# Exact RG equations, Quantum Gravity and Ghosts

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# Asymptotic safety and the Wilsonian Effective action

The Wilsonian action is an effective action valid at a scale $\Lambda$

$$I_\Lambda = \sum \bar{g}^A \Phi_A$$

$\Phi_A$ is a complete set of local operators consistent with field content and symmetries and up to redundancies

$$\bar{g}^A = \Lambda^{4-n_A} g^A$$

$g^A$ is dimensionless $n_A$ is dimension of operator

This action is local (or quasi-local) unlike the 1PI action which may be non-local. Also $I_\Lambda$ is to be used at energy scales $E \leq \Lambda$

$I_\Lambda$ is depends on the single scale $\Lambda$. To discuss physics at a lower scale $E \leq \Lambda' < \Lambda$ we need the Wilsonian action $I_{\Lambda'}$, obtained by integrating out modes between $\Lambda'$ and $\Lambda$ .

# A Polchinski type RG Equation

This process of integrating out gives an eqn of the form

$$I_{\Lambda'} = F[\Lambda', \Lambda : I_\Lambda]$$

$$\Lambda' \frac{d}{d\Lambda'} I_{\Lambda'} = \Lambda' \frac{d}{d\Lambda'} F[\Lambda', \Lambda : I_\Lambda]$$

Taking the limit $\Lambda' \to \Lambda$

$$\Lambda \frac{d}{d\Lambda} I_\Lambda = \Lambda' \frac{d}{d\Lambda'} F[\Lambda', \Lambda : I_\Lambda]|_{\Lambda'=\Lambda}$$

This is a first order differential equation – a beta function eqn for the Wilsonian action $I_\Lambda$ - given an initial action it determines the action at any other RG time as in the first eqn above.

# RG eqn for $I_\Lambda$

$$e^{-\Gamma(\phi_c)} = \int [d\phi] e^{-I[\phi] - J.(\phi - \phi_c)}|_{J = -\partial\Gamma/\partial\phi_c}.$$

$$\phi = \phi_c + \phi'$$

$$\begin{aligned} e^{-\Gamma(\phi_c)} &= \int [d\phi'] e^{-I[\phi_c + \phi'] - J.\phi'}|_{J = -\partial\Gamma/\partial\phi_c} \\ &= \int [d\phi'] e^{-\{I[\phi_c] + \frac{1}{2}\phi'.\frac{\delta^2 I}{\delta\phi_c^2}.\phi' + I_{\mathrm{i}}[\phi_c, \phi'] + (J + \frac{\delta I[\phi_c]}{\delta\phi_c}).\phi'\}}|_{J = -\delta\Gamma/\delta\phi_c} \\ &= e^{-I[\phi_c]} e^{-\frac{1}{2}\mathrm{Tr}\ln K[\phi_c]} e^{-I_{\mathrm{i}}[\phi_c, -\frac{\delta}{\delta \bar{J}}]} e^{\frac{1}{2}\bar{J}.K[\phi_c]^{-1}.\bar{J}}|_{\bar{J} = \delta I[\phi_c]/\partial\phi_c - \delta\Gamma/\delta\phi_c} \end{aligned}$$

$I_{\mathrm{i}}[\phi_c, \phi']$ contains all powers of $\phi'$ which are higher than quadratic

Use this formal structure to define a Wilsonian action $I_{\Lambda'}$ in terms of the "initial" action $I_\Lambda$ by regularizing the propagator and the one-loop effective action

$$K^{-1}_{\Lambda',\Lambda}(\phi_c; x, y) = < x| \int_{1/\Lambda^2}^{1/\Lambda'^2} ds e^{-\hat{K}[\phi_c]s} |y > \quad \ln K_{\Lambda',\Lambda}[\phi_c; x, y] = - < x| \int_{1/\Lambda^2}^{1/\Lambda'^2} \frac{ds}{s} e^{-\hat{K}[\phi_c]s} |y >$$

# RG eqn for $I_\Lambda$

$$e^{-I_{\Lambda'}(\phi_c)} = e^{-I_\Lambda[\phi_c]} e^{-\frac{1}{2}\mathrm{Tr}\ln K_{\Lambda',\Lambda}[\phi_c]} e^{-I_{i,\Lambda}[\phi_c, -\frac{\delta}{\delta J}]} e^{\frac{1}{2}\bar{J}.K_{\Lambda',\Lambda}[\phi_c]^{-1}.\bar{J}}|_{\bar{J}=\delta I_\Lambda[\phi_c]/\partial\phi_c - \delta I_{\Lambda',}/\delta\phi_c}$$

$$\Lambda' \to 0,\ I_{\Lambda'} \to \Gamma;\ \Lambda' \to \Lambda,\ K^{-1}_{\Lambda',\Lambda},\ \ln K_{\Lambda',\Lambda} \to 0, \Rightarrow I_{\Lambda'} \to I_\Lambda$$

$$\begin{aligned} -e^{-I_{\Lambda'}(\phi_c)}\dot{I}_{\Lambda'}[\phi_c] \quad = \quad & e^{-I_\Lambda[\phi_c]} e^{-\frac{1}{2}\mathrm{Tr}\ln K_{\Lambda',\Lambda}[\phi_c]} e^{-I_{i\Lambda}(\phi_c, -\frac{\delta}{\delta J})} \times \\ & (-\frac{1}{2}\frac{d}{dt}\mathrm{Tr}\ln K_{\Lambda',\Lambda}[\phi_c] + \dot{\bar{J}}.K^{-1}_{\Lambda',\Lambda}.\bar{J} + \frac{1}{2}\bar{J}.\dot{K}^{-1}_{\Lambda',,\Lambda}.\bar{J}) \times \\ & e^{\frac{1}{2}\bar{J}.K_{\Lambda',,\Lambda}[\phi_c]^{-1}.\bar{J}}|_{J=\delta I_\Lambda[\phi_c]/\partial\phi_c - \delta\Gamma_{\Lambda',}/\delta\phi_c} \end{aligned}$$

$$dt \equiv d\Lambda'/\Lambda'$$

Take limit $\Lambda' \to \Lambda$

$$\Lambda\frac{d}{d\Lambda}I_\Lambda[\phi_c] = \frac{1}{2}\Lambda'\frac{d}{d\Lambda'}\mathrm{Tr}\ln K_{\Lambda',\Lambda}[\phi_c]|_{\Lambda'=\Lambda} = \mathrm{Tr}\exp\{-\frac{1}{\Lambda^2}\frac{\delta}{\delta\phi_c}\otimes\frac{\delta}{\delta\phi_c}I_\Lambda[\phi_c]\}$$

# RG eqn for $I_\Lambda$

Alternatively use $dI_{\Lambda'}/d\Lambda=0$

$$
\begin{aligned}
0 \;=\; & e^{-I_\Lambda[\phi_c]} e^{-\frac{1}{2}\mathrm{Tr}\ln K_{\Lambda',\Lambda}[\phi_c]} e^{-I_{i\Lambda}[\phi_c,-\frac{\delta}{\delta J}]} \{-\Lambda \frac{d}{d\Lambda} I_\Lambda[\phi_c] - \Lambda \frac{d}{d\Lambda}\left(\frac{1}{2}\mathrm{Tr}\ln K_{\Lambda',\Lambda}[\phi_c] + I_{i\Lambda}[\phi_c, -\frac{\delta}{\delta \bar{J}}]\right) \\
& + \left(\Lambda \frac{d}{d\Lambda}\bar{J}\right).K^{-1}_{\Lambda',\Lambda}.\bar{J} + \frac{1}{2}\bar{J}.\left(\Lambda \frac{d}{d\Lambda} K^{-1}_{\Lambda',\Lambda}\right).\bar{J}\} \times \\
& e^{\frac{1}{2}\bar{J}.K_{\Lambda',\Lambda}[\phi_c]^{-1}.\bar{J}}|_{\bar{J}=\delta I_\Lambda[\phi_c]/\partial\phi_c - \delta I_{\Lambda'}/\delta\phi_c}
\end{aligned}
$$

$$\text{limit } \Lambda' \to \Lambda$$

$$\Lambda \frac{d}{d\Lambda} I_\Lambda[\phi_c] \;=\; -\frac{1}{2}\Lambda \frac{d}{d\Lambda}\mathrm{Tr}\ln K_{\Lambda',\Lambda}[\phi_c]|_{\Lambda'=\Lambda} = \mathrm{Tr}\exp\{-\frac{1}{\Lambda^2}\frac{\delta}{\delta\phi_c} \otimes \frac{\delta}{\delta\phi_c} I_\Lambda[\phi_c]\}$$

Gauge fixing: $K_{\Lambda',\Lambda}[\phi_c] \to K_{\Lambda',\Lambda}[\phi_c] + K^{\mathrm{GF}}_{\Lambda',\Lambda}[\phi_c,\alpha]$ Ghosts: $e^{+\frac{1}{2}\mathrm{Tr}\ln K^{\mathrm{ghost}}_{k,\Lambda}[\phi_c]}$

$$
\begin{aligned}
\Lambda \frac{d}{d\Lambda} I_\Lambda[\phi_c] \;=\; & \frac{1}{2}\Lambda' \frac{d}{d\Lambda'}\left\{\mathrm{Tr}\ln\left(K_{\Lambda',\Lambda}[\phi_c] + K^{\mathrm{GF}}_{\Lambda',\Lambda}[\phi_c,\alpha]\right) - \mathrm{Tr}\ln \mathrm{K}^{\mathrm{ghost}}_{\Lambda',\Lambda}\right\}|_{\Lambda'=\Lambda} \\
=\; & \mathrm{Tr}\exp\left\{-\frac{1}{\Lambda^2}\left(I^{(2)}_\Lambda[\phi_c] + I^{(2)\mathrm{GF}}_\Lambda[\phi_c,\alpha]\right)\right\} \\
& -\mathrm{Tr}\exp\left\{-\frac{1}{\Lambda^2} I^{(2)\mathrm{ghost}}_\Lambda[\phi_c,\alpha]\right\}
\end{aligned}
$$

# Wilsonian action to 1PI action?

The Wilsonian action $I_\Lambda = \sum \bar{g}(\Lambda)\Phi_A[\phi]$ is an expansion in an infinite set of local operators valid at scales $E \leq \Lambda$. The idea is that this an be used to compute say the S-matrix for scattering at that scale.

This means that there is no limit in which we can get the 1PI action. This needs to be computed from the Wilsonian action by doing the functional integral over modes below $\Lambda$ - in perturbation theory for instance.

The 1PI action is a non-local object – cannot be obtained from a local object unless we know how to do the sum!

The Wilsonian action at $\Lambda_{IR}$ however is used at scales $E \leq \Lambda_{IR}$ in the same way as the "classical action" which was useful at some high scale $\Lambda_{UV}$ when discussing physics close to that scale. The expectation is that one can compute the S-matrix at that scale to some reasonable accuracy by using low order perturbation theory. See for example the discussion in Weinberg's cosmology application 0911.3165.

# Gauge invariant observables-S matrix

$$I_\Lambda = \sum \bar{g}^A(\Lambda)\Phi_A[\phi]$$

$$I_\Lambda = \int d^4x\sqrt{g}[\Lambda^4 g_0(\Lambda) + \Lambda^2 g_1(\Lambda)R + (g_{2a}(\Lambda)R_{\mu\nu}R^{\mu\nu} + g_{2b}(\Lambda)R^2 + g_{3b}R_{....}R^{\cdots}) + \Lambda^{-2}(g_{3a}(\Lambda)RR_{\mu\nu}R^{\mu\nu} + \ldots) + (g_{3a}^{(1)}R\Box R + \ldots) + O(\Lambda^{-4})] + I_\Lambda^{\mathrm{GF}} + I_\Lambda^{\mathrm{ghost}}.$$

In general expect gauge fixing dependence in evolution eqns since K dependent on $\alpha$. Physical quantities should be gauge independent

No local gauge invariant obervables. S-matrix expected to be gauge invariant. Need to define dressed asymptotic states – i.e. well seperated asymptotic particle and graviton states need to have coherent graviton dressing as in QED

Fadeev+Kulish. Giddings, Akhoury,…SdA

work in progress

# Beta functions

$$\bar{g}^A = \Lambda^{4-n_A} g^A$$

$$\dot{g}^A + (4 - n_A) g^A = \Lambda^{n_A - 4} \mathrm{Tr} \exp\{-\frac{1}{\Lambda^2} \frac{\delta}{\delta \phi_c} \otimes \frac{\delta}{\delta \phi_c} I_\Lambda[\phi_c]\}|_{\Phi_A},\ A = 0, 1, 2, \ldots.$$

$$\dot{g}^A + (4 - n_A) g^A = \eta^A(\{g\})$$

Non-trivial fixed point $g^A = g^A_*$ not all zero solving

$$(4 - n_A) g^A = \eta^A(\{g\})$$

# Critical surface

$$\frac{d\delta g^A}{dt} = \sum_B \left( -(4-n_A)\delta^A_B + \frac{\partial \eta^A(\{g\})}{\partial g^B})|_{g_*} \right) \delta g^B \equiv \sum_B D^A{}_B(g_*)\delta g^B$$

$$\mathbf{D}\mathbf{u}^{(J)} = \theta^{(J)}\mathbf{u} \qquad \theta^{(0)}, \ldots \theta^{(R-1)} < 0,$$

$$\mathbf{u}^{(J)}(t) = e^{-|\theta^{(J)}|t}\mathbf{u}^{(J)}(0) \to 0,\ J = 0, \ldots R-1,\ t \to \infty$$

If the anomalous dimensions to not overwhelm the canonical dimensions then expect just a finite number of such “relevant” operators to be determined by experiment. All others (an infinite number of couplings) go to their fixed point values. Of course all this assumes the existence of such a non-trivial fixed point.

So if this is the case we have a predictive framework for a 4D QFT of quantum gravity coupled to the Standard Model assuming the above holds after coupling to the SM

# Gravity plus Scalar field

$$\begin{aligned} I_\Lambda &= \int d^4x\sqrt{g}[\Lambda^4 g_0(\Lambda) + \Lambda^2 g_1(\Lambda)R + (g_{2a}(\Lambda)R_{\mu\nu}R^{\mu\nu} + g_{2b}(\Lambda)R^2 + g_{3b}R_{....}R^{....}) \\ &\quad + \Lambda^{-2}(g_{3a}(\Lambda)RR_{\mu\nu}R^{\mu\nu} + \ldots) + (g_{3a}^{(1)} R\Box R + \ldots) + O(\Lambda^{-4})] \\ &\quad + \int d^4x\sqrt{g}[Z(\phi^2/\Lambda^2)\frac{1}{2}\phi(-\Box)\phi + V(\phi,\Lambda) + \xi(\phi,\Lambda)R + O(\partial^4)] \\ &\quad + I_\Lambda^{(\mathrm{G.F.})} + I_\Lambda^{(\mathrm{ghost})}. \end{aligned}$$

$$\begin{aligned} V(\phi,\Lambda) &= \frac{1}{2}\lambda_1(\Lambda)\Lambda^2\phi^2 + \frac{1}{4!}\lambda_2(\Lambda)\phi^4 + \frac{1}{6!}\lambda_3(\Lambda)\Lambda^{-2}\phi^6 + \ldots, \\ Z\left(\frac{\phi^2}{\Lambda^2}\right) &= Z_0 + \frac{1}{2}Z_1\frac{\phi^2}{\Lambda^2} + \ldots \\ \xi(\phi,\Lambda) &= \frac{1}{2}\xi_1\phi^2 + \frac{1}{4!}\xi_2\phi^4 + \ldots \end{aligned}$$

For earlier work on AS using the Wetterich eqn see reviews: Codello, Percacci + Rahmede 0805.2909; Reuter + Saueressig 1202.2274, Percacci ISBN 981723207175

# Beta function eqns

$$\frac{\delta}{\delta\phi_c} \otimes \frac{\delta}{\delta\phi_c} I_\Lambda[\phi_c] \equiv \mathbf{I}_\Lambda^{(2)} = -\nabla^2 \mathbf{I} + \mathbf{E}$$

$$\mathbf{E} = \Lambda^2 \mathbf{E}_0 + \hat{\mathbf{E}}$$

$$\mathbf{E}_0 = \begin{bmatrix} \frac{g_0}{g_1}\mathbf{I} & 0 \\ 0 & Z_0^{-1}\lambda_1(\Lambda) \end{bmatrix} \qquad \hat{\mathbf{E}} = \begin{bmatrix} \hat{O}_2 & \hat{O}_1 \\ \hat{O}_1^{\mathrm{T}} & \hat{\hat{O}}_2 \end{bmatrix}$$

$$\exp[-\Lambda^{-2}(-\nabla^2 \mathbf{I} + \mathbf{E})] = e^{-\mathbf{E}_0} \exp[-\Lambda^{-2}(-\nabla^2 \mathbf{I} + \hat{\mathbf{E}})]$$

Working with truncation up to two derivative terms can calculate beta functions (say in Landau gauge as in Reuter – 9605030
Codello, Percacci, Rahmede 0805.2909 )

# Heat Kernal expansion

$$
\begin{aligned}
\mathrm{Tr}e^{-\mathbf{K}/\Lambda^2} &= \frac{1}{(4\pi)^2}[\Lambda^4 B_0(\mathbf{K}) + \Lambda^2 B_2(\mathbf{K}) + B_4(\mathbf{K}) + \Lambda^{-2} B_6(\mathbf{K}) + \ldots]. \\
B_n(\mathbf{K}) &= \int d^4x\sqrt{g}\,\mathrm{tr}e^{-\mathbf{E}_0}\mathbf{b}_n.
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{b}_0 &= \mathbf{I}, \\
\mathbf{b}_2 &= \frac{R}{6}\mathbf{I} - \hat{\mathbf{E}}, \\
\mathbf{b}_4 &= \frac{1}{180}(R^{\cdots\cdot}R_{\cdots\cdot} - R^{\cdot\cdot}R_{\cdot\cdot} + \frac{5}{2}R^2 + 6\nabla^2 R)\mathbf{I} \\
&\quad -\frac{1}{6}R\hat{\mathbf{E}} + \frac{1}{2}\hat{\mathbf{E}}^2 - \frac{1}{6}\nabla^2\hat{\mathbf{E}}.
\end{aligned}
$$

# Gravity scalar beta functions

$$\dot{x} = \Lambda \tfrac{d}{d\Lambda} x, \qquad \hat{\lambda}_i = \lambda_i/(Z_0)^i \, \gamma_i = \ln Z_i$$

$\hat{\lambda}_i = \lambda_i/(Z_0)^i \, \gamma_i = \ln Z_i$

$$\begin{aligned}
\dot{g}_0 + 4g_0 &= \frac{1}{(4\pi)^2}[10e^{-g_0/g_1} - 4 + e^{-\hat{\lambda}_1}], \\
\dot{g}_1 + 2g_1 &= -\frac{1}{(4\pi)^2}\frac{1}{3}[13e^{-g_0/g_1} + 5 + \frac{1}{2}e^{-\hat{\lambda}_1}(1-6\hat{\xi}_1)], \\
\dot{\hat{\lambda}}_1 + \dot{\gamma}_0\hat{\lambda}_1 + 2\hat{\lambda}_1 &= -\frac{e^{-\hat{\lambda}_1}}{(4\pi)^2}[\frac{\hat{\lambda}_2}{2} + \frac{1}{8}\frac{\hat{\lambda}_1^2}{g_1}] - \frac{5}{(4\pi)^2}e^{-g_0/g_1}\frac{\hat{\lambda}_1}{g_1}.
\end{aligned}$$

Or with $g_N(\Lambda) = 2\kappa^2(\Lambda)\Lambda^2 = -\frac{1}{g_1(\Lambda)}, \; 2\lambda_{CC} = \Lambda^2 2\kappa^2 g_0 = -\frac{g_0}{g_1}.$

$$\begin{aligned}
\dot{\lambda}_{CC} + 2\lambda_{CC} &= \frac{g_N}{(4\pi)^2}\left[(5 - \frac{13}{3}\lambda_{CC})e^{2\lambda_{CC}} - (2 + \frac{5}{3}\lambda_{CC}) + e^{-\hat{\lambda}_1}(\frac{1}{2} - \frac{1}{6}\lambda_{CC}(1-6\hat{\xi}_1))\right] \\
\dot{g}_N - 2g_N &= -\frac{g_N^2}{(4\pi)^2}\frac{1}{3}\left[13e^{2\lambda_{CC}} + 5 + \frac{1}{2}e^{-\hat{\lambda}_1}(1-6\hat{\xi}_1)\right], \\
\dot{\hat{\lambda}}_1 + \dot{\gamma}_0\hat{\lambda}_1 + 2\hat{\lambda}_1 &= -\frac{e^{-\hat{\lambda}_1}}{(4\pi)^2}[\frac{\hat{\lambda}_2}{2} - \frac{1}{8}g_N\hat{\lambda}_1^2] + \frac{5}{(4\pi)^2}e^{2\lambda_{CC}}g_N\hat{\lambda}_1.
\end{aligned}$$

# Fixed point

Gaussian FP: $g_N = \lambda_{CC} = \lambda_1 = \lambda_2 = \xi_1 = \xi_2 = 0.$

There is also a non-trivial fixed point – for instance the second eqn is solved for

$$g_N^* = 6(4\pi)^2[13e^{2\lambda_{CC}} + 5 + \frac{1}{2}e^{-\hat{\lambda}_1}(1 - 6\hat{\xi}_1)]^{-1}$$

Giving a positive value for Newton's constant provided $\hat{\xi}_1$ is not too large. To get the other values need to solve some transcendental eqns.
The question is how stable are the fixed point eqns under inclusion of higher dimension operators.

Note in previous derivations there is a singularity $1/1 - 2\lambda_{CC})$

This is absent here and is clearly spurious coming from $e^{2\lambda_{CC}} = 1/e^{-2\lambda_{CC}}$

Expanding the denominator to leading order gives spurious singularity.

# Higher derivative term corrections to CC



$$\Lambda\frac{d}{d\Lambda}I_\Lambda[\phi_c] = \frac{1}{2}k\frac{d}{dk}\mathrm{Tr}\ln K_{k,\Lambda}[\phi_c]|_{k=\Lambda} = \mathrm{Tr}\exp\{-\frac{1}{\Lambda^2}\frac{\delta}{\delta\phi_c}\otimes\frac{\delta}{\delta\phi_c}I_\Lambda[\phi_c]\}$$

$$H_0(x;y) =< x|e^{-s_0\hat{\mathrm{p}}^2}|y >= \frac{1}{(2\pi)^4}\left(\frac{\pi}{s_0}\right)^2 e^{-(x-y)^2/s_0};\ s_0 \equiv \frac{1}{\Lambda^2},\ \hat{\mathrm{p}} = -i\partial$$

For CC sufficient to consider flat space – so write

$$\hat{\mathbf{K}} = \hat{\mathbf{p}}^2 + s_0h_1(\hat{\mathbf{p}}^2)^2 + s_0^2h_2(\hat{\mathbf{p}}^2)^3 + \ldots = \hat{\mathbf{p}}^2 + \sum_{n=1}^{\infty} s_0^n h_n(\hat{\mathbf{p}}^2)^{n+1}$$

$$\begin{aligned} H(x;x) &= < x|e^{-s_0\hat{\mathrm{K}}}|x >= e^{-\sum_{n=1}^{\infty} s_0^{n+1}h_n\frac{\partial^{n+1}}{\partial s^{n+1}}} < x|e^{-s\hat{\mathrm{p}}^2}|x > |_{s=s_0} \\ &= e^{-\sum_{n=1}^{\infty} s_0^{n+1}h_n\frac{\partial^{n+1}}{\partial s^{n+1}}}\frac{1}{(2\pi)^4}\left(\frac{\pi}{s}\right)^2 |_{s=s_0} \equiv \frac{1}{16\pi^2}G(\mathbf{h})\Lambda^4. \end{aligned}$$

# Higher derivative term corrections

In calculating effect of higher derivative terms to CC can set curvatures and background fields to zero after differentiating to define $\widehat{K}$ from $I_\Lambda$.

Tantamount to putting for example $\delta R \sim \nabla^2 \delta g$ and then taking $R \to 0$ after differentiation.

Contributions to $\widehat{K}$ will only come from E-H term ''$R^2$ '' terms and terms of the form $R(\nabla^2)^n R$.

In particular none of the terms $R^n, n > 2$ in a f($R$) trunction will contribute Ditto for $R^n_{\mu\nu}$, $R^n_{\mu\nu\lambda\sigma}$ etc for $n > 2$.

So RG eqn for the CC will be corrected to

$$\dot{g}_0 + 4g_0 = \frac{1}{16\pi^2}[10e^{-g_0/g_1} - 4 + e^{-\hat{\lambda}_1} + G(\mathbf{h})]$$

# Higher derivative corrections

Expect similar corrections to Newton's constant.

More complicated since one cannot do a flat space calculation

Nevertheless might expect a class of contributions from terms $k_n R^2 \Box^n R$

Expect modification of eqn from two derivative truncation to

$$\dot{g}_1 + 2g_1 = -\frac{1}{(4\pi)^2}\frac{1}{3}[13e^{-g_0/g_1} + 5 + H(\mathbf{k})]$$

H(k) is a function of the set of $k_n$

# Scalar field theory – flat space

Consider class of operators

$$\frac{1}{2}\phi \sum_{n=2}^{\infty} z_{n-1} s_0^{n-1} (-\Box)^n \phi$$

$$\hat{K} = \hat{\mathrm{p}}^2 + \sum_{n=1}^{\infty} z_n s_0^n (\hat{\mathrm{p}}^2)^{n+1}$$

If we ignore these higher derivative terms – get the RG eqn in the local potential approximation

$$\Lambda \frac{d}{d\Lambda} V_\Lambda(\phi) = \frac{\Lambda^4}{16\pi^2} e^{-Z_0^{-1} V''(\phi)/\Lambda^2}$$

Gives infinite set of equations that can be solved recursively!

$$\dot{\hat{\lambda}}_n + (n\dot{\gamma} + (4-2n))\hat{\lambda}_n = (2n)! \frac{\Lambda^4}{16\pi^2} e^{-Z_0^{-1} V''(\phi)/\Lambda^2}|_{\phi^{2n}}$$

# Scalar field theory

No reason to believe *a priori* that the z's are small

$$\Lambda \frac{d}{d\Lambda} V_\Lambda(\phi) = F(\mathbf{z}) \frac{\Lambda^4}{16\pi^2} e^{-Z_0^{-1} V''(\phi)/\Lambda^2}$$

$$F(\mathbf{z}) \frac{\Lambda^4}{16\pi^2} \equiv e^{-\sum_{n=1}^{\infty} s_0^{n+1} z_n \frac{\partial^{n+1}}{\partial s^{n+1}}} \frac{1}{(2\pi)^4} \left(\frac{\pi}{s}\right)^2 \Big|_{s=s_0\equiv 1/\Lambda^2}$$

Without knowledge of the $z_n$ one cannot extract useful information (beyond one-loop).

# Scalar fields + gravity

$$\begin{aligned}
\dot{\hat{\lambda}}_1 + \dot{\gamma}_0\hat{\lambda}_1 + 2\hat{\lambda}_1 &= -\frac{e^{-\hat{\lambda}_1}}{16\pi^2}F(\mathbf{z})[\frac{\hat{\lambda}_2}{2} - \frac{1}{8}g_N\hat{\lambda}_1^2] + \frac{5}{(4\pi)^2}e^{2\lambda_{CC}}g_N\hat{\lambda}_1. \\
\frac{1}{4!}(\dot{\lambda}_2 + \dot{\gamma}_0\hat{\lambda}_2) &= \frac{e^{-\hat{\lambda}_1}}{16\pi^2}F(\mathbf{z})(\frac{1}{8}\hat{\lambda}_2^2 - \frac{1}{4!}\hat{\lambda}_3 + \frac{1}{3}\frac{1}{g_1}\hat{\lambda}_2\hat{\lambda}_1) + \frac{4g_N e^{2\lambda_{CC}}}{(4\pi)^2}\frac{1}{4!}\hat{\lambda}_2
\end{aligned}$$

$$\begin{aligned}
\dot{\lambda}_{CC} + 2\lambda_{CC} &= \frac{g_N}{16\pi^2}[(5 - \frac{13}{3}\lambda_{CC})e^{2\lambda_{CC}} - (2 + \frac{5}{3}\lambda_{CC}) + e^{-\hat{\lambda}_1}(\frac{1}{2} - \frac{1}{6}\lambda_{CC}(1 - 6\hat{\xi}_1)) \\
&\quad + \frac{3G(\mathbf{h}) - 2\lambda_{CC}H(\mathbf{k})}{6}] \\
\dot{g}_N - 2g_N &= -\frac{g_N^2}{16\pi^2}\frac{1}{3}\left[13e^{2\lambda_{CC}} + 5 + \frac{1}{2}e^{-\hat{\lambda}_1}(1 - 6\hat{\xi}_1) + H(\mathbf{k})\right].
\end{aligned}$$

Without the higher derivative corrections $G, H, F$ these equations appear to admit a fixed point solution. However there appears to be no reason at this point to neglect these corrections – they need to calculated and shown to be small in order to argue that the two derivative fixed point is not destabilized.

# Ghost elimination



What happens in string theory?

- String theory is ghost free (at least around asymptotically flat backgrounds). This follows from the fact that there exists a gauge for the world sheet action – i.e. the non-linear sigma model in 2D – a light cone gauge in which the negative norm states are eliminated. This is gauge equivalent to a manifestly Lorentz invariant (Polyakov) formulation. Also the UV proper time cutoff that we imposed in QFT above – is built into the formalism due to modular invariance. This makes the theory UV finite as well!
- In fact there is an analog of the Polchinski RG eqn for string theory as well - (Brustein and SdA NPB 352 (1991) 451– which may be used to define a closed string field theory action eg Sen (2017)
- On the other hand there is a low energy expansion of the string theory effective action which is valid for $E < M_{string}$ in powers of derivatives and curvatures. This low energy effective action also appears to have ghost excitations but the ghost is at the string scale – where the expansion breaks down!
- The fact that these ghosts are spurious can also be directly demonstrated because all terms quadratic in curvatures (except for the Eulaer density) can be eliminated by field redefinitions Redlich and Deser PLB 1986

# Ghost elimination

- Well known that $R^2, R^2_{\mu\nu}, R^2_{\mu\nu\lambda\sigma}$ can be replaced by the Euler density by making a field redefinition $g_{\mu\nu} \to g_{\mu\nu} + b_0 R_{\mu\nu} + b_1 R g_{\mu\nu}$. In the Wilsonian action this is a cutoff dependent redefinition – so can be done at a given scale but flow will generate the original terms again.

- Redlich-Deser argument (by repeated use of Bianchi identities and integration by parts) shows that all terms quadratic in curvatures can be written

$$\mathcal{L} \sim \sqrt{g}[a_1 R_{\mu\nu} R^{\mu\nu} + a_2 R^2 + \sum_{n=1} (a_1^{(n)} R_{\mu\nu} \Box^n R^{\mu\nu} + a_2^{(n)} R \Box^n R)]$$

- These can all be removed at the cost of changing coefficients of higher powers of curvatures by generalizations of the above field redefinitions. i.e.

$$g_{\mu\nu} \to g_{\mu\nu} + b_0 R_{\mu\nu} + b_1 R g_{\mu\nu} + \sum_{n=1} (b_1^{(n)} \Box^n R_{\mu\nu} + b_2^{(n)} \Box^n R).$$

- i.e. All curvature squared terms (apart from the Euler density) can be removed at a given scale – so graviton propagator at this scale is the same as in the Einstein theory! Same is true of scalar ghosts – one can make an independent (but cutoff dependent) field redefinition on $\phi$ to get rid of all terms of the form $\phi\nabla^{2n}\phi$.

- The coefficients $a_i^{(n)}$, $n > 1$ are cutoff dependent hence so are the $b_i^{(n)}$. So in the new field basis we cannot assume cutoff independence of the fields as we did in our original basis. i.e. even if we set these terms to zero at a given scale the flow equation will generate them. So the additional terms in the flow eqns discussed earlier must be taken into account.

- The argument works only because the spurious ghosts appear only at the cutoff scale. At any fixed scale they can be removed at the cost of changing the higher order in curvatures.

- This is different from the program of renormalizable i.e. $R^2$ gravity which is defined in terms of the (low energy) Planck scale. Doing the field redefinition there will of course lead to a non-renormlaizable theory.

- Of course the whole program of ghost elimination will therefore make sense only if AS is true!

# Conclusions

- Is AS an alternative to string theory? i.e. is there a UV complete QFT of quantum gravity
- I've derived an exact RG eqn – essentially a background field and hence gauge invariant version of the Polchinski eqn – which gives an infinite set of coupled flow equations for the infinite set of couplings in the Wilsonian effective action.
- Lowest order truncations give results similar to that in earlier work by many authors – provides evidence for the existence of a non-trivial fixed point in QG. However spurious singularities are absent and the eqn appears easier to handle.
- I've shown that while $R^n,\ n > 2$ will not give contributions to the flow of the CC, $R\Box^n R$ terms will and furthermore all terms of the form $R^2\Box^n R$ will give contributions to flow of the Einstein term.
- The putative ghosts of the theory are at the cutoff scale and hence are spurious and can be removed by field redefinitions at a given scale. Of course for this to work at an arbitrary scale, the theory has to be asymptotically safe!

# Comparison to String theory



- String theory is a unified theory of QG+``standard model" – potentially more predictive
- Quantized Einstein gravity. This is more than a post-diction. The fact that low energy string theory is Einstein gravity is of course a post-diction though gravity was not put in ab initio. The fact that it is quantized (i.e. gravitons as quanta) is an actual model independent prediction! Similarly the existence of a dilaton and an antisymmetric tensor coupling with gravitational strength.
- There must be SUSY at some scale below the Planck scale.
- Existence of a landscape of different universes – the Multiverse. Testable? Perhaps!
- Have a class of models with low energy SUSY and predicting a whole tower of SUSY partners with an LSP around 1.2 TeV and Higgs at 125 GeV correlated with LSP saturating dark matter bound.
- UV complete models of inflation with Starobinsky like potentials.
- AS on the other hand does not require any physics beyond the SM – no susy no extra dimensions. If the LHC does not see any BSM then ….?