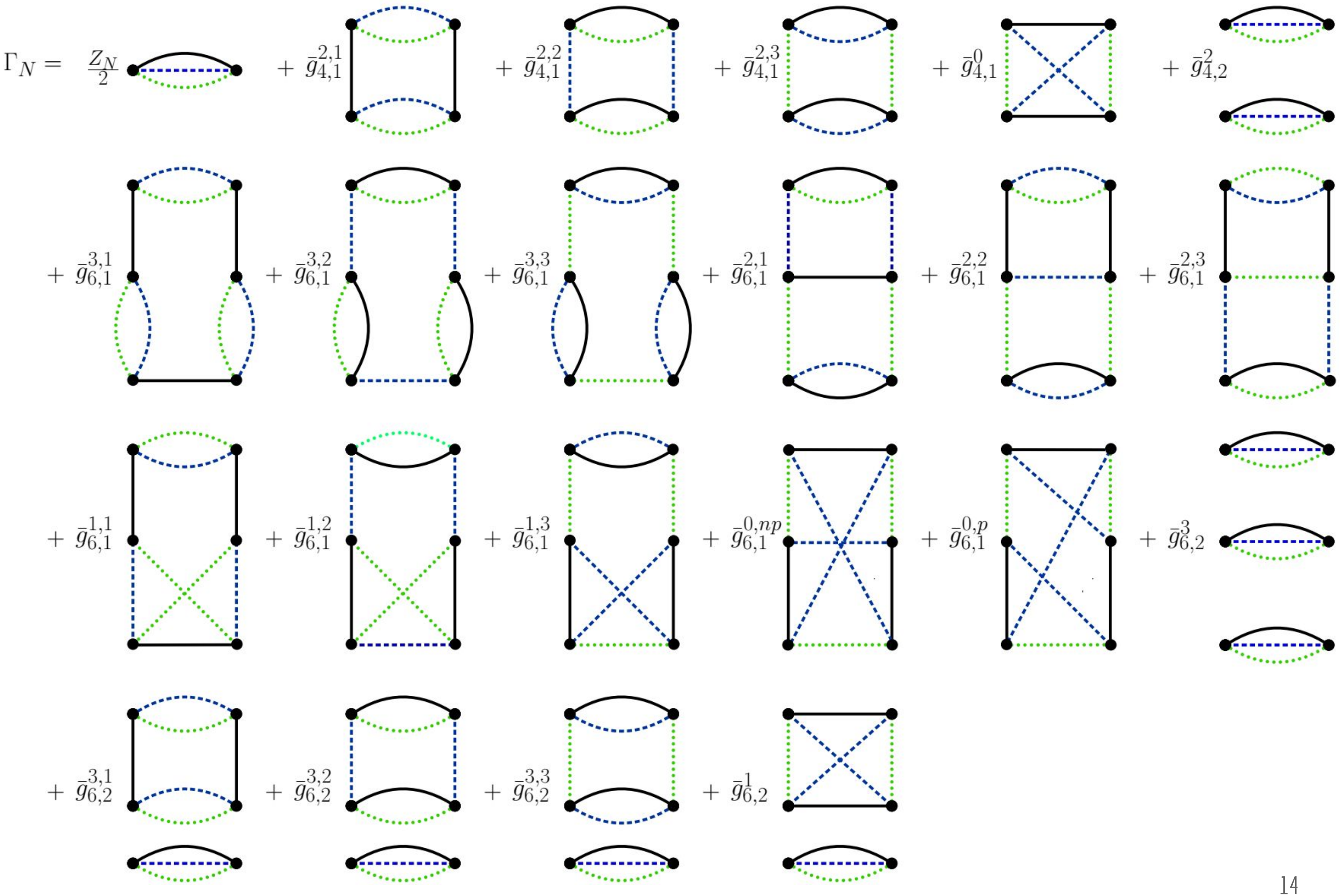
- FRGE cannot be solved exactly

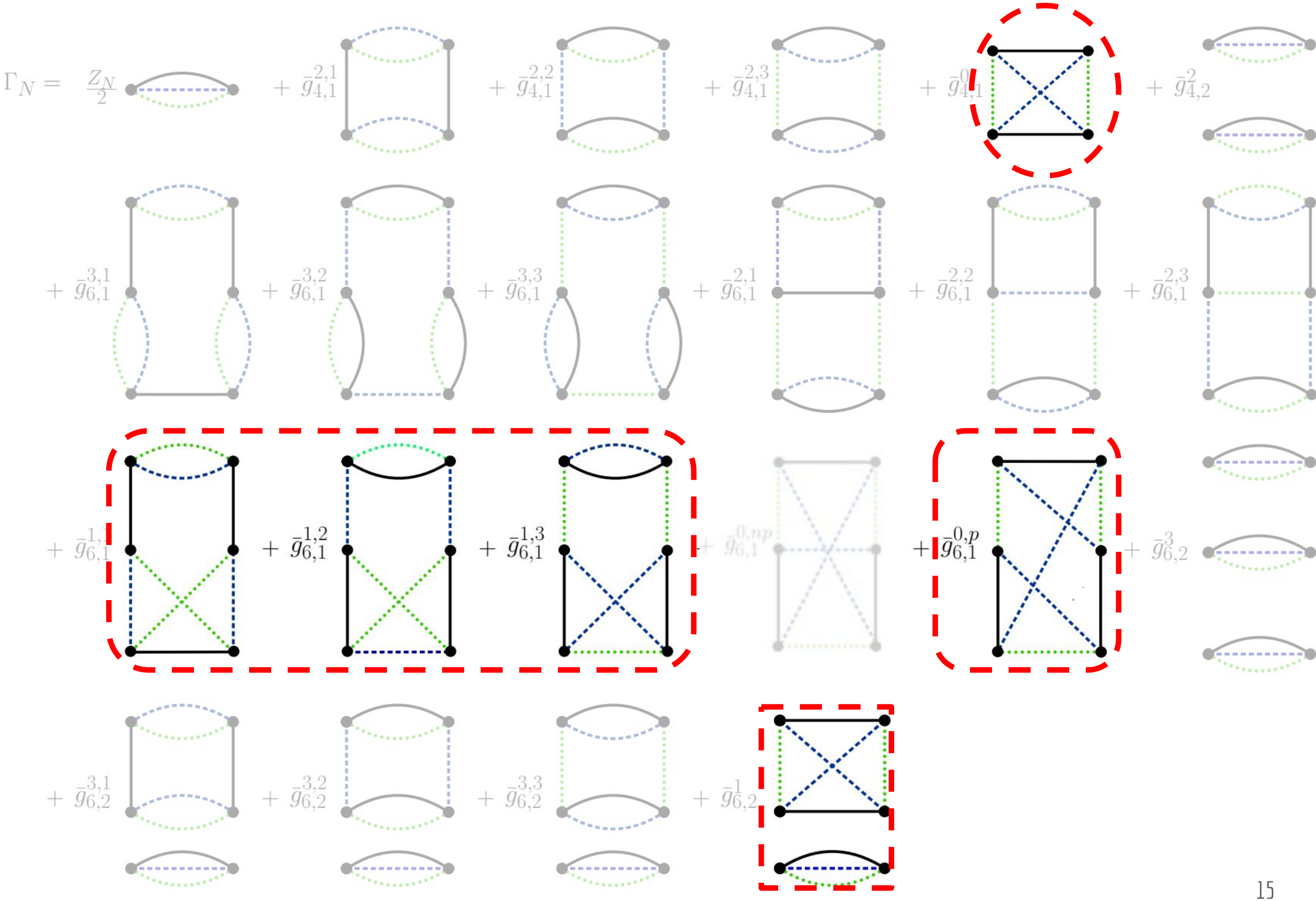
  → Need approximation/ truncation

- Include ALL interactions allowed by symmetry up to 6th order in *T*



- Dinstinguish interactions by prefered color


$\Gamma_N = \frac{Z_N}{2}$ + $\bar{g}^{2,1}_{4,1}$ + $\bar{g}^{2,2}_{4,1}$ + $\bar{g}^{2,3}_{4,1}$ + $\bar{g}^{0}_{4,1}$ + $\bar{g}^{2}_{4,2}$
+ $\bar{g}^{3,1}_{6,1}$ + $\bar{g}^{3,2}_{6,1}$ + $\bar{g}^{3,3}_{6,1}$ + $\bar{g}^{2,1}_{6,1}$ + $\bar{g}^{2,2}_{6,1}$ + $\bar{g}^{2,3}_{6,1}$
+ $\bar{g}^{1,1}_{6,1}$ + $\bar{g}^{1,2}_{6,1}$ + $\bar{g}^{1,3}_{6,1}$ + $\bar{g}^{0,np}_{6,1}$ + $\bar{g}^{0,p}_{6,1}$ + $\bar{g}^{3}_{6,2}$
+ $\bar{g}^{3,1}_{6,2}$ + $\bar{g}^{3,2}_{6,2}$ + $\bar{g}^{3,3}_{6,2}$ + $\bar{g}^{1}_{6,2}$



Γ_N = Z_N/2
+ ḡ^{2,1}_{4,1} + ḡ^{2,2}_{4,1} + ḡ^{2,3}_{4,1} + ḡ^{0}_{4,1} + ḡ^{2}_{4,2}
+ ḡ^{3,1}_{6,1} + ḡ^{3,2}_{6,1} + ḡ^{3,3}_{6,1} + ḡ^{2,1}_{6,1} + ḡ^{2,2}_{6,1} + ḡ^{2,3}_{6,1}
+ ḡ^{1,1}_{6,1} + ḡ^{1,2}_{6,1} + ḡ^{1,3}_{6,1} + ḡ^{0,np}_{6,1} + ḡ^{0,p}_{6,1} + ḡ^{3}_{6,2}
+ ḡ^{3,1}_{6,2} + ḡ^{3,2}_{6,2} + ḡ^{3,3}_{6,2} + ḡ^{1}_{6,2}

Ingredient II: Regulator

$$R_N(\{a_i\},\{b_i\}) = Z_N\, \delta_{a_1 b_1}\, \delta_{a_2 b_2}\, \delta_{a_3 b_3} \left(\frac{N^r}{a_1+a_2+a_3} - 1\right) \theta\left(\frac{N^r}{a_1+a_2+a_3} - 1\right)$$

$$a_1 + a_2 + a_3 < N: \quad R_N > 0$$

No notion of mass dimensionality

$$N < a_1 + a_2 + a_3: \quad R_N = 0$$

$$N \to N' \to \infty: \quad R_N \to \infty$$

⟹ All choices $r \geq 0$ seem to be allowed

Trusting the Fixed Point?

- Ensure stability of results by successively increasing truncation

  1. Starting Scheme: Quartic Order with $-\frac{N\partial_N Z_N}{Z_N} \equiv \eta = 0$
  2. Quartic Order with polynomial approximation for η
  3. Quartic Order with full non-polynomial η
  4. Sixth Order, same procedure for η

- Regulator bound $\eta < r$

- Assumption: Canonical guiding principle

Regulator Bound($\eta < r$) [Meibohm, Pawlowski, Reichert, 2015]

❖ Rewrite anomalous dimension

➢ $\eta = \frac{-N\partial_N Z_N}{Z_N} \implies Z_N \sim N^{-\eta}$

❖ Consider now our regulator

➢ $R_N(\{a_i\},\{b_i\}) = Z_N\, \delta_{a_1 b_1}\delta_{a_2 b_2}\delta_{a_3 b_3} \left(\frac{N^r}{a_1+a_2+a_3} - 1\right)\theta\left(\frac{N^r}{a_1+a_2+a_3} - 1\right)$

❖ Consider $N \to N' \to \infty$

N: IR-cutoff

N': UV-cutoff

➢ $\lim_{N\to N'\to\infty} R_N \sim Z_N\, N^r \sim N^{r-\eta}$

❖ Regulator needs to diverge in that limit

$\Longrightarrow$ $\boxed{\eta < r}$

Canonical Guiding Principle

- Idea: Increasing truncation should not induce new relevant directions

$$\left[\bar{g}^{0}_{4,1}\right] = -3/2 \quad \left[\bar{g}^{2,i}_{4,1}\right] = -2 \quad \left[\bar{g}^{2}_{4,2}\right] = -3$$

$$\left[\bar{g}^{1,i}_{6,1}\right] = -7/2 \quad \left[\bar{g}^{2,i}_{6,1}\right] = -4 \quad \left[\bar{g}^{3,i}_{6,1}\right] = -4$$

$$\left[\bar{g}^{0,np}_{6,1}\right] = -3 \quad \left[\bar{g}^{0,p}_{6,1}\right] = -3 \quad \left[\bar{g}^{3,i}_{6,2}\right] = -5$$

$$\left[\bar{g}^{1}_{6,2}\right] = -9/2 \quad \left[\bar{g}^{3}_{6,3}\right] = -6$$

$\Longrightarrow$ $\max(\theta) - \max(d_{\bar{g}}) \leq 5$

All couplings with scaling dimension > -5 are included

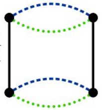
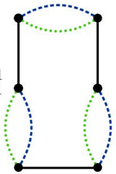
Next coupling with largest scaling dimension $[\bar{g}^{0}_{8,1}] = -5$

$\max(d_{\bar{g}}) = -3/2$ $\Longrightarrow$ $\max(\theta) \leq 3.5$

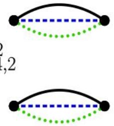
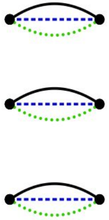
Results (r=1)

Dimensionally-reduced universality classes

- Cyclic Melonic singletrace FP

$\bar{g}^{2,1}_{4,1}$  $\bar{g}^{3,1}_{6,1}$ 

- Multitrace Bubble FP

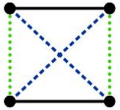
$\bar{g}^{2}_{4,2}$  $\bar{g}^{3}_{6,3}$ 

- Cyclic Melonic Multitrace FP

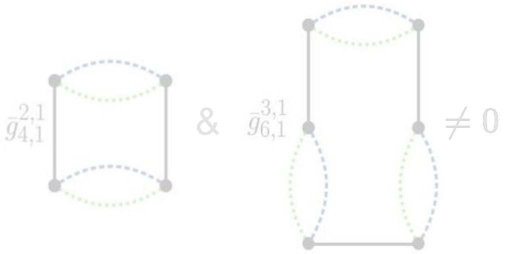
Cyclic Melons & Multi-Trace $\neq 0$

Candidates for universality classes for 3d Quantum Gravity

- Isocolored FP with tetrahedral interaction

$\bar{g}^{0}_{4,1}$ 

& No preferred color



Cyclic Melonic Singletrace FP

- 1 relevant directions @ Quartic and @ Hexic order for all approx. of η

(except @ All Orders, $\eta = 0$

Stable under extension of truncation 

$g_{4,1}^{2,1\,*} = -1$ $\theta_1 = 2$ $g_{4,1}^{2,1\,*} = -0.46$ $\theta_1 =$

$\theta_2 = 1$ $g_{6,1}^{3,1\,*} = -0.50$ $\theta_2 =$

Regulator Bound η 

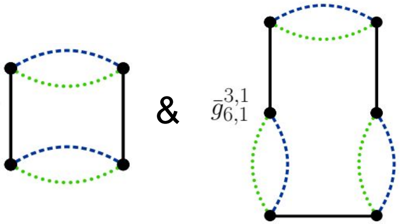
$g_{4,1}^{2,1\,*} = -0.43$ $\theta_1 = 2$ $g_{4,1}^{2,1\,*} = -0.30$ $\theta_1 =$

Canonical Guiding Principle $= -0.18$ 

η

$g_{4,1}^{2,1\,*} = -0.38$ $\theta_1 = 2.27$ $g_{4,1}^{2,1\,*} = -0.28$ $\theta_1 = 2.19$

$\theta_2 = -0.16$ $g_{6,1}^{3,1\,*} = -0.15$ $\theta_2 = -0.03$



$\bar{g}_{4,1}^{2,1}$ & $\bar{g}_{6,1}^{3,1}$ $\neq 0$

Cyclic Melonic Singletrace FP

- 1 relevant directions @ Quartic and @ Hexic order for all approx. of η

(except @ All Orders, $\eta = 0 \longrightarrow$ 2 rel. dir.)

Starting Scheme: Quartic Order with $\eta = 0$

$g_{4,1}^{2,1\,*} = -1$ $\qquad \theta_1 = 2$, $\theta_2 = 1$

2) Quartic Order with polyn. η

$g_{4,1}^{2,1\,*} = -0.43$ $\qquad \theta_1 = 2$, $\theta_2 = -0.14$

3) Quartic Order with full η

$g_{4,1}^{2,1\,*} = -0.38$ $\qquad \theta_1 = 2.27$, $\theta_2 = -0.16$

4) Hexic Order with $\eta = 0$

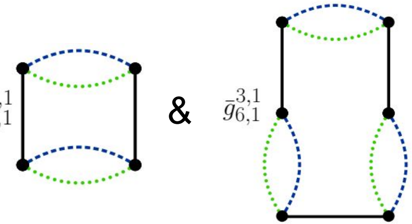
$g_{4,1}^{2,1\,*} = -0.46$, $g_{6,1}^{3,1\,*} = -0.50$ $\qquad \theta_1 = 2$, $\theta_2 = 0.53$

5) Hexic Order with polyn. η

$g_{4,1}^{2,1\,*} = -0.30$, $g_{6,1}^{3,1\,*} = -0.18$ $\qquad \theta_1 = 2$, $\theta_2 = -0.03$

6) Hexic Order with full η

$g_{4,1}^{2,1\,*} = -0.28$, $g_{6,1}^{3,1\,*} = -0.15$ $\qquad \theta_1 = 2.19$, $\theta_2 = -0.03$



$\bar{g}^{2,1}_{4,1}$ & $\bar{g}^{3,1}_{6,1}$ $\neq 0$

Cyclic Melonic Singletrace FP

- 1 relevant directions @ Quartic and @ Hexic order for all approx. of η

(except @ All Orders, $\eta = 0 \longrightarrow$ 2 rel. dir.)

Starting Scheme: Quartic Order with $\eta = 0$

$g^{2,1\,*}_{4,1} = -1$ $\quad \theta_1 = 2$, $\theta_2 = 1$ $\quad \eta = 0 < 1$

2) Quartic Order with polyn. η

$g^{2,1\,*}_{4,1} = -0.43$ $\quad \theta_1 = 2$, $\theta_2 = -0.14$ $\quad \eta = -0.57 < 1$

3) Quartic Order with full η

$g^{2,1\,*}_{4,1} = -0.38$ $\quad \theta_1 = 2.27$, $\theta_2 = -0.16$ $\quad \eta = -0.58 < 1$

4) Hexic Order with $\eta = 0$

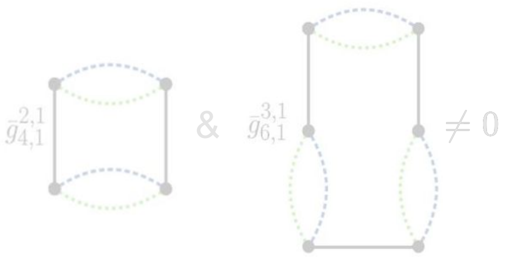
$g^{2,1\,*}_{4,1} = -0.46$, $g^{3,1\,*}_{6,1} = -0.50$ $\quad \theta_1 = 2$, $\theta_2 = 0.53$ $\quad \eta = 0 < 1$

5) Hexic Order with polyn. η

$g^{2,1\,*}_{4,1} = -0.30$, $g^{3,1\,*}_{6,1} = -0.18$ $\quad \theta_1 = 2$, $\theta_2 = -0.03$ $\quad \eta = -0.40$

6) Hexic Order with full η

$g^{2,1\,*}_{4,1} = -0.28$, $g^{3,1\,*}_{6,1} = -0.15$ $\quad \theta_1 = 2.19$, $\theta_2 = -0.03$ $\quad \eta = -0.41$



Cyclic-Melonic Singletrace FP

Recall: $\max(\theta) \leq 3.5$

- 1 relevant directions @ Quartic and @ Hexic order for all approx. of η

(except @ All Orders, $\eta = 0 \longrightarrow$ 2 rel. dir.)

Starting Scheme: Quartic Order with $\eta = 0$

$g_{4,1}^{2,1\,*} = -1$ $\theta_1 = 2$ $\theta_2 = 1$

2) Quartic Order with polyn. η

$g_{4,1}^{2,1\,*} = -0.43$ $\theta_1 = 2$ $\theta_2 = -0.14$

3) Quartic Order with full η

$g_{4,1}^{2,1\,*} = -0.38$ $\theta_1 = 2.27$ $\theta_2 = -0.16$

4) Hexic Order with $\eta = 0$

$g_{4,1}^{2,1\,*} = -0.46$ $g_{6,1}^{3,1\,*} = -0.50$ $\theta_1 = 2$ $\theta_2 = 0.53$

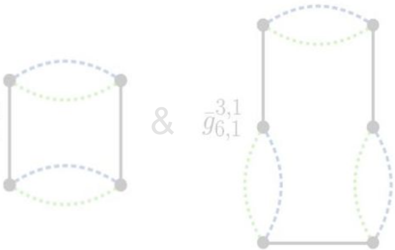
5) Hexic Order with polyn. η

$g_{4,1}^{2,1\,*} = -0.30$ $g_{6,1}^{3,1\,*} = -0.18$ $\theta_1 = 2$ $\theta_2 = -0.03$

6) Hexic Order with full η

$g_{4,1}^{2,1\,*} = -0.28$ $g_{6,1}^{3,1\,*} = -0.15$ $\theta_1 = 2.19$ $\theta_2 = -0.03$

Cyclic-Melonic Singletrace FP



$\bar{g}^{2,1}_{4,1}$ & $\bar{g}^{3,1}_{6,1}$ $\neq 0$

- 1 relevant directions @ Quartic and @ Hexic order for all approx. of η

(except @ All Orders, $\eta = 0 \longrightarrow$ 2 rel. dir.)

Stable under extension of truncation ✓

$g^{2,1\,*}_{4,1} = -1$ $\theta_1 = 2$ $\theta_2 = 1$

$g^{2,1\,*}_{4,1} = -0.46$ $g^{3,1\,*}_{6,1} = -0.50$ $\theta_1 = 2$ $\theta_2 = 0.53$

Regulator Bound ✓

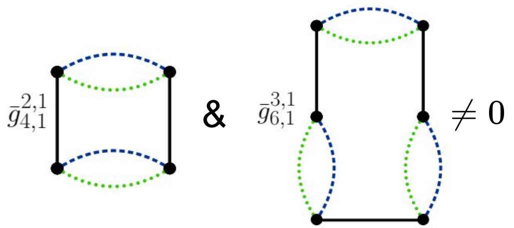
$g^{2,1\,*}_{4,1} = -0.43$ $\theta_1 = 2$

$g^{2,1\,*}_{4,1} = -0.30$ $g^{3,1\,*}_{6,1} = -0.18$ $\theta_1 = 2$

Canonical Guiding Principle ✓

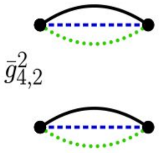
$g^{2,1\,*}_{4,1} = -0.38$ $\theta_1 = 2.27$ $\theta_2 = -0.16$

$g^{2,1\,*}_{4,1} = -0.28$ $g^{3,1\,*}_{6,1} = -0.15$ $\theta_1 = 2.19$ $\theta_2 = -0.03$

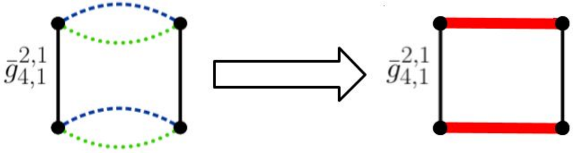


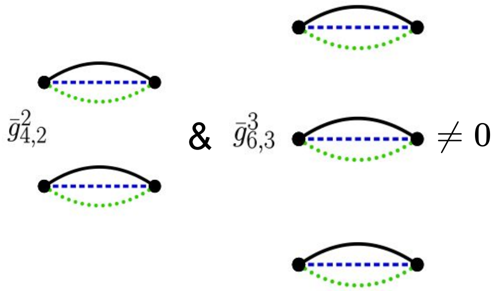
Cyclic Melonic Singletrace FP

- Also found in complex model
- Distinguishing colours crucial in decoupling



- Dimensional Reduction





Multitrace Bubble FP

Starting Scheme: Quartic Order with $\eta = 0$

$g^2_{4,2}{}^* = -3.75$

$\theta_1 = 3$

$\theta_2 = -1.5$

2) Quartic Order with polyn. η

$g^2_{4,2}{}^* = -1.67$

$\theta_1 = 3$

$\theta_2 = 0.17$

$\eta = -0.83$

3) Quartic Order with full η

$g^2_{4,2}{}^* = -1.44$

$\theta_1 = 3.47$

$\theta_2 = 0.18$

$\eta = -0.84$

4) Hexic Order with $\eta = 0$

$g^2_{4,2}{}^* = -1.73$

$g^3_{6,3}{}^* = -7.45$

$\theta_1 = 3$

$\theta_2 = -1.5$

5) Hexic Order with polyn. η

$g^2_{4,2}{}^* = -1.16$

$g^3_{6,3}{}^* = -2.82$

$\theta_1 = 3$

$\theta_2 = -0.34$

$\eta = -0.58$

6) Hexic Order with full η

$g^2_{4,2}{}^* = -1.05$

$g^3_{6,3}{}^* = -2.27$

$\theta_1 = 3.32$

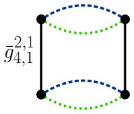
$\theta_2 = -0.33$

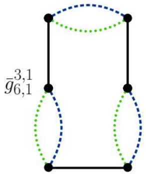
$\eta = -0.59$

Results (r=1)

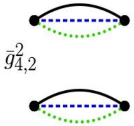
Dimensionally-reduced universality classes

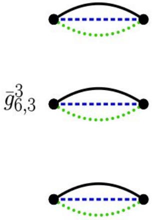
- Cyclic Melonic singletrace FP





- Multitrace Bubble FP



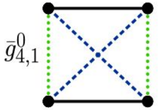


- Cyclic Melonic Multitrace FP

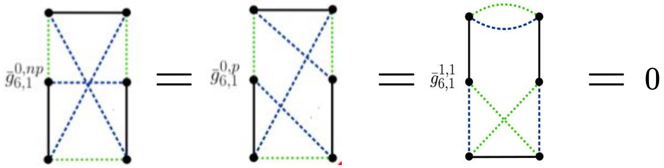
Cyclic Melons & Multi-Trace $\neq 0$

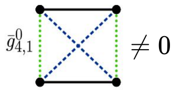
Candidates for universality classes for 3d Quantum Gravity

- Isocolored FP with tetrahedral interaction

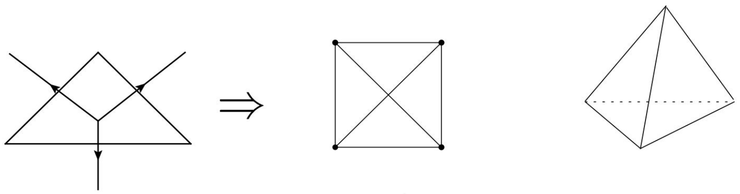


& No preferred color

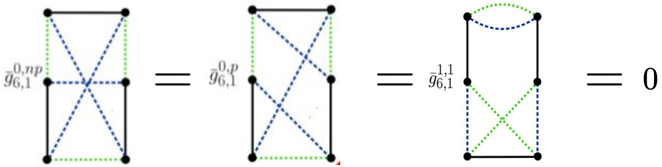


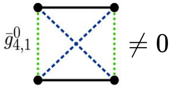


Isocolored FP
with tetrahedral interaction

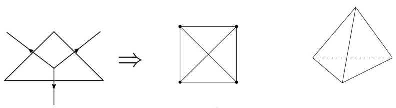


NEW!!! Not featured in the complex model





Isocolored FP with tetrahedral interaction



Starting Scheme: Quartic Order with $\eta = 0$

Not present ($\beta_{g^0_{4,1}} = (\frac{3}{2} + 2\eta) g^0_{4,1}$)

2) Quartic Order with polyn. η

$\theta_1 = 2.98$

$\theta_{2,3} = -0.28 \pm 0.22\, i$

$\eta = -0.75$

3) Quartic Order with full η

$\theta_1 = 2.60$

$\theta_{2,3} = -0.27 \pm 0.21\, i$

$\eta = -0.75$

4) Hexic Order with $\eta = 0$

$\theta_{1,2} = 1.95 \pm 0.69\, i$

$\theta_{3,4} = 0.38$

$\theta_{5,6} = -0.03 \pm 5.96\, i$

5) Hexic Order with polyn. η

$\theta_{1,2} = 1.46 \pm 1.39\, i$

$\theta_{3,4} = 0.15$

$\theta_{5,6} = -0.11 \pm 4.83\, i$

$\eta = -0.32$

6) Hexic Order with full η

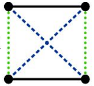
$\theta_{1,2} = 1.3 \pm 1.56\, i$

$\theta_{3,4} = 0.13$

$\theta_{5,6} = -0.02 \pm 5.10\, i$

$\eta = -0.33$

Conclusions

- Indications for two types of universality classes
  - Dimensional Reduction
  - Candidate for 3d Quantum Gravity ?
- New universality class not featured in the complex model $\bar{g}^0_{4,1}$ 
- Asymptotic Safety and Dynamical Triangulations two sides of the same medal?
- What’s Next? Rank-4 Real Tensor Model
  (A. Eichhorn, J.L., A. Pereira and A. Sikandar)