RG flow with a covariant foliation

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Foliating spacetimes I

- ingredients for foliation structure:
  - normalised timelike vector n_μ $g^{\mu\nu} n_\mu n_\nu = 1$
  - spatial metric orthogonal to vector $\sigma_{\mu\nu}$ $g^{\mu\nu} n_\mu \sigma_{\nu\rho} = 0$
  - dictionary 4-metric to foliation variables $g_{\mu\nu} = \sigma_{\mu\nu} + n_\mu n_\nu$

- note: **quadratic** relationship as opposed to non-polynomial relationship if lapse and shift are used:

$$g_{\mu\nu} = \begin{pmatrix} N^2 + \Sigma^{ij} N_i N_j & N_j \\ N_i & \Sigma_{ij} \end{pmatrix}$$

Foliating spacetimes II - curvature

- curvature tensors can be decomposed into temporal and spatial parts:
  - intrinsic Riemann tensor

$$^{(3)}R_{\mu\nu\rho\sigma}[\sigma]$$

  - extrinsic curvature

$$K_{\mu\nu} = \frac{1}{2}\left(n^\alpha D_\alpha \sigma_{\mu\nu} + D_\mu n_\nu + D_\nu n_\mu\right)$$

  - acceleration vector

$$\mathcal{A}_\mu = n^\alpha D_\alpha n_\mu$$

- Gauss-Codazzi relation:

$$R = {}^{(3)}R - K^{\mu\nu}K_{\mu\nu} + K^2 - 2D_\mu(n^\mu K)$$

Constructing an RG flow with foliation I

$$\dot{\Gamma}_k = \frac{1}{2}\mathrm{STr}\left[\left(\Gamma_k^{(2)} + \mathfrak{R}_k\right)^{-1} \dot{\mathfrak{R}}_k\right]$$

- aim: construct FRG flow for metric with foliation structure
- background field formalism, metric language:

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$$

- one-loop structure of FRG relies on **quadratic** regulator, want to preserve that

$$n_\mu = \bar{n}_\mu + \hat{n}_\mu$$
$$\sigma_{\mu\nu} = \bar{\sigma}_{\mu\nu} + \hat{\sigma}_{\mu\nu} - \hat{\mathbf{n}}_\mu \hat{\mathbf{n}}_\nu$$

- **quadratic** parameterisation of spatial metric yields **linear** relation between metric and foliated language:

$$h_{\mu\nu} = \hat{\sigma}_{\mu\nu} + \bar{n}_\mu \hat{n}_\nu + \hat{n}_\mu \bar{n}_\nu$$

Intermezzo: foliation gauge fixing

- going from metric language to foliation language, we added degrees of freedom: 10 (symmetric matrix) vs. 14 (matrix+vector)
- however: full quantum fields have to satisfy their constraints:

$$g^{\mu\nu} n_\mu n_\nu = 1$$

$$g^{\mu\nu} n_\mu \sigma_{\nu\rho} = 0$$

- easiest solution:

$$\mathfrak{F}_\nu = \bar{n}^\mu \hat{\sigma}_{\mu\nu} - \bar{n}^\mu \hat{n}_\mu \hat{n}_\nu = 0$$

- implement this with Lagrange multiplier, similar to gauge fixing

Constructing an RG flow with foliation II

- recipe to construct the flow:
  - calculate second variation of effective average action in metric language (including standard gauge fixing and regulator)
  - use *linear* map to express metric fluctuation in terms of foliation fluctuations
  - add foliation gauge fixing
  - use *standard* heat kernel techniques to calculate the trace
- result: **flow on background with foliation structure which preserves background diffeomorphism invariance**

Hořava-Lifshitz gravity

Hořava, 0901.3775

- idea: break Lorentz symmetry to allow perturbative quantisation, similar to many condensed matter systems

- anisotropic scaling with dynamical critical exponent z:

$$x \to \alpha x\,, \qquad t \to \alpha^z t$$

- propagator structure:

$$\frac{1}{\omega^2 - \mathbf{k}^2 - g\,(\mathbf{k}^2)^z}$$

- consequence: improved UV behaviour, recently proven to be perturbatively renormalisable (for some concrete model)

Barvinsky, Blas, Herrero-Valea, Sibiryakov, Steinwachs, 1512.02250

Hořava-Lifshitz gravity - action

Hořava, 0901.3775

- allow for second order temporal and higher order spatial derivatives
- "kinetic" part:

$$S_k = \frac{1}{16\pi G_N}\int \mathrm{d}^{(3+1)}x\sqrt{|\det g|}\,\left(K^{\mu\nu}K_{\mu\nu} - \lambda K^2\right)$$

- "potential" part:

$$S_V = \frac{1}{16\pi G_N}\int \mathrm{d}^{(3+1)}x\sqrt{|\det g|}\,V[\sigma]$$

- potential includes terms like

$${}^{(3)}R^3\,,\,{}^{(3)}R\,{}^{(3)}\Delta\,{}^{(3)}R\,,\,{}^{(3)}R_{\mu\nu}\,{}^{(3)}\Delta\,{}^{(3)}R^{\mu\nu}\,,\ldots$$

Hořava-Lifshitz gravity - mechanism

Hořava, 0901.3775

- UV: cubic terms in potential dominate, anisotropic scaling, perturbative renormalisation
- IR: cubic terms negligible, **reduces to standard GR?**

$\downarrow$

$\boxed{\textbf{study RG flow!}}$

Flow of breaking terms

- with foliation structure at hand, can calculate flow of background diffeomorphism breaking terms

- consider flow of Einstein-Hilbert plus all other second order derivative terms:

$$\Gamma_{\mathrm{break}} = \frac{1}{16\pi G_N} \int \mathrm{d}^4 x \sqrt{|g|} \left[k_0 K^2 + k_2 K_{\mu\nu} K^{\mu\nu} + a_1 \mathcal{A}_\mu \mathcal{A}^\mu \right]$$

- use harmonic gauge and single metric approximation to simplify calculations

- restrict to linear order in breaking couplings

- calculation of flows: xAct

Flow equations

- beta functions of Newton's&cosmological constant: Reuter flow + corrections from breaking terms

- flow of breaking couplings (to linear order in all couplings):

$$\dot{k}_0 = -\frac{g}{24\pi}(22a_1 + 19k_0 - 3k_2)$$

$$\dot{k}_2 = \frac{g}{24\pi}(22a_1 - 3k_0 - 49k_2)$$

$$\dot{a}_1 = -\frac{g}{6\pi}(25a_1 + 6k_0 + 12k_2)$$

- are the breaking terms relevant?

Relevance of Lorentz symmetry I

- consider inner product of beta functions with coordinate vector:

$$(k_0, k_2, a_1) \cdot (\dot{k}_0, \dot{k}_2, \dot{a}_1) = -\frac{g}{24\pi}\left[\frac{23}{2}(2a_1 + k_0)^2 + \frac{13}{2}(2a_1 + k_2)^2 + 28a_1^2 + \frac{15}{2}k_0^2 + \frac{85}{2}k_2^2\right] \leq 0$$

- points towards center, i.e. **Lorentz breaking increases towards the IR**
- Is this the death of Lorentz symmetry breaking quantum gravity theories? Competing effects:
  - relevance enhances couplings
  - prefactor g goes to zero quickly, flow dies out in IR

Relevance of Lorentz symmetry II

- diagonalise flow of breaking couplings:

$$\dot{d}_1 \approx -0.21 g\, d_1$$

$$\dot{d}_{2,3} \approx (-1.01 \pm 0.11\mathbf{i}) g\, d_{2,3}$$

- can be solved analytically:

$$d_{1,k} \approx d_{1,\Lambda} \exp\left[0.21 \int_k^\Lambda \frac{\mathrm{d}k}{k} g\right]$$

$$d_{2,3,k} \approx d_{2,3,\Lambda} \exp\left[(1.01 \mp 0.11\mathbf{i}) \int_k^\Lambda \frac{\mathrm{d}k}{k} g\right]$$

Relevance of Lorentz symmetry III

- assumption: IR is governed by GR, Newton's constant runs classically up to Planck scale

$$g \approx \frac{k^2}{M_{\rm Pl}^2}$$

- corresponding magnification of breaking couplings:

$$d_{1,0} \approx 1.11 d_{1,M_{\rm Pl}}\,, \quad d_{2,3,0} \approx 1.66 d_{2,3,M_{\rm Pl}}$$

$$\begin{pmatrix} k_{0,0} \\ k_{2,0} \\ a_{1,0} \end{pmatrix} \approx \begin{pmatrix} 1.15 & 0.01 & 0.22 \\ 0.01 & 1.35 & -0.24 \\ 0.24 & 0.52 & 1.92 \end{pmatrix} \begin{pmatrix} k_{0,M_{\rm Pl}} \\ k_{2,M_{\rm Pl}} \\ a_{1,M_{\rm Pl}} \end{pmatrix}$$

Conclusion&Outlook

- first background diffeomorphism invariant flow equation with access to foliation structure
  - FRG Asymptotic Safety = CDT? (“Euclidean QG *is* good enough, if done right?”)
- Lorentz symmetry is technically relevant, but practically marginal: not enough to rule out Hořava-Lifshitz quantum gravity
- obvious points of extensions:
  - non-perturbative dependence on breaking couplings
  - higher order terms
  - matter couplings
  - bimetric
  - arbitrary gauge fixing
  - …