Asymptotic safety and Conformal Standard Model

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Asymptotic Safety Seminar

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Motivation

- To predict properties of new particles in Conformal Standard Model / Higgs portal / 2HDM models
- To understand the origin of the Softly Broken Conformal Symmetry Mechanism - hierarchy problem
- To compare the data with the ones required for non-minimal Higgs inflation / other inflation mechanisms

Asymptotic safety

Renormalisation group equations

In the quantum field theory the couplings change with energy ("run") due to renormalisation group equations:

$$k\frac{\partial g_i(k)}{\partial k} = \beta_i\left(\{g_i(k)\}\right), \tag{1}$$

where β functions are calculated for a given theory. Standard possibilities:

- Landau pole: $g \to \infty$ for some μ_0. Example: QED.
- $g \to \infty$ for $\mu \to \infty$. Example: Square of Higgs mass
- asymptotic freedom, $\lim_{\mu\to\infty} g = 0, \forall_i \beta_i(g^*) = 0$. Theory has a UV fixed point. Example: QCD.

Non-standard possibilities

Non standard possibilities are:

- Asymptotic safety, $\lim_{\mu\to\infty} g \neq 0, \forall_i \beta_i(g^*) = 0$. Theory has a UV fixed point. Example: Weinberg hypothesis: Gravity.
- Oscillating g. Theory has a limit cycle. Quantum mechanics: $-g/r^2$ potential [12].

Fixed points

Fixed point for a given coupling can be:

- repulsive. Example: QED
- attractive. Example: QCD

For repulsive fixed point there is only one IR value of a parameter, which will result in asymptotic safe (free) theory! For attractive fixed point there is a range of allowed parameters.

Standard Model with gravitational corrections

For the Standard Model beta functions one can calculate the gravitational corrections:

$$\beta(g_j) = \beta_{SM}(g_j) + \beta_{grav}(g_j, k), \tag{2}$$

where due to universal nature of gravitational interactions the β_{grav} are given by:

$$\beta_{grav}(g_j, k) = \frac{a_j k^2}{M_P^2 + 2\xi k^2} g_j, \tag{3}$$

with $\xi \approx 0.024$. The a_j are unknown parameters, however they can be calculated. Then, depending on a sign of a_j, we have repelling/attracting fixed point at 0 in the perturbative region of couplings.

Standard Model with gravitational corrections: Higgs mass

With the assumption of asymptotic safety of gravity the Higgs mass (self coupling) was calculated by Mikhail Shaposhnikov and Christof Wetterlich using this approach. They obtained the correct value

$$m_H = 126\mathrm{GeV}$$

two years before the detection.

Higgs Portal Models / Conformal Standard Model

Higgs Portal Models

- Sterile complex (real) scalar ϕ coupled to Higgs doublet:

$$\mathcal{L}_{scalar} = (D_\mu H)^\dagger (D^\mu H) + (\partial_\mu \phi^\star \partial^\mu \phi) - V(H, \phi). \tag{4}$$

$$\begin{aligned} V(H,\phi) = & -m_1^2 H^\dagger H - m_2^2 \phi^\star \phi + \lambda_1 (H^\dagger H)^2 \\ & + \lambda_2 (\phi^\star \phi)^2 + 2\lambda_3 (H^\dagger H) \phi^\star \phi. \end{aligned} \tag{5}$$

- The scalar particles are combined from two mass states:

$$m_1^2 = \lambda_1 v_H^2 + \lambda_3 v_\phi^2, \qquad m_2^2 = \lambda_3 v_H^2 + \lambda_2 v_\phi^2, \tag{6}$$

and the lighter is identified with Higgs particle.

Conformal Standard Model

Conformal Standard Model also includes right handed neutrinos coupled to ϕ with the coupling y_M:

$$\mathcal{L} \ni \frac{1}{2} Y^M_{ji} \phi N^{j\alpha} N^i_\alpha, \tag{7}$$

where $Y^M_{ij} = y_M \delta_{ij}$. To resolve the baryogenesis problem via resonant leptogenesis the right handed neutrinos have to be unstable:

$$M_N > y_M v_\phi / \sqrt{2}. \tag{8}$$

Furthermore this model can resolve the SM problems like: hierarchy problem and has dark matter candidate, minoron with mass: v^2/M_P, inflation.

Softly Broken Conformal Symmetry

We consider an effective theory valid below Λ. We split the bare parameters mass and self-coupling into renormalised parameters and counter-terms:

$$m_B^2(\Lambda) = m_R^2 - f_{\mathrm{quad}}(\Lambda, \mu, \lambda_R)\Lambda^2 + m_R^2 g\left(\lambda_R, \log\left(\frac{\Lambda}{\mu}\right)\right), \quad (9)$$

where $g\left(\lambda_R, \log\left(\frac{\Lambda}{\mu}\right)\right)$ is some function. Assume that the quadratic divergences depends only on bare couplings:

$$f_{\mathrm{quad}}(\Lambda, \mu, \lambda_R) = f_{quad}(\lambda_B(\Lambda)). \quad (10)$$

So if $f_{\mathrm{quad}}(\lambda_B) = 0$ at certain scale, then the hierarchy problem is solved.

Conformal Standard Model and Softly Broken Conformal Symmetry

For CSM the $\hat{f}_i^{\mathrm{quad}} = 16\pi^2 f_i^{\mathrm{quad}}$ are:

$$\begin{aligned}
\hat{f}_1^{\mathrm{quad}}(\lambda, g, y) &= 6\lambda_1 + 2\lambda_3 + \frac{9}{4}g_2^2 + \frac{3}{4}g_1^2 - 6y_t^2, && (11)\\
\hat{f}_2^{\mathrm{quad}}(\lambda, g, y) &= 4\lambda_2 + 4\lambda_3 - 3y_M^2. && (12)
\end{aligned}$$

For the Conformal Standard Model $\Lambda \lesssim M_P$ is sufficient, while the Standard Model requires $\Lambda \gg M_P$.

Inflation in Conformal Standard Model

Lagrangian in the Jordan frame [6]:

$$\mathcal{L} = D_\mu H^\dagger D^\mu H + \partial_\mu \phi \partial^\mu \phi^* - \frac{\left(M_P^2 + \xi_1 H^\dagger H + \xi_2 |\phi|^2\right)}{2} R - V_J(H, \phi), \tag{13}$$

with $\xi_i > 0$. We get the standard result that:

$$n_s \simeq 1 - \frac{2}{N} \simeq 0.97, \tag{14}$$

and:

$$r \simeq 12/N^2 \simeq 0.0033, \tag{15}$$

however it requires that $\xi_1, \xi_2 \sim \mathcal{O}(10^4)$.

Values of parameters

We run following couplings: g_1, g_2, g_3 (Gauge couplings),y_t (top Yukawa coupling), $\lambda_1, \lambda_2, \lambda_3, y_M$. The CSM beta functions are $\hat{\beta} = 16\pi^2\beta$:

$$
\begin{aligned}
\hat{\beta}_{g_1} &= \tfrac{41}{6} g_1^3, \\
\hat{\beta}_{g_2} &= -\tfrac{19}{6} g_2^3, \\
\hat{\beta}_{g_3} &= -7 g_3^3, \\
\hat{\beta}_{y_t} &= y_t \left(\tfrac{9}{2} y_t^2 - 8 g_3^2 - \tfrac{9}{4} g_2^2 - \tfrac{17}{12} g_1^2 \right), \\
\hat{\beta}_{\lambda_1} &= 24\lambda_1^2 + 4\lambda_3^2 - 3\lambda_1 \left(3g_2^2 + g_1^2 - 4y_t^2 \right) \\
&\quad + \tfrac{9}{8} g_2^4 + \tfrac{3}{4} g_2^2 g_1^2 + \tfrac{3}{8} g_1^4 - 6 y_t^4, \\
\hat{\beta}_{\lambda_2} &= \left(20\lambda_2^2 + 8\lambda_3^2 + 6\lambda_2 y_M^2 - 3 y_M^4 \right), \\
\hat{\beta}_{\lambda_3} &= \tfrac{1}{2}\lambda_3 \left[24\lambda_1 + 16\lambda_2 + 16\lambda_3 \right. \\
&\quad \left. - \left(9g_2^2 + 3g_1^2 \right) + 6 y_M^2 + 12 y_t^2 \right], \\
\hat{\beta}_{y_M} &= \tfrac{5}{2} y_M^3.
\end{aligned}
\tag{16}
$$

Conditions for low energy coupling values and a_i values

We impose two conditions:

- absence of Landau poles
- $\lambda_1(\mu) > 0, \quad \lambda_2(\mu) > 0, \quad \lambda_3(\mu) > -\sqrt{\lambda_2(\mu)\lambda_1(\mu)}.$

Furthermore we take:

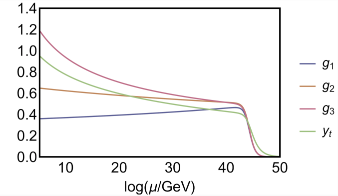
$$a_{g_i} = a_{y_M} = -1, a_{y_t} = -0.5, a_{\lambda_1} = +3 \tag{17}$$

and

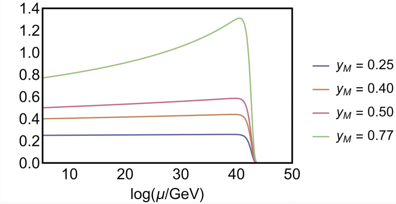
$$a_{\lambda_2} = \pm 3, a_{\lambda_3} = \pm 3, \tag{18}$$

hence we have four possibilities.

Gauge and Yukawa couplings running



(a) g_1, g_2, g_3, y_t



(b) y_M running

The low-energy values at $\mu_0 = 173.34$ are taken as:
$g_1(\mu_0) = 0.35940$, $g_2(\mu_0) = 0.64754$, $g_3(\mu_0) = 1.1888$ and $y_t(\mu_0) = 0.95113$.

Coefficient: $a_{\lambda_3} = +3$

For $a_{\lambda_3} = +3$, we get that: $\lambda_3 = 0$. So SM and ϕ decouple.

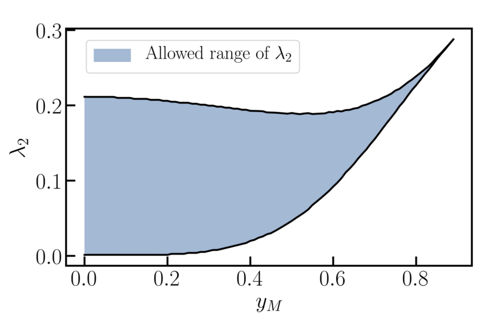


Figure 1: λ_2 dependence on y_M

The case $a_{\lambda_2} = +3$ follows the lower bound of the plot.

Coefficients $a_{\lambda_2} = a_{\lambda_3} = -3$, set of allowed couplings $\lambda_2, \lambda_3, y_M$

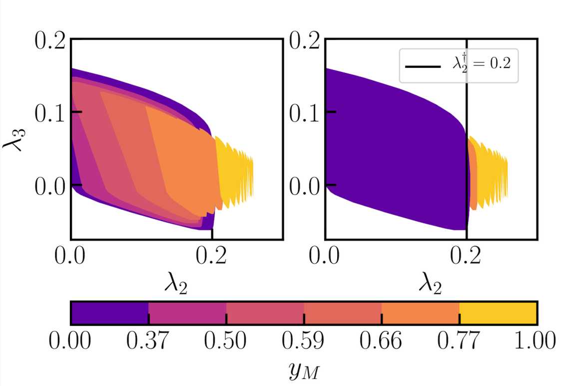


Figure 2: Maximal (left) and minimal (right) $y_M(\lambda_3, \lambda_2)$, $a_{\lambda_2} = -3, a_{\lambda_3} = -3$

Coefficient: $a_{\lambda_2} = a_{\lambda_3} = -3$, allowed λ_1

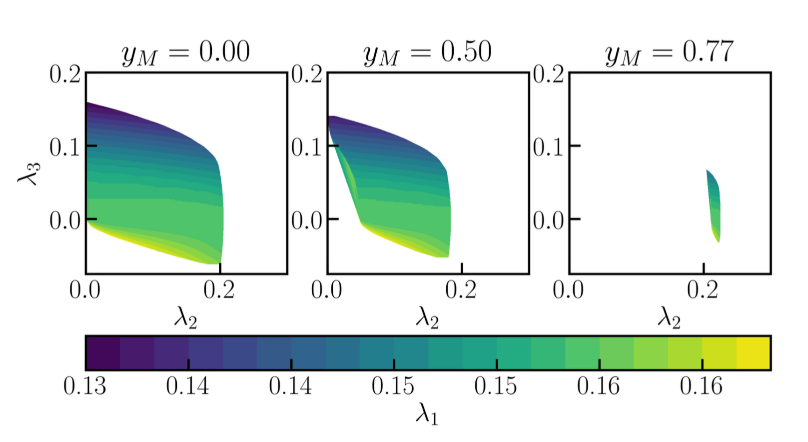
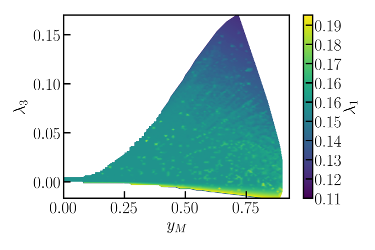
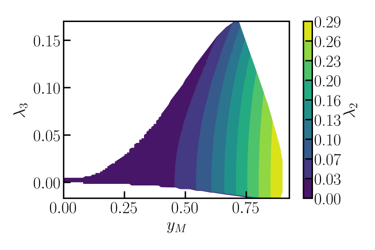


Figure 3: Plot of $\lambda_1(\lambda_2, \lambda_3, y_M)$

Coefficients $a_{\lambda_3} = -3, a_{\lambda_2} = +3$



(a) λ_1 dependence on λ_3, y_M



(b) λ_2 dependence on λ_3, y_M

Conditions on m_2

One can parametrize the discrepancies from SM as:

$$\tan\beta = \frac{\lambda_0 - \lambda_1}{\lambda_3}\frac{v_H}{v_\phi}. \tag{19}$$

- $|\tan\beta| < 0.35$.
- Global stability condition of the potential at μ_0: $\lambda_3(\mu_0) < \sqrt{\lambda_2(\mu_0)\lambda_1(\mu_0)}$,
- un-stability condition for the second particle: $m_2 > 2m_1$.

Second scalar particle mass

If we take the tree level relations:

$$m_1^2 = \lambda_1 v_H^2 + \lambda_3 v_\phi^2, \tag{20}$$
$$m_2^2 = \lambda_2 v_\phi^2 + \lambda_3 v_H^2, \tag{21}$$

and: $m_H = 136\text{GeV}$, $v_H = 226$ GeV. Then are able to constrain the second scalar mass as:

$$270 \text{ GeV} < m_2 < 328 \text{ GeV} \quad \text{and} \quad y_M > 0.71. \tag{22}$$

Mass restrictions for $a_{\lambda_2} = +3, a_{\lambda_3} = -3$

We found that there are only two sets of parameters satisfying the imposed conditions

$$y_M = 0.84, m_2 = 275, v_\phi = 538, M_N = 319, \tag{23}$$

and

$$y_M = 0.85, m_2 = 296, v_\phi = 574, M_N = 345. \tag{24}$$

Further restrictions

We also calculate the neutrinos masses:

$$M_N = 342^{+41}_{-41}\ \mathrm{GeV} \tag{25}$$

and they satisfy the leptogenesis condition($M_N > y_M v_\phi/\sqrt{2}$). We calculated (assuming the $v_\phi = v$) mass of dark matter candidate as: 10^{-4} eV. For $y_M = 0.0$ we found out that:

$$m_2 = 160^{+103}_{-100}\ \mathrm{GeV}, \tag{26}$$

so classically it is stable.

Further restrictions(2)

1. We checked that the analysed parameters satisfy the Softly Broken Conformal Symmetry requirements at M_P with couplings going to zero (but nowhere else).
2. We analyzed the running of the β functions for m_2 and m_1, where we took $a_{m_i} = -1$. It gives no new bounds on m_2 and lambda-couplings.

Comparison with experimental data

- The excess of events with four charged leptons at $E \sim 325$ GeV seen by the CDF [9] and CMS [8] Collaborations can be identified with a detection of a new 'sterile' scalar particle proposed by the Conformal Standard Model [7].
- The hypothetical heavy boson mass is measured to be around 272 GeV (in the $270 - 320$ GeV range), according to [10, 11].

Summary

Summary and further work

Take home message:

- Standard Model supplemented by the gravitational corrections can be a fundamental theory, yet not a complete one
- Applying the gravitational corrections can give the quantitive predictions for new particles, like for Conformal Standard Model, which can be tested in near future

Further work:

- The remaining a_i's have to be calculated
- The (higher)-loop corrections have to be taken into account

Thank you for your attention

Talk based on article: arxiv.org/abs/1810.08461

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Bibliography

- S. Weinberg General Relativity: An Einstein centenary survey, Hawking, S.W., Israel, W (eds.). (1979) Cambridge University Press, pages 790-831.
- M. Shaposhnikov and C. Wetterich Phys. Lett. B **683**, 196 (2010) doi:10.1016/j.physletb.2009.12.022 [arXiv:0912.0208 [hep-th]].
- A. Eichhorn, arXiv:1810.07615 [hep-th].

- P. H. Chankowski, A. Lewandowski, K. A. Meissner, and H. Nicolai, Mod. Phys. Lett. **A30** (2015) 1550006.
- A. Lewandowski, K. A. Meissner and H. Nicolai, Phys. Rev. D **97**, no. 3, 035024 (2018) doi:10.1103/PhysRevD.97.035024 [arXiv:1710.06149 [hep-ph]].
- J.H. Kwapisz and K.A. Meissner, year 2017, arXiv 1712.03778,Acta Physica Polonica B, Vol. 49, No. 2

- K. A. Meissner and H. Nicolai, Phys. Lett. B **718**, 943 (2013) doi:10.1016/j.physletb.2012.11.012.
- S. Chatrchyan *et al.* [CMS Collaboration], Phys. Rev. Lett. **108**, 111804 (2012) doi:10.1103/PhysRevLett.108.111804.
- T. Aaltonen *et al.* [CDF Collaboration], Phys. Rev. D **85**, 012008 (2012) doi:10.1103/PhysRevD.85.012008.
- S. von Buddenbrock, N. Chakrabarty, A. S. Cornell, D. Kar, M. Kumar, T. Mandal, B. Mellado, B. Mukhopadhyaya, and R. G. Reed, (2015), arXiv:1506.00612 [hep-ph].

- S. von Buddenbrock, N. Chakrabarty, A. S. Cornell, D. Kar, M. Kumar, T. Mandal, B. Mellado, B. Mukhopadhyaya, R. G. Reed, and X. Ruan, Eur. Phys. J. C76, 580 (2016).
- S. M. Dawid, R. Gonsior, J. Kwapisz, K. Serafin, M. Tobolski and S. D. G?azek, Phys. Lett. B **777**, 260 (2018) doi:10.1016/j.physletb.2017.12.028 [arXiv:1704.08206 [quant-ph]].